Problem set 5

Ge 108

October 30, 2008

1 Going to China

If we jump through a hole which goes through the center of the earth, how fast are we going when we reach the center? Assume, again, that the earth has a uniform density. Use the same differential equation method that we used in class to calculate the velocity of a rock falling towards to sun (pg. 83-85), and be careful to use the right sign for your velocities and accelerations.

If we had instead written this as a single second order differential equation, we would have had something with a form like

$$\frac{d^2r}{dt^2} = a(r).$$

Write the correct differential equation out explicitly. Now try a solution of the form $r = r_0 \cos \omega t$. What is ω in terms of known parameters. How long does it take to get from here to China?

2 Going to Paris

Forget the Chunnel, let's dig a hole that goes straight from here to Paris and grease it down so it's frictionless. Use the notation of Figure 1. What is the linear distance to Paris? What is the gravitational field as a function of s? What is the Paris-ward component of acceleration as a function of s? What is the second-order differential equation describing the motion? How long does it take to get to Paris?

3 Comet orbits

Comets travel on large elliptical orbits like that in Figure 2. At aphelion (the point furthest from the sun) the comet travels at a velocity of v_a. What is its velocity as a function of distance from the sun? When is it moving fastest? Why?

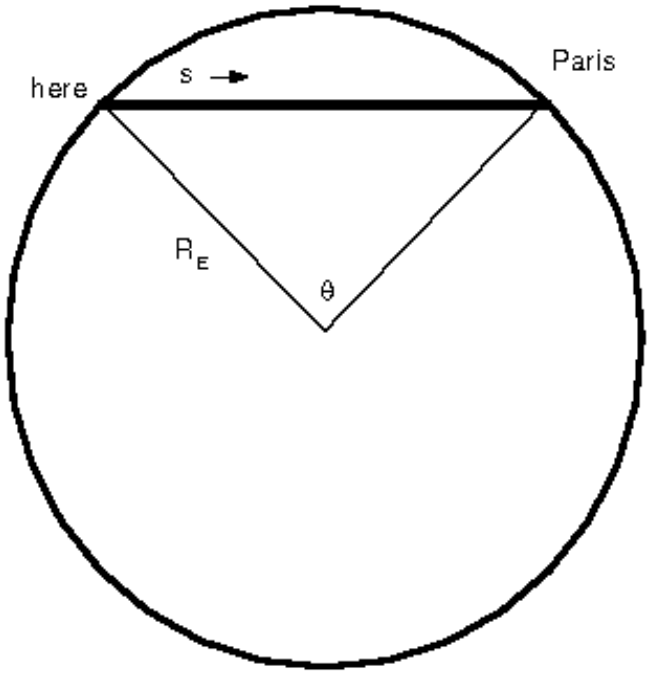

Figure 1: Tunneling for baguettes.

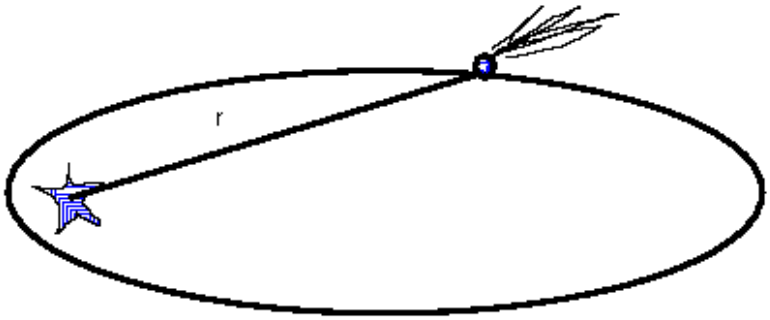

Figure 2: A comet orbit.