Ge/Ay 132

Problem set # 1

You are allowed to use the class notes to solve these problems, but no other books. The questions are worth 25, 20, 25 and 30 points, respectively. Due Friday, October 19th. Please discuss any need for an extension with one of us (Geoff or Cam) in advance. Collaboration on the numerical/fitting parts of the problem set is permitted, but do write up your own explanations. Don't hesitate to see us if you find ambiguities. Corrections or clarifications will be posted on the Ge/Ay 132 web site.

1. Give the spectroscopic terms arising from the following configurations, using LS coupling. Include parity and J values. Give arguments for deriving the results.

a) 3p 4s
b) 4d 5d
c) $3d^2$ (Hint: only five terms of those found under b) remain)
d) $3p^4$ 4s
e) $2p^2$ 4p
f) $3p^3$ (Hint: only three terms of those found under e) remain. Give arguments which three are likely to remain, rather than writing out completely all ot the possible determinants!)

2. Consider the ionized oxygen atom with the ground configuration $1s^22s^22p^3$, and the excited configuration $1s^22s2p^4$.

a) What is the order of the LS terms arising from the ground and excited configurations (see also problem 1)? What is the expected order of the fine–structure levels within each of the terms? Draw a schematic energy level diagram.
b) List all transitions that are electric dipole or magnetic dipole allowed between these states considering the strict LS coupling rules outlined in the class notes. We'll look at this ion later in the course, specifically in H II regions. Can O^+ (O II to astronomers) be easily detected in cold gas?

3. The state of hydrogen, either as H I (H), H II (H^+), or H_2 dominates the physical characteristics of the interstellar medium and therefore determines the boundaries between objects. For the process H I + hν → H II, the optical depth can be expressed as:

$$d\tau \quad = \quad n_{H\ I} \times \frac{\pi e^2}{m_e c} \times \phi(\nu) \times g_{nf} \times dL$$

where the lineshape function $\phi(\nu)$ is given by

$$\phi(\nu) \quad = \quad 2\nu_L^2/\nu^3 \qquad \nu > \nu_L \qquad , \qquad 0 \text{ else.}$$

ν_L is the frequency of the Lyman limit (at 13.6 eV), L is the physical pathlength, and the Gaunt factor g_{nf} may be written as

$$g_{nf} \quad = \quad 13.856\pi \, \frac{\nu_L}{\nu} \times \frac{e^{-4z cot^{-1} z}}{1 \; - \; e^{-2\pi z}}$$

where $z^2 \quad = \quad \nu_L/(\nu \; - \; \nu_L)$.

a. Sketch the qualitative appearance of τ as a function of frequency and find the path length for which $\tau_{abs} = 1$ for photons at 912 Å and 180 Å assuming a constant density of neutral H atoms of 1 cm^{-3} (the Gaunt factor g_{nf} peaks at 180 Å). Is the H I → H II transition sharp or broad compared to interstellar cloud sizes (typically a few parsecs) at these wavelengths?

b. The ionization fraction of hydrogen gas exposed to UV light should be density dependent. To show this, consider the following: Assume that a dilute, uniform gas of density n=n(H) + $n(\mathrm{H}^+)$ at temperature $T = 8 \times 10^3$ K with $n(\mathrm{H}^+)$=$n(e)$ is exposed to isotropic radiation of intensity $J_\nu = 4\pi I_\nu = 10^{-20}(\nu/\nu_o)^{-1}$ ergs s^{-1} cm^{-2} Hz^{-1}. You may assume that all neutral atoms are in the ground state. Calculate the rate of photoionization of H in s^{-1}.

c. Assume that photoionization of H is balanced by radiative recombination of H^+ and e with a thermal rate coefficient $\alpha_H = 1.9 \times 10^{-10} T^{-0.7}$ cm^3 s^{-1}. Calculate the fractional ionization x=$n(\mathrm{H}^+)/n$ for n=0.005, 1.0 and 200.0 cm^{-3}. What are the corresponding ionization temperatures in thermodynamic equilibrium that would give the same degree of ionization you calculate for the various densities?

4. The accompanying table is a calculated curve of growth for the He I line at 3888.65 Å. The assumed temperature for the doppler line shape calculation is 12,500 K, and the absorption oscillator strength is 0.06446. It arises from an excited long–lived state and it has been observed in absorption in the spectra of stars in the Orion nebula and in the Carina nebula.

a. What levels of He I are most likely involved? Why?

b. Write down the formula for the equivalent width of this line in the optically thin limit, and test its accuracy for (lower level) column densities of 3×10^{10}, 10^{12} and 3×10^{13} cm^{-2}.

c. Draw a graph of the curve of growth in the form log W_λ vs. log N. What is the doppler parameter for the temperature assumed? Take a point on the 'flat' part of the curve-of-growth and verify the optical depth+Voigt function corrections to the predicted equivalent width using this doppler parameter. For extra credit, generate the curve of growth by running this calculation over the relevant column density range.

d. The measured equivalent widths in the Carina Nebula are W=0.8, 0.198, 0.098 and 0.058 Å. What are the corresponding column densities of He I in the lower level of the transition? If W is uncertain by 15%, what are the uncertainties in N (approximately)?

Calculated curve of growth for He I 3888.65 Å for T_D=12500 K
Log N is in cm^{-2} and W_λ is in Å.

log N	W_λ	log N	W_λ
11.0	0.00086	16.5	0.7293
11.5	0.00272	17.0	0.7888
12.0	0.00852	17.5	0.8569
12.5	0.02621	18.0	0.9563
13.0	0.07611	18.5	1.145
13.5	0.1886	19.0	1.547
14.0	0.3387	19.5	2.375
14.5	0.4495	20.0	3.967
15.0	0.5335	20.5	6.891
15.5	0.6052	21.0	12.154
16.0	0.6694	21.5	21.553