1 Fundamental Equation of Radiative Transfer

\[ \mu \frac{dI(\tau, \mu, \phi)}{d \tau} = I(\tau, \mu, \phi) - J(\tau, \mu, \phi) \]  

(1)

\[ J(\tau, \mu, \phi) = \frac{\omega}{4\pi} \int_{-\pi}^{\pi} \int_{-1}^{1} P(\mu, \mu', \phi - \phi') I(\tau, \mu', \phi') d\phi' d\mu' \]  

(2)

2 Phase Matrix \( P \) and Scattering Matrix \( F \)

- Phase matrix usually known with reference to scattering plane: scattering matrix \( F \)
- Many different planes of scattering in multiple scattering problems
- Necessary to choose one common plane of reference for Stokes parameters
- Normally use local meridian plane, specified by local normal and direction of emergence
- \( P \) related to \( F \) through a coordinate transformation (from the scattering plane to the local meridian plane)
- \( F \), in general, a function of all the angles
- Special cases:
  - randomly oriented particles, each with a plane of symmetry
  - randomly oriented asymmetric particles and their mirror images in equal numbers
  - Rayleigh and Mie scattering
- For special cases, \( F \) is a function only of scattering angle \( \Theta \) and has the form
\[
\begin{bmatrix}
F_{11} & F_{12} & 0 & 0 \\
F_{12} & F_{22} & 0 & 0 \\
0 & 0 & F_{33} & F_{34} \\
0 & 0 & -F_{34} & F_{44}
\end{bmatrix}
\]

- **Isotropic Rayleigh scattering:**
  \[
  F_{11} = F_{22} = \frac{3}{4} (1 + \cos^2 \Theta)
  \]
  \[
  F_{33} = F_{44} = \frac{3}{2} \cos \Theta
  \]
  \[
  F_{12} = F_{21} = -\frac{3}{4} \sin^2 \Theta
  \]

- **Mie scattering:**
  \[
  F_{11} = F_{22}, F_{33} = F_{44}
  \]

3. **Scattering Behavior in Different Regimes [Fig. 5, Hansen and Travis]**

- Rayleigh scattering
  - strong positive linear polarization \((- F_{21} / F_{11})\), with a maximum at 90° scattering angle

- Geometric optics
  - small scattering angles: phase function large and linear polarization small because diffracted light is predominant
  - other than diffraction, most of the light scattered into forward hemisphere due to rays passing through particle with two refractions => negative linear polarization (Fresnel’s equations)
  - positive linear polarization at ~ 80-120° scattering angle: reflection from outside of particles
  - positive polarization maximum at ~ 150° scattering angle (primary rainbow): internal reflection – scattering angle has a maximum as a function of the incident angle on the particle; weaker feature at ~ 120°
scattering angle (secondary rainbow): two internal reflections – scattering
gle has a minimum as a function of the incident angle on the particle
  • maximum in backscattering direction (glory): incident edge rays

  • Transition region in between
    • linear polarization complicated function of size parameter
    • as size parameter decreases, degree to which paths of separate light rays
      can be localized decreases
    • secondary rainbow, with a more detailed ray path, lost from polarization
      before primary rainbow
    • primary rainbow becomes blurred with decreasing size parameter

4 Effect of Nonsphericity [Figure 1, Mishchenko et al.]

Define \( \rho \equiv F_{11}(spheroid) / F_{11}(sphere) \). There are five distinct regions:

  • Nearly direct forward scattering
    • \( \rho \sim 1 \)
    • region least sensitive to particle sphericity because diffraction dominates
    • and is determined by the average area of the particle geometrical cross
      section
  • \( \sim 10 \) to 30° scattering angle
    • \( \rho > 1 \)
    • ratio increases with increasing aspect ratio \( \varepsilon \)
  • \( \sim 30 \) to 90° scattering angle
    • \( \rho < 1 \)
    • region becomes narrower with increasing \( \varepsilon \)
    • ratio decreases with increasing \( \varepsilon \)
  • \( \sim 90 \) to 150° scattering angle
    • \( \rho \gg 1 \)
    • strongly enhanced side scattering as opposed to deep and wide side
      scattering minimum in spherical particles
region becomes wider with increasing $\varepsilon$

- $\sim 150$ to $180^\circ$ scattering angle
  - $\rho \ll 1$
  - strong rainbow and glory features suppressed by nonsphericity

In general, the polarization generated by spheroids is more neutral than that for spheres and shows less variability with size parameter and scattering angle [Figure 10.6, Mishchenko et al.].

- $>\sim 60^\circ$ scattering angle
  - degree of linear polarization $p$ is strongly $\varepsilon$-dependent
  - deviation from spherical behavior becomes more pronounced with increasing $\varepsilon$
  - Lorenz-Mie theory inappropriate for nonspherical particles in this region

- $<\sim 60^\circ$ scattering angle
  - $p$ weakly dependent on particle shape

- $\sim 120^\circ$ scattering angle
  - most prominent polarization feature for spheroids: bridge of positive polarization, which separates two regions of negative or neutral polarization at small and large scattering angles
  - bridge absent in spherical particles

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5 Global Climatology of Aerosol Types [Figure 2]

- Seven basic aerosol types: sulfate(land/water), seasalt, carbonaceous, black carbon, mineral dust (accumulated/coarse)
- Each mixing group is a combination of 4 aerosol components
- Lognormal distribution
Phase Function for Kahn Mixing Group Types

Linear Polarization for Kahn Mixing Group Types
Radiative Effect I: Weighting Functions in 0.76 µm O₂ A Band

![Graph showing weighting functions in the 0.76 µm O₂ A Band.](image)

Radiative Effect II: Weighting Functions in 1.61 µm CO₂ Band

![Graph showing weighting functions in the 1.61 µm CO₂ Band.](image)
Radiative Effect III: Weighting Functions in 2.06 $\mu$m CO$_2$ Band

Reducing Mixing Group Types Using SSA and Extinction Behavior
distribution is adequately characterized by the effective size parameter, $x_{\text{eff}}$, which is a certain average size parameter for the distribution. The width of the size distribution does have a noticeable influence on Mie results, for which phase effects are exactly computed. However, the width of the distribution (2.11), is sufficient to wash out interference effects except for one mentioned below.

The concentration of light near $\alpha = 0^\circ$ is the diffraction ($f=0$), which is unpolarized for the assumed case of unpolarized incident light. The external reflection ($f=1$) does not leave any apparent feature in the intensity, but it is strongly polarized and causes the broad positive polarization for $\alpha \sim 80^\circ$ to $120^\circ$. The energy contained in the twice
Figure 1. Ratios of the nonspherical to spherical phase functions denoted in lower panels of Plate 3 as "all spheroids" and "spheres."
Figure 10.6. Elements of the normalized Stokes scattering matrix for log normal, gamma, and modified power law size distributions of randomly oriented oblate spheroids with an axis ratio \( a/b = 1.6 \). All three distributions of the surface-equivalent-sphere radii have the same effective radius \( r_{\text{eff}} = 1.5 \mu \text{m} \) and effective variance \( \sigma_{\text{eff}} = 0.1 \). The power exponent of the modified

(1994c) showed that, in practice, many plausible size distributions of spherical and nonspherical particles can be adequately represented by just two parameters, viz., the effective radius and the effective variance, defined by Eqs. (5.248) and (5.249), respectively. This means that different size distributions that have the same values of \( r_{\text{eff}} \) and \( \sigma_{\text{eff}} \) can be expected to have similar dimensionless scattering and absorption characteristics, as illustrated by Table 10.1 and Fig. 10.6. In this regard, the power
Figure 2. Global Climatology of Aerosols (Kahn et al., 2001)