Bidirectional Reflectance:
An Overview with Remote Sensing Applications & Measurement Recommendations

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Abstract

Overhead multi- and hyperspectral imaging has enabled a variety of quantitative remote sensing techniques, such as the determination of the abundance and health of vegetation through the Normalized Difference Vegetation Index (NDVI). In addition, anomaly and target detection have been made possible by advanced matched filter algorithms. However, the derived surface reflectance from which all these techniques rely upon is generally directional, and as such depends upon the incident solar and receiving detector angles. This fact is rarely taken into account. The bidirectional reflectance distribution function (BRDF) may be used to compensate for the anisotropic reflectance of a material. However, quantification and employment of BRDF in remote sensing is challenging.

The theoretical framework necessary to understand BRDF in the visible to near-infrared (VNIR) spectral region is discussed, along with BRDF measurement techniques. Some popular BRDF models, which enable interpolation of measured data are reviewed. The most general form of the BRDF, that which includes polarization, is examined in detail. A viable polarimetric BRDF measurement technique is presented, along with models which may be used to extrapolate the measured data. Potential applications of spectro-polarimetric BRDF to remote sensing is reviewed. Finally, a recommendation is made for a simple outdoor BRDF measurement system which enables the application of spatial resolution-dependent BRDF variation toward hyperspectral algorithms and synthetic image generation.

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1 Introduction

Hyperspectral remote sensing has enabled imaging spectroscopy whereby specific materials may be identified or detected based upon knowledge of their spectral reflectance. Prior to employing hyperspectral algorithms, the raw image data must first be corrected for atmospheric effects in an attempt to determine the material spectral reflectance, $\rho(\lambda)$. However, reflectance is generally directional, and as such depends upon the incident solar and receiving detector angles. Some materials may be approximated as completely diffuse (Lambertian), in which case the object radiance does not vary with view angle [1]. This is often a poor approximation resulting in an inaccurate derivation of the surface reflectance, which in turn detrimentally impacts quantitative remote sensing and the quantification of such parameters as Normalized Difference Vegetation Index (NDVI), leaf area index (LAI) and the performance of target-detection algorithms.

The bidirectional reflectance distribution function (BRDF) quantifies the geometric radiance distribution which results from light incident in any direction. The term bidirectional is used as it is a function of the incident and reflected light directions. It is also a distribution function in the classical sense, as integration over the hemisphere results in the reflectance, $\rho$, which ranges from 0 to 1.

The theoretical basis necessary for understanding optical scatter and hence BRDF is first discussed, which includes the relevant framework necessary for polarized BRDF. BRDF measurement techniques are reviewed which include specific examples of laboratory and field instruments. Next, BRDF models are summarized and their strengths and weaknesses discussed. Polarized BRDF models, most of which have their origin in scalar BRDF models, are discussed and reviewed. The generalized scattering (Mueller) matrix is developed which is the most general form of the BRDF. The use of BRDF data in remote sensing is discussed through the development of the governing radiometric equation. Finally, a recommendation on a protocol for outdoor BRDF measurements is made, which inherently captures the spatial-dependent BRDF variance of a material.
2 Theory

The theoretical background necessary to discuss BRDF and in particular, polarimetric 
BRDF is now presented. Some basic properties of electromagnetic waves are introduced, 
which leads to defining the Fresnel equations, which in turn governs polarized reflectance. 
BRDF and the more general polarimetric BRDF are reviewed, along with the math struc-
ture necessary to quantify and propagate polarized radiance.

2.1 Electromagnetic Waves

A generalized electromagnetic plane wave travelling in the $z$ direction may be expressed in 
terms of the orthogonal $x$ and $y$ components of the electric field vector as

$$\tilde{E}(z, t) = (E_{0x} \hat{i} + E_{0y} \hat{j}) e^{i(\omega t - \vec{k} \cdot \vec{z})} \quad (1)$$

where $\tilde{E}$ is the magnitude and direction of the electric field [V/m] in the $x$-$y$ plane as a 
function of position, $z$, and time, $t$. $E_{0x}$ and $E_{0y}$ are the complex electric field amplitudes 
projected onto the $x$- and $y$-axes. The propagation direction vector, $\vec{k}$, is given by $\frac{2\pi}{\lambda}$ 
where $\lambda$ is the wavelength. The angular frequency is given by $\omega$ which is $\frac{2\pi c}{\lambda}$ where $c$ is 
the speed of propagation of light in a vacuum. The variables which are of primary concern 
for BRDF are the magnitudes and phases of the electric field components, $E_{0x}$ and $E_{0y}$ and 
the wavelength, $\lambda$.

The quantity of an electromagnetic wave that is actually measured by sensors is the 
irradiance, $E$, or the power per unit area [W/m$^2$] incident upon a detector. The irradiance 
is equal to

$$E = \frac{c \varepsilon_0 |\tilde{E}|^2}{2} \quad (2)$$

where $\varepsilon_0$ is the permittivity of free space equal to $\frac{10^7}{4 \pi c^2}$ or $\sim 8.85 \times 10^{-12} \left[\frac{c^2}{\varepsilon_0 \mu_0}\right]$ in mks 
units.\(^1\)

The polarization of an electromagnetic wave describes the direction of $\tilde{E}(z, t)$ in Eq (1), 
which is determined by the relative magnitudes of $E_{0x}$ and $E_{0y}$ and their phase difference. 
For instance, if the magnitude of $E_{0x}$ and $E_{0y}$ are equal and the complex component of each 
is zero, then the electric field oscillates in a single plane which is oriented at $\eta = \frac{\pi}{4}$ or $45^\circ$

\(^1\)Throughout this document, radiometric quantities such as irradiance will generally be treated in spectral 
terms. That is, the irradiance per $\Delta \lambda$. For brevity, the spectral dependence may not always be explicitly 
included in the notation.
Figure 1: Linear polarization at $\eta = \frac{\pi}{4}$ formed by equal amplitude $E_{0x}$ and $E_{0y}$ components which are also in phase.

Given the magnitude of solar irradiance and sensor response/noise levels, it is necessary to integrate many cycles of an electromagnetic wave—where the frequency, $\nu$ in the VNIR is $\approx 10^{14}$ Hz—to register a meaningful signal. It is for this reason that the observed polarization in incoherent imaging is the net time-averaged orientation of the electric field vector, $\vec{E}(z,t)$. This is expressed as

$$\langle \vec{E} \rangle = \frac{1}{T} \int_{0}^{T} \vec{E}^*(z,t)\vec{E}(z,t) dt = \left\langle \vec{E}^*(z,t)\vec{E}(z,t) \right\rangle$$

(3)

where $T \gg \nu^{-1}$ and $^*$ is the complex conjugate.

Natural light is mostly randomly polarized, that is there is no net preference of the electric field vector. Solar irradiance is also randomly polarized, but reflection from surfaces and scattering by aerosols impart polarization. As will be seen, the polarization of reflected light is governed by the optical properties of the material reflecting the light.

In order to quantify polarimetric radiometry, scalar flux values such as the irradiance, $E$, must be specified as a vector, $\vec{E}$, which contains the polarization information. A four element Stokes vector is traditionally used for this specification, which completely characterizes the polarization. The first element of the Stokes vector ($S_0$) corresponds to standard scalar radiometric flux values (e.g., radiance or irradiance). The second and third Stokes elements ($S_1$ and $S_2$) relate to linear polarization. The orientation of linear polarization is often referred to in relation to the material surface upon which light is incident and reflected. When the electric field is polarized parallel to the surface (or perpendicular to
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Figure 2: Stokes vector examples.

The plane of incidence defined by the plane containing the surface normal and the incident irradiance), $S_1$ has a value of 1, and decreases to a value of -1 as the polarization is completely vertical. The third element, $S_2$, is similar, except ranging from +1 at $\eta = +45^\circ$ polarization to -1 at $\eta = +135^\circ$. The fourth element conveys the magnitude and direction of any circular polarization. Examples of Stokes vectors are shown Figure 2.

Some polarization relationships and metrics may now be defined which are derived from the Stokes vectors. First, it is noted that $S_0 \leq \sqrt{S_1^2 + S_2^2 + S_3^2}$, with equality holding only for completely polarized light. The degree of polarization (DOP) and degree of linear polarization (DOLP) are given by

$$DOP = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}$$

$$DOLP = \frac{\sqrt{S_1^2 + S_2^2}}{S_0}$$

Circular polarization will not be considered in detail here, due to the small magnitude present in the VNIR from reflected solar radiation [2, p. 486]. This approximation results in $S_3 \approx 0$ which makes $DOLP \approx DOP$.

2.2 Fresnel Equations

Reflection and scattering of electromagnetic energy is governed by Maxwell’s equations and boundary conditions imposed at the interface of the dissimilar materials. When light encounters an object, or more generally a new medium with a different index of refraction, $\hat{n}$, it is either reflected, transmitted or absorbed. Consistent with the conservation of energy,
the sum of the reflectance ($\rho$), absorptance ($\alpha$) and transmittance ($\tau$) must be unity. For absorptance to occur, a finite distance must be traversed in the new medium.

Application of boundary conditions to Maxwell’s equations at the interface between dissimilar mediums may be made to derive the Fresnel equations. The Fresnel equations provide the reflection and hence transmission magnitudes of an electromagnetic wave incident on a material. Absorptance is not an issue here, as only the infinitesimal interface between the two mediums is considered. (Absorptance would occur with additional propagation by the transmitted fraction of energy). The reflection and transmission are a function of the angle and polarization of the incident electromagnetic wave. For the purposes of applying the equations, the orthogonal polarization components are defined relative to the plane of incidence. The component of the electric field perpendicular to the plane of incidence (and hence parallel to the material surface) is called $S$-polarization. The other component is parallel to the plane of incidence and is termed $P$-polarization. The Fresnel equations are presented without derivation, which may be found in many popular optics books [3, 4].

The reflection and transmission coefficients for the $S$-polarization component are given by

$$
\begin{align}
  r_S &= \frac{E_r S}{E_i S} = \frac{-\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \\
  t_S &= \frac{E_t S}{E_i S} = \frac{2 \sin \theta_t \cos \theta_i}{\sin(\theta_i + \theta_t)}
\end{align}
$$

where $\theta_t$ is the transmission angle given by Snell’s law as

$$
\theta_t = \sin^{-1} \left( \frac{\hat{n}_i}{\hat{n}_t} \sin \theta_i \right)
$$

where $\hat{n}_i$ and $\hat{n}_t$ are the complex indices of refraction of the incident and transmitted mediums.

In a similar manner, the reflection and transmission coefficients for the $P$-polarization component are given by

$$
\begin{align}
  r_P &= \frac{E_r P}{E_i P} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \\
  t_P &= \frac{E_t P}{E_i P} = \frac{2 \cos \theta_i \sin \theta_t}{\sin(\theta_i + \theta_t) \cos(\theta_i - \theta_t)}
\end{align}
$$

However, these equations provide the relative magnitude of the orthogonal electric field
components, while irradiance is proportional to the square of the electric field magnitude (Eq (2)). Therefore, the reflection and transmission ($R$ and $T$) are given by the square of Eqs (6 & 8). The total reflection is given by the sum of each of the polarized components, as is the total transmittance.

$$R_F = r_S^2 + r_P^2 \quad (9a)$$
$$T_F = t_S^2 + t_P^2 \quad (9b)$$

To illustrate the Fresnel equations, an example is given of for a typical glass, where $\hat{n}_t = 1.5 + 0i$ and the incident light is in air, $\hat{n}_i = 1$. For this case, the reflected components as a function of the angle of incidence, $\theta_i$, are shown in Figure 3.

Notice from Figure 3 that there is a point at which the $P$-component reflectance is 0. This corresponds to Brewster’s angle and is equal to $\tan^{-1}(\hat{n}_t/\hat{n}_i)$. At this angle the reflected radiance is completely polarized, which results in $DOP = 1$.

It is also of interest to observe that the Fresnel equations are not an explicit function of wavelength. The index of refraction, $\hat{n}$, is wavelength dependent, but the index variation across the VNIR region for most materials is minimal. As a result, Fresnel reflectance and transmittance are largely color neutral, meaning the spectral content of the reflected and transmitted energy is similar to that of the source.
2.3 Optical Scatter

In the preceding example of Fresnel reflectance, the magnitude of reflectance was completely determined based upon the optical properties of the materials and the angle of incidence. In addition, the reflected energy is only directed in the plane of incidence at the reflected angle, \( \theta_r \), where \( \theta_r = \theta_i \) per the law of reflection. However, this is only true for perfectly planar surfaces which also have no internal scatter.

A quick look around is all it takes to realize that most surfaces are not perfect “mirror” surfaces,\(^2\) and even mirror surfaces are not perfect. Also obvious is the fact that objects have color different than the illumination source, which is not accounted for by the Fresnel equations.

Two effects are responsible for energy reflected or more generally scattered outside the \( \theta_r = \theta_i \) reflectance angle. First, all materials have some level of surface roughness. This results in a distribution of localized surface normals which are oriented in multiple directions, similar to individual sequins on a dress. Therefore, the Fresnel reflectance is actually distributed around a reflection angle according to the “roughness” of the material. The second and usually more significant phenomena directing energy out the scattering angle is *internal scatter*. Once light has entered a material, multiple internal scattering results in distributing the energy around the hemisphere. The internal scattering sources are also responsible for color by the selective absorption of wavelengths. Figure 4 illustrates this complex interaction.

In Figure 4, several possible ray paths are noted. Incident irradiance, \( \vec{E}_i \) may be reflected off the front surface of the material according to the local surface normal (\( \hat{N}_i \)) per the Fresnel reflection equation giving \( R_F \) (type A photons). Transmitted Fresnel irradiance, \( T_F \) may then interact with a myriad of particles and molecules having selective absorption. After these single and multiple interactions, the energy may then re-emerge from the surface, again according to the Fresnel equations (type B photons). In most cases the incident medium is air, which results in the real part of the refractive index of the transmitted medium being greater than the incident medium or \( \Re\{\hat{n}_t\} > \Re\{\hat{n}_i\} \). This in turn results in total internal reflection for upward scattered radiance exceeding the critical angle relative to the local surface normal (as most have experienced, only a small area of the sky is visible when looking up swimming underwater). Of course after re-emerging from the surface, additional interactions with adjacent facets may also occur (type C photons). In

\(^2\)An interesting thought experiment is to consider a world in which all surfaces were perfect mirror surfaces. In this world, only sources of illumination would be visible and no objects could be discerned!
Figure 4: Detailed view of light scatter from material.
this manner, the scattering properties of a material are determined.

A few important conclusions may be made. The multiple scattering within a material has the net effect of depolarizing the fraction of transmitted irradiance, $T_F$, which results in the diffuse component of scatter being highly randomly polarized. Also the scattered radiance from dark materials, or those which highly absorb $T_F$, have a higher relative Fresnel reflection component, $R_F$, since the magnitude of re-emerging scattered energy is low. This results in the degree of polarization (DOP) being inversely proportional to a material’s reflectance—recall $R_F$ is comprised of two orthogonal polarization components from Eq (9).

2.4 BRDF Overview

A means of characterizing this directional scatter is needed, which is the BRDF. BRDF may be thought of as quantitatively defining the qualitatively property of “shininess”. A material may be described as being “diffuse” or “specular”; for example, a mirror is highly specular, and hence scatters minimal energy outside of the reflection angle. On the other hand, a projector screen is highly diffuse, where the apparent brightness (radiance) of the screen is the same regardless of viewing orientation. Examples of specular and diffuse scatter may be seen in Figure 5 where the geometric radiance distribution from different classes of objects is illustrated. The same principle is also illustrated in Figure 6 showing computer-generated spheres with different surface finishes.

Specifically, BRDF quantifies the radiance scattered into all directions from a surface illuminated by a source from any direction above the hemisphere of the material. The BRDF is given by

$$f_r(\theta_i, \phi_i; \theta_r, \phi_r; \lambda) = \frac{dL_r(\theta_r, \phi_r)}{dE(\theta_i, \phi_i)}$$

where $L_r$ is the surface leaving spectral radiance $[W m^{-2} sr^{-1} \mu m]$ and $E$ is the spectral irradiance $[W m^{-2} sr^{-1} \mu m]$ which results in BRDF having units of $sr^{-1}$.

Half the battle in comprehending BRDF (and radiometry in general) is understanding the nomenclature and geometry. The nomenclature used here is that recommended by Nicodemus [7,8], which has subsequently been adopted by many authors. The National Bureau of Standards monograph by Nicodemus is a seminal document on BRDF [8].

The BRDF is a function of the incident zenith and azimuth angles ($\theta_i$ and $\phi_i$), the reflected zenith and azimuth angles ($\theta_r$ and $\phi_r$) and the wavelength ($\lambda$). The zenith angles are defined relative to the local surface normal, which is $\theta_i = 0^\circ$. Most materials have
Figure 5: BRDF examples illustrating the extremes (specular and diffuse, left) to the more realistic (right). From Schott [5] without permission.

Figure 6: Computer-rendered spheres with increasingly specular surfaces (left to right). From [6] without permission.
azimuthal or rotational symmetry about the surface normal. This reduces the degrees of freedom by one, enabling the azimuth angle to be characterized by only the difference between $\phi_i$ and $\phi_r$. By convention, $\phi_i$ will be designated as $\phi = 180^\circ$ and the reflected or scattering azimuth angle defined relative to this orientation. Forward scattering is therefore $\phi = 0^\circ$. This reduces the BRDF specification for rotationally symmetric materials to $f_r(\theta_i; \theta_r, \phi; \lambda)$. The geometry is illustrated in Figure 7.

Note from Figure 7 that the source and detector occupy a solid angle, $d\omega$. BRDF is theoretically specified for a point source and detector, as well as an infinitesimal surface area, but practical measurement considerations results in some averaging over the source and detector solid angles $\omega_i$ and $\omega_s$, and surface area $A$. The averaging is most critical when the BRDF varies greatly as a function of angle, such as is the case with a highly specular or mirror-like material around the scattered specular lobe.

When high angular resolution is required to resolve the specular peak of mirror-like materials, the solid angle subtended by the detector may be minimized by increasing the material-to-detector distance or decreasing the detector size. For diffuse materials, the angular resolution is not as critical, since there are usually only modest changes in BRDF with reflection angle. The detector signal to noise can become an issue as one makes spectral BRDF measurements where a $\Delta \lambda$ of 10 nm may be desired, commensurate with the spectral bins of many hyperspectral sensors. Signal strength may also become problematic when measuring highly specular materials outside the specular lobe. However, in this circumstance the low signal is usually not of interest in remote sensing applications.
2.5 Texture

While it has been stated that BRDF is ideally measured from a point source and zero-area detector, the same is not necessarily true for the infinitesimal material surface area. In theory, BRDF measurements from increasing microscopic spatial extents on a material surface may be scaled up to reconstitute the macroscopic BRDF. For instance, a 1 ft × 1 ft painted metal plate may have numerous areas with imperfections, such as chips or bubbles in the paint. If the illumination area and/or the detector FOV fails to encompass many of the small pits and bubbles in the sample, then the individual bubble and pit BRDF characteristics aren’t sufficiently averaged out and influence the macro-level BRDF. Such small scale variability within a defined material is an example of texture and is a significant topic for further discussion. The spatially varying BRDF of a material has been referred to as the bidirectional texture function (BTF) [9] or the bidirectional reflectance variance function (BRVF) [10]. The notation of BTF will be used here. The BTF may be defined in terms of the BRDF, but with the addition of spatial-extent terms such that

\[
BTF(\Delta x, \Delta y) = f_r(\Delta x, \Delta y; \theta_i; \phi_i; \theta_r, \phi_r; \lambda) \tag{11}
\]

where \(\Delta x\) and \(\Delta y\) are at a scale in which inhomogeneities in the sample are manifested. It follows that the BRDF for a specified reflection angle may be defined from the BTF as

\[
f_r(\theta_r, \phi_r) = \frac{1}{A} \int_A BTF \, dx \, dy \tag{12}
\]

where \(A\) is the area over the material for integration. BTF may be thought of as “micro-scale” BRDF, which when averaged over a surface gives the BRDF for that material. For example, consider the BRDF of a “blade of grass.” To solve for the BRDF of a “grass field,” the orientations of individual blades of grass must be considered, which collectively constitute the grass field BRDF. The BRDF distribution of individual grass blades and their orientation therefore impact the BTF of a grass field. As \(\Delta x\) and \(\Delta y\) increase to the point where individual blades of grass are averaged out, then the BTF approaches the BRDF.

The issue of BTF is at the heart of BRDF employment methods. One may consider BTF as simply the contributions of distinct materials having varying surface orientations relative to the global material in question. For the painted plate example, individual BRDF may be acquired for the small chipped areas and for the individual bubbles. Then, the BRDF of the painted metal plate as a whole could be reconstituted based upon the individual BRDF.
contributions of 1) pristine painted areas, 2) chipped areas and 3) paint bubble areas. Of course, some density of chipped and bubble areas would have to be prescribed. For the sake of convenience, on overall BRDF is usually defined for the painted plate as a whole, and the variability is captured by such measures as the BTF. This becomes a matter of necessity also, as the “composite” BRDF may not be a simple linear superposition of the individual components.

It should be noted that BTF is a function of the spatial resolution at which an object is viewed. If a single radiometer is viewing the entire metal plate, then there is no variability in BRDF for the metal plate. However, if an imaging system is viewing the plate such that it is resolved with a $10 \times 10$ pixel area, then it is likely that variance will be present across the metal plate depending on the spatial distribution of the aforementioned imperfections. The issue of BTF will be revisited frequently.

BRDF is actually a subset of the more general bidirectional scattering distribution function (BSDF). Accompanying BRDF are the transmissive (BTDF) and volume (BVDF) scattering functions which apply to materials having those scattering features [11]. The discussion will be limited to BRDF, but it is worth noting that radiance contributions in what remote sensing ascribes to BRDF have elements from these other scattering sources (e.g., vegetative canopies).

In general, man-made surfaces are likely to have a higher BRDF value in the forward scattering plane ($\phi = 0^\circ$) near a reflectance angle equal to the incidence angle per the law of reflectance. However, this is generally not the case for natural surfaces with significant structure such as vegetation. A so-called “hot spot” is present in the backscattering direction of the illuminating source which may produce BRDF 2 to 10 times that of a diffuse reflection angles. The source of the hot-spot is primarily due to no self-shadowing being visible when looking at the target from the illumination angle. As the view angle moves away from the illumination position, self-shadowing by the material, such as from leaves of vegetation, result in decreased radiance. Coherent backscatter is also responsible for the hot spot phenomenon, but only dominates when the structure size of the material is on the order of the incident wavelength.

2.6 Reflectance

It is worthwhile at this point to explore the definition of reflectance. A material’s reflectance, $\rho$, is usually the parameter of interest one attempts to derive in remote sensing. The reflectance is defined as the ratio of the power leaving a surface to the power incident
upon a surface. Reflectance is therefore unitless and has a range limited to $0 \leq \rho \leq 1$. However, reflectance is generally dependent upon the incident angle of the irradiance, and as such is actually a function of BRDF. Nicodemus notes nine different reflectance specifications [8], but the value of current interest is the directional hemispherical reflectance (DHR). The DHR, $\rho_{DHR}(\theta_i, \phi_i, \lambda)$, is the total reflectance resulting from a light source limited to a small angular extent, such as the sun (i.e., directional). As with other radiometric terms, the DHR is also a function of wavelength, though the explicit dependence is not shown here. In terms of the BRDF, DHR is given by

$$\rho_{DHR}(\theta_i, \phi_i, \lambda) = \int_{2\pi} f_r(\theta_i, \phi_i; \theta_r, \phi_r; \lambda) \, d\Omega_r$$  \hspace{1cm} (13)$$

where $d\Omega_r$ is the projected solid reflected angle defined by

$$d\Omega_r = \cos \theta_r \sin \theta_r \, d\theta_r \, d\phi_r.$$  \hspace{1cm} (14)$$

For materials having azimuthal symmetry, the dependence of $\rho_{DHR}$ with $\phi_i$ is eliminated.

The spectral reflectance library data used as “truth” in remote sensing applications are therefore dependent upon the measurement configuration. Application of this data to the same material under different illumination conditions may result in errors.

DHR is synonymous with the surface albedo, which plays an important role in determining global thermal balances for the earth. DHR for man-made material surfaces are often conveniently measured with an integrating sphere. However, for natural materials and landscapes, an accurate albedo (DHR) estimate is only achievable through BRDF measurements and application of Eq (13).

Using the concept of DHR, a crude approximation to a surface BRDF may be made by specifying the diffuse reflectance. The diffuse reflectance, $\rho_d$, is the DHR minus a small solid angle which encompasses the specular lobe. Similar to this value is the total integrated scatter (TIS) which is also prevalent in the literature. The ratio of diffuse reflectance to DHR may then be used as a measure of a diffuseness, $d$, of a material or

$$d = \frac{\rho_d}{\rho_{DHR}}.$$  \hspace{1cm} (15)$$

A perfect Lambertian surface would therefore have $d = 1$, while a perfect mirror would have $d = 0$. Used in this manner, the radianc leaving a surface when viewing the surface outside the specularly reflected lobe may be approximated as $L_r \approx \frac{d}{\pi}$. 
2.7 Polarimetric BRDF

Polarimetric BRDF is a more generalized case of the scalar BRDF. In addition to quantifying the magnitude of the directional scattering, the polarization of the scattering is characterized. In general, the change in polarized radiometric flux upon transmission or reflection from a medium is given by

\[ \mathbf{S}_r = \mathbf{M} \mathbf{S}_i \]  

(16)

where \( \mathbf{M} \) is a polarization transfer function called the \( \textit{Mueller matrix} \). The Mueller matrix is a \( 4 \times 4 \) matrix which completely characterizes the polarized reflection or transmission properties of a medium for any incident Stokes vector, \( \mathbf{S}_i \). Most often in remote sensing, interest will be limited to the reflection properties, or \( \mathbf{M}_r \).

Invoking the assumption that circular polarization is not present in a significant amount upon reflection from most surfaces \cite[p. 486]{2} reduces the Mueller matrix to a \( 3 \times 3 \) matrix and the Stokes vector to a three element vector. With this reduction in dimensionality, Eq (16) may be explicitly written as

\[
\begin{bmatrix}
S_{0r} \\
S_{1r} \\
S_{2r}
\end{bmatrix} =
\begin{bmatrix}
m_{00} & m_{01} & m_{02} \\
m_{10} & m_{11} & m_{12} \\
m_{20} & m_{21} & m_{22}
\end{bmatrix}
\begin{bmatrix}
S_{0i} \\
S_{1i} \\
S_{2i}
\end{bmatrix}
\]  

(17)

As described in Section 2.1, the radiometric quantities of Eq (10) become Stokes vectors. The generalized polarimetric BRDF is thus represented as

\[
\mathbf{M}_r(\theta_i, \phi_i; \theta_r, \phi_r; \lambda) = \frac{d\mathbf{L}_r(\theta_r, \phi_r)}{d\mathbf{E}(\theta_i, \phi_i)}
\]  

(18)

Mueller matrix notation is most often used to describe transmissive mediums (such as optics) which results in unitless Mueller matrices. Also, the matrix is frequently normalized such that the \( m_{00} \) element is 1 and the multiplicative constant dropped—this notation readily represents the medium’s polarization characteristics at the expense of the net transmission. When representing BRDF using Mueller matrices, the matrix has units of \( \text{sr}^{-1} \) as expected, and multiplicative constants should be maintained. In this manner the \( m_{00} \) element remains equivalent to the scalar BRDF value.

A few representative Mueller matrices for ideal transmissive polarization filters are provided in Eq (19), which will be referenced when discussing polarimetric BRDF mea-
measurements (§ 3.6).

\[
M_\oplus = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad M_{\odot} = \frac{1}{2} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

\[
M_\oslash = \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad M_{\oslash} = \frac{1}{2} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}
\]

\[
M_\odot = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad M_{\text{dep}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

(19a) (19b) (19c)

The subscripts \(\oplus\), \(\odot\), \(\oslash\), and \(\odot\) represent the linear transmission orientation and are for horizontal, vertical, +45° and -45° (or +135°), respectively. A filter having no effect is the identity matrix, shown as \(M_\odot\). A completely depolarizing filter is given by \(M_{\text{dep}}\).

As with the scalar BRDF, energy conservation is maintained by recalling the relationship with DHR from Eq (13).

\[
\rho_{DHR}(\theta_i, \phi_i) = \int_{2\pi} m_{00}(\theta_i, \phi_i; \theta_r, \phi_r; \lambda) \, d\Omega_r
\]

(20)

A good review of polarized BRDF representations is provided by Flynn [12].
3 Measurement

The key elements of any optical scatter measurement are the sample material or object to be measured, the illumination source and the detector. Most BRDF measurement devices employ one or more goniometric arms which provide angular positioning of the source and/or detector element. In some cases, the sample orientation may also be changed in order to achieve the full hemispherical range of source and detector orientations.

Commercial BRDF measurement systems have been developed and are available from at least two manufacturers. However, most measurement needs are satisfied with systems customized to the user’s unique application. It is for this reason, in part, that very few BRDF databases exist. Measurements taken by a particular group often have inadequate material and experimental conditions described, and have tailored features which are not easily adaptable to a new user’s interest.

Newer approaches in BRDF measurement often incorporate imaging techniques which enable the simultaneous sampling of multiple angles, greatly decreasing the required number of measurements. Imaging systems also enable characterization of the bidirectional texture function (BTF). Other novel techniques have also been developed and will be briefly explored. The impetus for most of the newer measurement approaches are for improved rendering in computer animation and simulation, in which there is a significant commercial market.

Outdoor BRDF measurements are common for remote sensing due to the large spatial scales of the materials involved, as well as the inability to bring representative materials into the lab, such as undisturbed live vegetation. Approaches toward outdoor BRDF measurements will be reviewed, as well as the means to handle some of the challenges outdoor measurements present.

Finally, the measurements required to capture the most general form of BRDF, the polarimetric BRDF will be reviewed. The foregoing measurement techniques may all be adapted to polarimetric measurements, with varying levels of complexity.

3.1 Commercial Devices

A small number of commercial devices are available for measuring directional reflectance. Surface Optics Corporation markets the SOC 200 Bidirectional Reflectometer, which is a full BRDF measurement device. The SOC 200 allows the use of laser sources, or a broad source combined with a filter wheel system. Such a system may cost > $500 K [13] and is

\[\text{www.surfaceoptics.com/index.htm}\]
usually not warranted for most research needs.

Surface Optics also offers other hand-held portable devices which function in the mid-to far-infrared. Termed the “SOC 600,” it is advertised as a handheld imaging reflectometer and operates at 3–5 $\mu$m or 8–12 $\mu$m with angles of incidence up to 85°. A pre-production version of the SOC 600 is described in [14] and was developed with funding in part from the Air Force Research Lab. The pre-production version has a measurement area of only 4 $\text{mm}^2$ and uses a microbolometer imaging array with an ellipsoidal mirror to sample multiple reflection angles using the 7.75 lb measurement head.

An alternate Surface Optics Corp. device, the SOC 250 was designed for portability and operation over the VNIR (400–1100 nm) or in the IR (3.0–12.0 $\mu$m) range [15]. The measurement head weighs $\sim 60$ lbs and is accompanied by a power supply and laptop computer, which provides automated positioning of the source and detector. However, the size of the device requires short detector and illumination standoff distances, as well as a small measurement area.

Schmitt Industries, Inc., offers similar products including the CASI© (Complete Angle Scattering Instrument) which uses a laser source and has a high angular resolution. (0.001°) A portable device, the $\mu$Scan© provides measurements using a laser diode and fixed angle of incidence.

The portable commercial devices are typically used in quality assurance applications to determine if surfaces meet a required specification. They are often not suitable for material geometric and spectral BRDF characterization, in particular where larger sample sizes are required.

### 3.2 Conventional Laboratory Measurements

The most common and traditional means of measuring BRDF is to use an illumination source of small angular extent and a corresponding radiometer to measure the scattered radiance. Several means of acquiring the necessary source and detector angular sampling are invoked by using a goniometer. For most systems, it is easiest to fix the source position and vary the detector location to sample $\theta_r$ and $\phi_r$. The incident angle, $\theta_i$ is sampled by moving the target sample, which is usually a relatively small, planar sample. In other circumstances where the detector system may be large, such as a spectrometer, the detector position is fixed and the source and the source and sample are moved to sample the hemisphere [16].

\[\text{www.schmitt-ind.com/products-services-measurement-systems-casi.shtml}\]
Illumination sources may either be lasers, or a broad-band source coupled with spectral filters at the source or detector to enable spectral measurements. Often the data acquisition process is automated, whereby the angular position of the detector and material is changed to cover the prescribed BRDF measurement sampling density. The number of required measurements is significant. For an isotropic material (no azimuth dependency) and sampling at $10^\circ$ increments in both $\theta_i$, $\theta_r$ and $\phi$, the number of required measurements is exceeds 1500 per spectral band $(9 \ (0^\circ \leq \theta_i \leq +80^\circ) \times 9 \ (0^\circ \leq \theta_r \leq +80^\circ) \times 19 \ (0^\circ \leq \phi \leq +180^\circ))$. If the material does not have azimuthal symmetry, an additional multiplier of 72 or more than 100,000 measurement per spectral band is required! This simple calculation illustrates the challenge in adequately measuring BRDF.

Lab measurements intended for remote sensing applications are particularly challenging. The heterogeneity or texture of most natural materials occurs at a spatial scale much larger than the typical sample size which is used in the laboratory. For this reason, natural materials are best measured over larger spatial scales and in their natural, undisturbed states by outdoor measurement techniques (§ 3.4). However, BRDF measurements of man-made materials, which often constitute “targets” in spectral detection algorithms, may be more accurately measured in the controlled lab environment. A review of BRDF lab measurements with a remote sensing perspective is provided by Sandmeier [17].

3.3 Camera-based Measurements

The use of focal planes to make BRDF measurements greatly increases the measurement efficiency. Rather than having a single detector, and hence a single bistatic angle for each measurement, multiple reflection angles may be simultaneously acquired. Several permutations on this concept may be employed. BRDF measurement techniques using focal planes may be grouped into three basic approaches:

1. Wide field of view (FOV) sampling many reflecting angles

2. Tailored optics systems to image the material (many variants)

3. Narrow FOV used similar to a single element detector

The wide FOV systems rely upon a large uniform material area for making measurements. Discrete scattering angles are obtained from each pixel of the imaging system, which enables an efficient, dense sampling of scattering angles. Of course, spatial inhomogeneities in the material erroneously manifest themselves as a BRDF change so caution
must be used. Several outdoor systems make use of this measurement approach and will be addressed separately in § 3.4.

The second basic imaging configuration, tailored optics systems, encompasses a number of measurement concepts and are among the most creative. The overall approach is to re-image the material surface in a manner which enables efficient changes to the system, such as the incident angle of illumination and multiple viewing geometries. Two such approaches are included here, one which images an infinitesimal surface point, and another employing a kaleidoscope which provides multiple discrete scattering angles while resolving the surface. The most significant disadvantages of these systems are the limitations imposed upon the illuminating source and the sample size—outdoor measurements using the sun would be difficult. Also, since these systems use reflective optics having multiple reflections or varying reflectance angles, measuring the polarimetric BRDF is problematic due to the polarization dependency of the system.

Finally, a narrow FOV imaging system may be used, analogous in the manner of a single element detector. Implicit in this measurement technique is the BTF from the image data. However, as with a single detector, many measurements or images must be acquired to cover the scattering hemisphere. Therefore, this technique must heavily rely upon BRDF models to inter/extrapolate the data. This technique is adaptable to both the lab and field. For field use, calibration and stray light mitigation are readily employed. Systems using this approach have not been noted in the literature, likely due to the single scattering angle sampling. However, it is this technique which the author recommends for high spatial-resolution remote sensing, as will be seen in Section 6. These three basic imaging approaches to BRDF measurement, along with their relative merits are illustrated in Figure 8.

The following approaches are those of the “tailored optics systems.” As previously mentioned, wide FOV systems will be addressed in § 3.4 and the new narrow FOV systems in § 6.

Marschner reports a system which images an object of known shape, such as a sphere or a cone, covered with a desired material to be measured [18]. The shape of the object inherently provides the multiple viewing geometries rather than using optics. Viewing the object while having a single illumination source enables the direct measurement of a large number of incident and scattering angles with a single image. Scanning either the source or the camera in a single plane enables sampling the full hemisphere of source and detector angular positions. A diagram illustrating this approach is shown in Figure 9. Appropriate coordinate transformations relating the local surface normal to the illumination and obser-
Figure 8: Three fundamental approaches toward focal-plane based BRDF measurements.

Figure 9: Image-based BRDF measurement system from [18] (without permission).

...
angle ($\theta_i$) are possible with a simple planar translation of the light source aperture. In a similar manner, multiple image points on the material surface are obtained by translating the parabolic mirror above the surface. The multiple image points enable texture (BTF) measurements or the spatial “micro-scale” variation of BRDF (§2.5). A schematic overview of the Dana measurement technique is provided in Figure 10. A similar approach is also reported by Apel [21].

Another unique approach has been implemented by Han [22] which employs imaging through a kaleidoscope to enable the simultaneous measurement of the BRDF and BTF. A tapered kaleidoscope having front-surface mirrors is used to image a material, which creates a virtual sphere consisting of multiple, tapered facets corresponding to different viewing zenith angles of the object. The effect is equivalent to having multiple camera angles imaging the same surface area. A digital projector provides the light source and the incident illumination angles are controlled by selectively turning on groups of pixels in the digital projector. A beam splitter is used to merge the optical paths of the illumination source and the camera. The taper angle of the kaleidoscope controls the number of facets viewed, or equivalently the number of scattering angles sampled. A smaller taper angle increases the number of facets, but has the disadvantage of lower spatial sampling when imaged by the camera. The converse is true of a larger taper angle. This concept is illustrated in Figure 11.
3.4 Field Measurements

Portable BRDF devices suitable for outdoor measurements are attractive for a number of reasons. The use of portable devices arises out of necessity when measurements must be made which are extremely difficult, if not impossible to replicate in the lab. Natural materials may be heterogeneous over spatial extents significantly larger than what may be measured in the lab. Vegetation is a classic example of one such material, whether it is grass or a leaf canopy. Direct measurement of materials in their natural state and at larger spatial scales eliminates the requirement to scale-up individual material BRDFs which are often interactive, such as leaf transmittance and multiple leaf adjacency effects. Having the use of the sun as the source is advantageous as well. A good review of BRDF field measurements in the VNIR is given by Walthall [23].

An obvious challenge to outdoor measurements is cooperative weather and stray light. A nice day may eventually be found (even in Rochester, NY), but the downwelled radiance outside the solar disk is always an error source in the measurements. In addition, the magnitude and distribution of this error source changes depending on local atmospheric conditions, such as extent of cloud cover. This error source obviously has a spectral dependence, as the blue sky testifies. A good discussion of outdoor measurement errors and
minimization techniques is provided by Sandmeier [24] with some quantitative assessments provided by [25, 26]. Techniques for minimizing these errors will be discussed concurrent with the recommended BRDF measurement approach in § 6.1.4. Finally, the source zenith position is not easily adjustable (unless you’re Clark Kent).

In most circumstances, outdoor measurements are made over a sample spatial extent much greater than that made with lab measurements. The basic criteria regarding what constitutes a sufficient area for measurement still applies (as discussed in Section 2). High spatial frequency inhomogeneities in the material must be adequately averaged out over the measured sample area. For example, if the material of interest is grass, then the measured area must encompass many individual blades of grass which integrates out the individual blade detail. An indication of adequate sample size is when the resulting BRDF value is relatively insensitive to changes in the sample area in the field of view (FOV) of the instrument, or equivalently as the variance in the BTF approaches zero for increasing $\Delta x$ and $\Delta y$.

A highly relevant challenge, though outside the scope of this treatment, is generating a sufficiently accurate and meaningful descriptive characterization of the material, which is critical for natural materials. It is by these descriptive labels that the acquired data will be selected and used in subsequent analysis, synthetic image generation, etc. A simple descriptor such as “Paint XYZ on Aluminum” is not sufficient when ascribing BRDF to inhomogeneous targets.\footnote{Actually, adequately describing “simple” materials is very challenging also. Added to the description of “Paint XYZ on Aluminum” should also be information such as application method, surface condition and paint thickness—and again a picture doesn’t hurt.} It is suggested that a robust meta-data set always accompany such measurements. This meta-data should include photographs of various viewing geometries of the sample, as well as detailed verbal descriptors.

A review is now provided of some field devices reported in the literature. Two fundamental designs may be employed. A traditional “lab-like” system where the sensor is moved around a hemisphere above the target, or one in which the sensor does not translate, but acquires different view angles from the fixed position. With the latter type system, the target area must be sufficiently uniform such that views of each area are each representative of one another.

3.4.1 Mobile Sensor Designs

The most direct approach toward field BRDF measurements is to emulate a laboratory setup by using a goniometer. With the illumination source (the sun) and target orientation
on the ground fixed, the goniometer serves to move the detector to sampling positions throughout the hemisphere.

One such example of a system is FIGOS (field goniometer system) built by the Remote Sensing Lab of the University of Zürich [27, 28]. The system consists of a “zenith” arc of 2 m radius which rests on a circular frame of 4 m diameter—the azimuthal arc (Figure 12). The sensor is a spectroradiometer providing coverage from 300–2450 nm with spectral resolution of 1.5 nm in the VIS and 8.4 nm in the SWIR providing a total of 704 bands. The PC-controlled spectroradiometer, with a FOV of 2°, is driven along the zenith arc by a DC motor to enable sampling $\theta_r$. The entire zenith arc assembly is rotated on the azimuth arc, providing $\phi_r$ sampling. The entire system weighs some 500 lbs and may be set up by a team of two in $\sim$ 90 min. Measurements are made at a resolution of $\Delta \theta_r = 15^\circ$ with a range of $\pm 75^\circ$ and $\Delta \phi_r = 30^\circ$. The 11 measurements made in each azimuth position require 3 min for a total acquisition time of 18 min for all 66 hemispherical measurements. The mechanical structure cost $\sim$ $125,000 (1994 USD) [28].

A nearly identical goniometer, the Sandmeier Field Goniometer (SFG) was constructed by NASA Ames based upon the FIGOS design. However, this field goniometer is fully automated and the acquisition time for the same angular sampling scheme as FIGOS ($\Delta \theta_r = 15^\circ, \Delta \phi_r = 30^\circ$) is completed in less than 10 min [24]. Figure 12 pictures the SFG and FIGOS systems.

Another goniometer advertised as having outdoor measurement capability was constructed by ONERA in France. However, this system is much more massive with a weight approaching 2000 lbs, but with individual components disassembled to $\sim$125 lbs. The system provides spectral coverage from 400–1000 nm with a $\Delta \lambda$ of 3 nm. The target area is imaged with a bundle of 59 fiber optics and a fore optic. The fiber optics mixes the incident
polarization of the scattered radiance, thus eliminating the polarization dependency of the
diffraction grating in the spectrometer [29,30]. This also enables polarimetric BRDF mea-
surements using the device, to be addressed in § 3.6. The instrument is shown in Figure
13.

The previous goniometer systems provide high angular precision and rapid sampling
of the scattering hemisphere. They are suitable for highly accurate characterization of
field materials. However, both systems require significant support for transport and setup,
which is exacerbated by having to time the weather conditions for suitable measurement
periods. A much more simple measurement technique is often warranted which still provides
meaningful BRDF data. Representing this opposite extreme are simple measurements made
with a radiometer attached to a hand-held boom. The angular position of such a device
may be estimated based on trigonometry of the height and distance from the measured
area. Measurements of only a few geometric positions provides an understanding of the
BRDF anisotropy, or departure from a Lambertian surface. A simple boom system is shown
in Figure 14.

3.4.2 Immobile Sensor Designs

An alternative approach to a sensor being repositioned around the hemisphere is a fixed
sensor which changes the view angle over a large homogeneous measurement area. One such
BRDF measurement device is PARABOLA (Portable Apparatus for Rapid Acquisition of
Bidirectional Observations of Land and Atmosphere), which has been used in various forms
by NASA-Goddard since the mid-1980s [31]. Such a sensor is often mounted high on a mast
or a lift in order for the FOV to average out spatial inhomogeneities in the landscape (Figure
15).
Figure 14: A simple hand-held boom BRDF measurement. From NASA (http://modarch.gsfc.nasa.gov/MODIS/LAND/VAL/prove/grass/prove.html) without permission.

Figure 15: The Parabola III system showing the sensor, and the sensor mounted on a boom for field measurements. From NASA (http://modarch.gsfc.nasa.gov/MODIS/LAND/VAL/prove/grass/prove.html) without permission.
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A similar device based upon this employment technique was designed and built by the Digital Imaging and Remote Sensing Laboratory at the Rochester Institute of Technology. The device uses a diffraction grating and a 2D CCD array which enables 10 nm sampling from 400–950 nm with a FOV of 1.72° × 0.06°, which was limited by the number of photosites on the focal plane. Using multiple scans, the effective FOV is increased to 1.72° × 1.6° [32].

As stated earlier, if the texture of the landscape is not sufficiently averaged out within the FOV of the sensor, errors are present in the BRDF data. An equivalent statement is that the area, $A$, in Eq (12) is not of sufficient magnitude. This is most prevalent for measurements near nadir ($\theta_r \sim 0^\circ$) which has the smallest spatial footprint, or ground field of view (GFOV). Nonetheless, it is a simple and useful technique for making measurements without the use of large field goniometers.

An equivalent approach may be made with an imaging system as discussed in § 3.3. Rather than scanning a radiometer to acquire the multiple view angles, a wide FOV camera lens may be used. The University of Arizona uses such a system which assists in the vicarious radiance calibration of Landsat satellites [33, 34, 35]. Linear CCDs may also be used to make a conical push-broom scan around the target area [36, 37].

3.5 Overhead BRDF Measurement

Finally, new payloads have enabled BRDF measurement from satellites and aircraft. In these instances the atmosphere provides an additional measurement uncertainty in deriving the surface-leaving radiance. One such instrument is POLDER (Polarization and Directionality of the Earth Reflectances), which has a wide field of view (GSD of 6 km) and is flown on the ADEOS satellite [38]. Derivation of POLDER BRDF products is described in [39]. A BRDF product of evergreen trees derived from POLDER is shown in Figure 16.

Similar data is provided by NASA’s MODIS (Moderate Resolution Imaging Spectroradiometer) instruments, flown on Terra (EOS AM-1) and Aqua (EOS PM-1), having a GSD of 250-1000 m depending upon the spectral band. MODIS scans ±55° thus enabling multiple angular views with successive passes. Initial BRDF products from the instrument were available in the year 2000 [41].

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6http://smsc.cnes.fr/POLDER/
7http://modis.gsfc.nasa.gov/
3.6 Polarimetric BRDF Measurement

As noted in § 2.7, one quantifies the reflective Mueller matrix, $M_r$, in making polarimetric BRDF measurements [Eq (18)]. In polarimetric BRDF measurements, the scattered or reflected Stokes radiance vector, $\vec{L}$, must be quantified such that

$$\vec{L}(\theta_r, \phi) = M_r(\theta_i, \phi, \theta_r)\vec{E}(\theta_i)$$

However, without the use of any polarization filtering, the detector only measures the magnitude of the irradiance and radiance as in the case of the scalar BRDF or

$$L_0(\theta_r, \phi) = m_{00}(\theta_i, \phi, \theta_r)E_0(\theta_i)$$

where the “0” subscript denotes the first element of the Stokes vector, which is the total flux and $m_{00}$ is the upper left element of the Mueller matrix equivalent to the scalar BRDF.

Clearly, additional measurements are needed to characterize the other 15 elements of the Mueller matrix. When considering linear polarization, this requirement is reduced to determining the remaining 8 elements of the $3 \times 3$ Muller matrix. These additional elements of the array may be determined by linear combinations of incident irradiance polarization states, $\vec{E}$, and received polarization radiance states, $\vec{L}$.

The most generalized means of acquiring the matrix elements is through presenting multiple incident polarization states, and measuring the output for each incident state, thus building a system of linear equations. The polarization filters which create incident polarization states are termed *generators* while those which filter the output are called *analyzers*. The presentation of $i$ incident polarization states onto the sample and their
polarized radiance measurements may be represented as

\[
\mathbf{M}_r \left[ \vec{E}_1 \vec{E}_2 \cdots \vec{E}_{i-1} \vec{E}_i \right] = \left[ \vec{L}_1 \vec{L}_2 \cdots \vec{L}_{i-1} \vec{L}_i \right] \tag{23}
\]

where \( \left[ \vec{E}_1 \vec{E}_2 \cdots \vec{E}_{i-1} \vec{E}_i \right] \) is a \( 4 \times i \) matrix consisting of irradiance column Stokes vectors and \( \left[ \vec{L}_1 \vec{L}_2 \cdots \vec{L}_{i-1} \vec{L}_i \right] \) is the equivalent radiance representation. Rewriting these terms as matrix quantities \( \mathbf{E} \) and \( \mathbf{L} \), the new expression is

\[
\mathbf{M}_r \mathbf{E} = \mathbf{L} \tag{24}
\]

where it is seen that

\[
\mathbf{M}_r = \mathbf{L} \mathbf{E}^{-1} \tag{25}
\]

However, inversion of \( \mathbf{E} \) is only possible when it is a nonsingular square matrix. For the general case where \( i > 4 \), the pseudoinverse of \( \mathbf{E} \) is sought, \( \mathbf{E}^\# \), which provides a least squares estimate in the presence of random noise. The pseudoinverse is given by

\[
\mathbf{E}^\# = (\mathbf{E}^T \mathbf{E})^{-1} \mathbf{E}^T \tag{26}
\]

where \( T \) is the transpose of the matrix. The Mueller matrix is therefore solved by

\[
\mathbf{M}_r = \mathbf{L} \mathbf{E}^\# \tag{27}
\]

In this manner the full Mueller matrix may be determined for each permutation of \( \theta_i, \theta_r, \phi \) and \( \lambda \) as one would measure the scalar BRDF.

For practical measurement considerations, one would like an efficient set of input and output polarization states to minimize the number of measurements. The equation describing this measurement setup is given as

\[
\vec{L} = \mathbf{M}_A \mathbf{M}_r \mathbf{M}_G \vec{E} \tag{28}
\]

where \( \mathbf{M}_G \) is the generator filter over the source and \( \mathbf{M}_A \) is the analyzer filter over the detector. Note that the generator and analyzer Mueller matrices have no units, but the reflectance Mueller matrix \( \mathbf{M}_r \) has BRDF units, \( \text{sr}^{-1} \) (see also § 2.7).

Referencing the polarization filters provided by Eq (19), a simple example is constructed
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for generator and analyzer linear horizontal filters. The equation is given by

\[ \vec{L} = \mathbf{M}_\oplus \mathbf{M}_r \mathbf{M}_\oplus \vec{E} \]  (29)

or explicitly as

\[
\begin{bmatrix}
L_0 \\
L_1 \\
L_2
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
1 & 1 & 0 \\
1 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
m_{00} & m_{01} & m_{02} \\
m_{10} & m_{11} & m_{12} \\
m_{20} & m_{21} & m_{22}
\end{bmatrix} \frac{1}{2} \begin{bmatrix}
1 & 1 & 0 \\
1 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
E_0 \\
E_1 \\
E_2
\end{bmatrix}
\]  (30)

which reduces to

\[
\begin{bmatrix}
L_0 \\
L_1 \\
L_2
\end{bmatrix} = \frac{1}{4} \begin{bmatrix}
(m_{00} + m_{01} + m_{10} + m_{11}) (E_0 + E_1) \\
(m_{00} + m_{01} + m_{10} + m_{11}) (E_0 + E_1) \\
0
\end{bmatrix}
\]  (31)

However, it is only the \( L_0 \) Stokes component that the detector will be measuring. If the original source is highly randomly polarized, then \( E_1 \ll E_0 \) and the measurement yields

\[ L = \frac{(m_{00} + m_{01} + m_{10} + m_{11}) E_0}{4} \]  (32)

In a similar manner, other permutations of generator and analyzer polarization states produce additional linear combinations of the Mueller matrix elements. A summary of such combinations is provided by Bicket [42]. To quantify the \( 3 \times 3 \) subset of \( \mathbf{M}_r \) which relates to linear polarization, a total of 9 measurement permutations is required which include the three generator and analyzer states of horizontal, \( +45^\circ \) and no (or random) polarization. These states are represented symbolically as \( \ominus, \odot \) and \( \oslash \), respectively. Using this symbolic representation, Eq (32) may be recast as

\[ \ominus \ominus = \frac{L}{E_0} = \frac{m_{00} + m_{01} + m_{10} + m_{11}}{4} \]  (33)

where the first \( "\ominus" \) represents the generator state, and the second \( "\ominus" \) is the polarization...
of the analyzer. Using this notation, the Mueller matrix elements may be shown to equal

\[
\begin{align*}
m_{00} &= \circ \circ \\
m_{01} &= 2 \circ \circ - \circ \circ \\
m_{02} &= 2 \bullet \circ - \circ \circ \\
m_{10} &= 2 \circ \circ - \circ \circ \\
m_{11} &= 4 \circ \circ - 2 \circ \circ - 2 \circ \circ + \circ \circ \\
m_{12} &= 4 \circ \circ - 2 \circ \circ - 2 \circ \circ + \circ \circ \\
m_{20} &= 2 \circ \circ - \circ \circ \\
m_{21} &= 4 \circ \circ - 2 \circ \circ - 2 \circ \circ + \circ \circ \\
m_{22} &= 4 \circ \circ - 2 \circ \circ - 2 \circ \circ + \circ \circ 
\end{align*}
\]

Often the symmetry of the material results in \( m_{01} = m_{10} \), \( m_{02} = m_{20} \) and \( m_{12} = -m_{21} \).

Careful error analysis must also accompany this measurement process. Errors may be introduced from several factors. The illuminating source may have some inherent polarization and the detector may have a polarization-dependent response. Some errors are always present from polarization filters, which are not perfect. All filters have a finite extinction coefficient, or the transmittance along one polarization axis relative to the other axis. This amounts to some leakage of the opposite polarization state which becomes an error source. This leakage is also spectrally-dependent. Finally, the fidelity of filter alignment results in an error source. Propagation of Mueller matrix element errors has been addressed by several authors [43, 44, 45, 46].

\section*{3.7 BRDF Databases}

As stated at the beginning of this section, there are few BRDF databases which satisfy the requirements of diverse users. That being said, there are some extensive BRDF data which may be accessed and used. Given the time and expense involved in making BRDF measurements, the use of existing BRDF data is attractive.

The Columbia-Utrecht Reflectance and Texture (CUReT) Database is a joint effort between Columbia University of New York and Utrecht University of The Netherlands, with funding provided in part by the European Commission and the National Science Foundation. It’s emphasis is on computer graphics rendering and as such the spectral data is limited to RGB channels. It contains more than 60 different materials, with measurements
for each consisting of more than 200 incident and scattering angle combinations. Details on the database are in [9] and the database is accessible via the internet.\footnote{http://www1.cs.columbia.edu/CAVE/curet/} Data files for a material are typically 100s of MB and fits are made to the data using the Oren-Nayar BRDF model \cite{47}.

Cornell University’s computer graphics program maintains a similar database, also internet accessible.\footnote{http://www.graphics.cornell.edu/online/measurements/reflectance/index.html} The database primarily contains paint samples which are measured with high spectral ($\leq 10$ nm) and geometric resolution, but is limited to visible (VIS) wavelengths.

The Nonconventional Exploitation Factors (NEF) BRDF database contains BRDF measurements of more than 400 materials which are fit to a modified Maxwell-Beard BRDF model \cite{48}. The measurement protocol used in acquiring the data is that recommended by Maxwell \cite{49}. Laser sources from the UV to LWIR are used to obtain the scattering in the plane of incidence as well as the cross scattering ($\phi_r = \pm 90^\circ$). Spectral interpolation is accomplished by a high spectral resolution DHR measurement, where it is assumed that the spectral BRDF change with orientation is a slowly varying function \cite{48}. Materials covered in the NEF are grouped into twelve general categories which include asphalt, brick, camouflage, composite, concrete, fabric, water, metal, paint, rubber, soil and wood. The database has been employed in rendering computer graphics \cite{6, 50} as part of a project by the National Institute of Standards and Technology.\footnote{http://math.nist.gov/~FHunt/appearance/nefdsimages.html}

Surface Optics Corporation, discussed previously in Section 3.1, maintains an optical properties database which primarily includes DHR measurements. The data may be purchased in whole or in part and includes some 180 materials grouped into categories which include construction materials, fabrics, paints, rocks, soils and vegetation. Only a subset of the materials includes BRDF data which was acquired over three broad bands: VIS, MWIR and LWIR.\footnote{http://www.surfaceoptics.com/brochures/SOC_Measured_Data_Doc.pdf}

Finally, numerous spacecraft and optical component BRDF data are consolidated in SOLEXIS\textsuperscript{TM}, a database created and maintained by Stellar Optics Research International Corporation (SORIC).\footnote{http://www.soric.com} The database appears to primarily contain in-plane scattering measurements, and the emphasis is on stray light for thermo-optical components. Data was collected from a large number of public and private sources \cite{51}.
4 Models

As has been seen, an infinite number of measurements is required to fully quantify BRDF. The difficulty in making and managing BRDF datasets necessitates the use of BRDF models. BRDF models are motivated from several factors:

1. **Compactness**: Individual material data sets with high angular and spectral sampling may easily exceed 100 MB, and if a scene is considered which contains hundreds of materials, the volume of data is unmanageable. A BRDF model provides a concise means of storing the data.

2. **Interpolation**: Often only sparse hemispherical sampling has been measured for samples, in which case a means of inter- or extrapolating the measured values is required.

3. **Prediction**: No BRDF measurements have been made for a material, but the physical attributes of the material are known which enable prediction of the BRDF.

4. **Information Extraction**: In fields such as remote sensing and semiconductor processing, BRDF models may be linked to physical attributes such as leaf area index which provide target information.

Virtually all BRDF models satisfy the need for compactness, and most provide some means of interpolation. Models providing prediction without any measured data are first-principles, physics based and are attractive since empirical data is not needed. Many models are prediction models which use a modest amount of empirical data, to which some constants or parameters are fit. Finally, models which provide information extraction are usually tailored to a specific target classes of interest, such as erectophile vegetation or conifer forests.

There are seemingly an infinite number of BRDF models, derived from many researcher’s dissatisfaction with attempting to apply existing models to their specific interest area. Surprisingly (at least to a physical scientist), the computer graphics community has made substantial contributions to the field as processing power has enabled three-dimensional rendering of objects. Of course, there is a significant commercial market for these applications which continues to drive development.

The BRDF models covered in this section are limited to those for homogeneous materials. That is, models which describe a highly uniform surface with minimal texture such as common with man made materials. Thus they are suitable for describing surfaces which are often “target materials” in spectral algorithms. Heterogeneous material BRDF models,
such as those commonly used in remote sensing for complex vegetation canopies are very
distinct from the homogeneous material models. These models may be used to describe
“background materials” in spectral algorithms. It is rare to find a remote sensing publica-
tion which references homogeneous BRDF models common to the optics and radiometry
community. Discussion of remote sensing BRDF models is postponed until § 5.4.

BRDF models may be classified in a number of ways. One classification is based upon
the treatment of the optics. Geometric optics models are in general more approachable, but
the underlying ray model assumptions break down as surface roughness dimensions decrease
and become proportional to or less than the wavelength. Models based on physical optics
provide a much more thorough treatment through field equations, but result in complicated
expressions.

BRDF models may also be classified as physical or empirical. Physical models rely
upon first-principle physics of electromagnetic energy and material interactions, and require
inputs such as surface roughness parameters and the complex index of refraction. Empirical
models rely solely upon measured BRDF values, while semi-empirical models incorporate
some measured data, but may have significant elements of physics-based principles. These
semi-empirical models are perhaps the most common and versatile.

Many BRDF models divide a surface into microfacets, for which the distribution of the
individual microfacet normals drives the specular and diffuse scattering contributions, as
previously discussed when considering optical scatter and illustrated in Figure 4. These
models require the use of spherical trigonometry which relates the local microfacet coordi-
nate system to the material surface coordinate system.

Recently, polarized BRDF (pBRDF) models have been developed which predict the
polarized radiance as discussed in § 2.7. Such models are a prerequisite to the quantitative
analysis of polarimetric images in remote sensing. The polarized models are usually enabled
by using the Mueller matrix representation of the microfacet reflections. Two of the pBRDF
models reported in the literature have been enabled by adapting an existing BRDF model
to include polarization effects.

Finally, the question of required accuracy must be addressed. A systems engineering
approach to this question is appropriate, as the required accuracy of any BRDF model
depends upon the specific application, as well as the magnitudes of other radiometric
errors present in the remote sensing imaging chain. It is suggested that the accuracy of
many of the models discussed below is more than sufficient in remote sensing applications.
Consider a spectral target detection algorithm. An assumption must be made regarding
the orientation of the target relative to the local horizon. The most obvious assumption is
that the target is on level ground. However, deviations of $\pm 20^\circ$ are not difficult to consider (e.g., the slope of the front of a vehicle, a vehicle on a modest hill, etc.). More discussion on target orientation will be provided in § 5.

4.1 Early BRDF Models

Though often not considered a BRDF model, the Lambertian assumption ascribes a constant BRDF for all incident and reflecting geometries [1]. It is simply

$$f_r = \frac{\rho}{\pi}$$ (35)

where $\rho$ is the reflectance. This is traditional treatment of reflectance in remote sensing.

One of the earliest variable BRDF models was proposed by Minnaert in 1941 to account for darkening near the lunar limb [52]. It is represented by

$$f_r = \frac{\rho (\cos \theta_r \cos \theta_i)^{k-1}}{\pi}$$ (36)

where $k$ is the “limb darkening” parameter. Note for $k = 1$, it is equivalent to Lambert’s BRDF.

Astronomical observation was responsible for significant BRDF work. In particular, physical explanations of the “hot spot” effect for planetary bodies was sought. Analysis was also performed in attempts to better understand the surface of the moon for preparation for the lunar landings. Toward this end, Hapke developed what is known as the Hapke/Lommel-Seeliger BRDF model which accounted for opposition effects [53]. This work concluded that the lunar surface was composed of fine, compacted dust, and the model would later form the basis of the popular semi-empirical model by Maxwell-Beard.

4.2 Empirical Models

The most straightforward means of producing BRDF data for all incident and scattering angles is simply by interpolation of empirical data, which may be viewed by some as circumventing a BRDF model altogether. Here, no physical basis of the scattering is considered, and the only inputs are the empirical measurements. This approach is attractive due to the simplicity.

Such an approach has been used by the University of Zurich’s Remote Sensing Laboratories for outdoor measurements using the BRDF measurement system described in
Figure 17: BRDF data for grass at 600 nm interpolated using spherical Delaunay triangulation with ~70 measurements (left) and ~400 measurements (right). Data acquired with the FIGOS system (see § 3.4.1). From [54] without permission.

Section 3.4 [27,28]. An example of fitting BRDF data for lawn grass is shown in Figure 17 at 600 nm for an incident solar angle of $\theta_i = 35^\circ$. Here, interpolation is accomplished by spherical Delaunay triangulation, with the left figure having a sampling of $\Delta \theta_r = 15^\circ; \Delta \phi = 30^\circ$ (~70 measurements) and the right figure having a ~sixfold increase in sampling at $\Delta \theta_r = 5^\circ; \Delta \phi = 15^\circ$ (~400 measurements). While there is a marked change in the peak magnitude around the “hot spot” or retroreflection position, the other angular positions in the coarser sampling appear to have only minor variations.

The data shown in Figure 17 was made using an IDL software package from the University of Zurich called GONIO. A copy of GONIO has been acquired, which also includes an ENVI interface. GONIO handles multispectral BRDF data and provides interpolation of sparse data sets via IDL’s Triangulate function. Further exploration of this package is warranted to investigate the utility it may have for the DIRS group.

Interpolation of measured data is highly accurate so long as measurements are made with reasonable sampling densities. However, as the geometric sampling density increases, so does the accuracy, but at the expense of massive data storage requirements (see § 3.2). Measured BRDF values may be decomposed into appropriate basis functions having spherical or circular symmetry, greatly reducing the storage requirements for measured data sets.

Spherical harmonics may be used as a basis set to represent an arbitrary BRDF, analogous to the manner in which a Fourier series may be used to represent a function. However, 

$^{13}$http://www.geo.unizh.ch/rsl/research/SpectroLab/goniometry/index.shtml
a significant number of coefficients may be required for accurate representation, and ringing 
may be present from series truncation [55]. As an alternative to spherical harmonics, 
the hemisphere may be projected onto a single plane and Zernike polynomials used [56]. 
In a similar manner, spherical wavelets may also be used [57]. Other ideas for efficient 
representation include transforming BRDF variables by taking advantage of symmetries to 
reduce the number of basis function coefficients required [58].

4.3 Semi-empirical Models

4.3.1 Torrance-Sparrow

In 1967 Torrance and Sparrow presented one of the first BRDF models to capture such 
phenomena as the off-specular peak, as well as providing good predictions as $\theta_r \to 90^\circ$ [59]. 
The Torrance-Sparrow (T-S) model is developed on the basis of geometrical optics, and as 
such it requires the RMS surface roughness, $\sigma_m$, to be comparable to or greater than the 
wavelength considered ($\sigma_m/\lambda \gtrsim 1.0$). As with many BRDF models, the contributions of in-
dividual microfacet reflections to the overall material BRDF is considered. Each microfacet 
of area $A_f$ is treated as a specular surface, for which the surface normals are distributed 
according to a gaussian probability distribution, $P(\alpha)$. The diffuse BRDF component of 
the BRDF arises from multiple microfacet reflections or internal scattering. Therefore, the 
reflected radiance, $L_r$, may be expressed as the sum of the specular and diffuse components 

$$L_r = L_{r,s} + L_{r,d} \quad (37)$$

with the diffuse component given by 

$$L_{r,d} = a L_i \cos \theta_i \quad (38)$$

where $a$ is a constant.

The specular reflection is obtained by considering the Fresnel reflection, $F$, off each mi-
crofacet according to Eqs (6 & 8). The significant advancement made by the T-S model was 
the introduction of a geometric attenuation factor, $G$, which enables masking and shadow-
ing. Masking is the blockage of specular reflections by adjacent microfacets while shadowing 
is the blockage of the illumination source onto a microfacet by adjacent microfacets. The
resulting BRDF from the T-S model is given as

\[ f_r = \frac{F(\theta_i', \hat{n}) A_f G(\theta_i,p, \theta_r,p) P(\alpha)}{4 \cos \theta_i \cos \theta_r} + \frac{a}{d\omega_i} \]  

(39)

where the second term is the diffuse component and the primed coordinate system is relevant to the microfacet normal. The \( \theta_{X,p} \) coordinates result from projections of \( \theta_i \) and \( \theta_r \) onto the plane determined by the facet and surface normals. \( \hat{n} \) is the complex index of refraction given by \( n - ik \).

So what parameters are required for producing T-S BRDF predictions? To obtain the Fresnel reflectance, \( \hat{n} \) is required. A roughness parameter, \( c \), which relates the distribution of facet slopes relevant to the normal plane is required. Note that \( c \) is manifested in \( P(\alpha) \) which is proportional to \( \exp(-c^2\alpha^2) \). T-S used a value of 0.05 which was justified based on fitting the data to experimentally-determined BRDF. While T-S makes use of first principles to model the BRDF, it nonetheless requires a parameter which they fit to experimental data.

### 4.3.2 Maxwell-Beard

The development of the Maxwell-Beard (M-B) BRDF model was originally motivated for use on painted surfaces [49]. The model development emphasizes BRDF prediction from IR laser sources (1–4 \( \mu m \)) with varying polarization states. As with the Torrance-Sparrow model, separate diffuse and specular contributions to the BRDF are considered, which Maxwell and Beard term surface and volume contributions.

With the surface model, only single reflections from the microfacet surface are considered. The distribution of the microfacets are obtained through a monostatic scan in which the detector and illumination source are co-located, or as close to the same position without subtending each other. The surface normals of each microfacet are defined as being oriented in the \( (\theta_N, \phi_N) \) direction. The measured signal of the monostatic scan may then be related to the density of microfacets which fall within the detector solid angle, giving a density of \( \Xi(\theta_N, \phi_N) \) which has units of \( \text{sr}^{-1} \). Reflection from the microfacets is given by the Fresnel reflectance, Eq (9).

In terms of these experimentally measured parameters, the surface model component of the BRDF may be expressed as

\[ f_{r \text{surf}}(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{R_F(\beta) f_r(\theta_N) \cos^2 \theta_N}{R_F(0) \cos \theta_i \cos \theta_r} \]  

(40)
where $R_F$ is the Fresnel reflectance, expressed in terms of half the angle between the source and receiver, where $2\beta$ is the bistatic angle or angle between the source and receiver. $f_r(\theta_\hat{N})$ is the BRDF from the monostatic scan through the zenith position (i.e., $-90^\circ \leq \theta_i = \theta_r \leq 90^\circ$). It is given by

$$f_r(\theta_\hat{N}) = \frac{R_F(0)\Xi(\theta_\hat{N}, \phi_\hat{N})}{4 \cos \theta_i \cos \theta_r}$$

from which the microfacet density function is obtained.

All information needed for Eq (40) is experimentally obtained. However, the Fresnel reflectance requires the complex index of refraction, $\hat{n}$, of the material. M-B assumed the surfaces were dielectrics, a reasonable assumption for the paint samples they were modelling, which makes $k \approx 0$ or $\hat{n} = n$. A value of $n$ in their study was estimated as $n = 1.65$ and was based upon experience with paint samples [49, § 7.2, p. 56]. As an alternative, M-B indicate the value of $n$ may be calculated based upon Brewster’s angle, $\theta_B$, or the angle of incidence where the $P$-polarization component is minimum (§ 2.2).

Using Eq (40), M-B then derived what the monostatic BRDF should be with a fixed incident angle. Systematic variations were found which were attributed to shadowing and masking of the microfacets, previously addressed in the Torrance-Sparrow model discussion. However, M-B developed their own empirically-derived function to account for shadowing and obscuration (SO), which they found superior to the Torrance-Sparrow function. The SO function has two free parameters, $\tau$ and $\Omega$ [49, p. 10] and is given by

$$SO(\tau, \Omega) = \frac{1 + \frac{\theta_\hat{N}}{\Omega} e^{-2\beta/\tau}}{1 + \frac{\phi_\hat{N}}{\Omega} \left(1 + \frac{\phi_\hat{N}}{\Omega} \right)}$$

where $\phi_\hat{N}$ is a “factor calculated from the geometry, which adjusts the fall-off rate of the shadowing and obscuration function in the forward-scattered direction” [49, p. 10]. With this modification, the M-B surface BRDF is given by

$$f_{\text{surf}}(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{R_F(\beta) f_r(\theta_\hat{N}) \cos^2 \theta_\hat{N}}{R_F(0) \cos \theta_i \cos \theta_r} \times SO(\tau, \Omega)$$

Maxwell and Beard then develop a volume component of the model. The non-Lambertian volume component development was motivated by experimental observation that the diffuse scatter component was in fact not Lambertian, both from the angular dependency and the lack of complete depolarization. The non-Lambertian volume component accounts for subsurface scatter, or the type B and C photons in Figure 4. Derivation of this volume component considers the exponential loss via scattering of energy as the light propagates.
into the medium, as well as the exponential loss of energy as the light propagates back to the surface. It is assumed there is no net transmission of energy through the surface, and absorption in the medium is not explicitly considered. Given these considerations, the parametric volume component of the BRDF is given as

\[ f_{\text{vol}} = 2 \rho V f(\beta) g(\hat{\theta}_N) \cos \theta_i + \cos \theta_r \]  

(44)

where \( f(\beta) \) and \( g(\theta_N) \) collectively include the \( \beta \) and \( \hat{N} \) dependencies, and are treated as free parameters for adjustment based upon the empirical data. \( \rho V \) is a constant which equals \( f_{\text{vol}} \) when \( \theta_i = \theta_r = 0^\circ \) and \( f(\beta) = g(\theta_N) = 1 \). It is experimentally obtained by measuring the BRDF at \( \theta_i = \theta_r = 0^\circ \) with the incident light polarized orthogonal to the detector filter [49, pp. 16, 57]. However, the computer model implemented by M-B kept the free parameters \( f(\beta) = g(\theta_N) = 1 \), and simply cite these parameters may provide flexibility in future model development [49, p. 57].

The complete M-B BRDF model is given by the sum of the surface and volume components.

\[ f_r(\theta_i, \phi_i; \theta_r, \phi_r) = f_{\text{surf}} + f_{\text{vol}} \]

(45)

\[ = R_F(\beta) f_r(\theta_N) \cos^2 \theta_N \cos \theta_i \cos \theta_r + 2 \rho V f(\beta) g(\theta_N) \cos \theta_i + \cos \theta_r \]

(46)

So how is the model implemented? The monostatic scan is a necessary for determining the microfacet surface normal distribution function. For the microfacet Fresnel reflectance, an estimate of \( \hat{n} \) is required which may be estimated based upon Brewster's angle derived from experimental data. Using these as inputs, three parameters are fit to empirically-measured BRDF: two which model the shadowing and obscuration, \( \tau \) and \( \Omega \); and one which gives the volume component of the scatter, \( \rho V \). Values of these parameters used in the M-B paper for a green and tan paint were \( \tau = 15 \), \( \Omega = 40 \) and \( \rho V = 0.007 \) and 0.05. The author has no idea how \( \phi_N \) is determined for inclusion in the \( SO \) function, Eq (42)!

The Nonconventional Exploitation Factors (NEF) BRDF database and system (§ 3.7) uses a modified version of the M-B model [48, pp. 21-27]. The \( SO \) function is simplified and does not include the term in parentheses in Eq 42, hence eliminating the mysterious \( \phi_N \) term. The volume scattering parameters, \( f(\beta) \) and \( g(\theta_N) \) are also dropped. However, the NEF version allows the simultaneous inclusion of a Lambertian (\( \rho_d \)) as well as the non-Lambertian volume component of scatter. These modifications result in perhaps a more
useful form of the Maxwell-Beard model given by

\[ f_r(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{R_F(\beta)}{R_F(0)} \cos \theta_i \cos \theta_r \left( \frac{1 + \frac{\theta_i}{\Pi} e^{-2\beta/\tau}}{1 + \frac{\theta_r}{\Pi}} \right) + \rho_D + \frac{2\rho_V}{\cos \theta_i + \cos \theta_r} \]  

(47)

4.3.3 Sandford-Robertson

The Sandford-Robertson (S-R) BRDF model has its origins in infrared signature prediction for aircraft. Though developed for IR emittance, Kirchoff’s law enables the application to reflected energy as well since \( \varepsilon_0 = 1 - \rho_h \). The model is presented as given by Conant [60] and Jafolla [61], since the original document by Sandford is not easily acquired [62]. The model serves as the BRDF model for the SPIRITS image generation and radiometry code (§ 4.7).

The model uses four parameters: 1) strength of the diffuse reflectance (\( \rho_d \)), 2) the hemispherical-averaged emittance (\( \varepsilon \)), or equivalently, \( 1 - \rho_h \), 3) the rate of emittance decrease toward grazing-angle (\( b \)) and 4) the width of the specular lobe (\( e \)). Like the previous models, the BRDF is composed of a summation of specular and diffuse components.

From Kirchoff’s law, the hemispherical reflectance from a direction \( \theta \) equals

\[ \rho_h(\theta) = 1 - \varepsilon(\theta) \]

\[ = \rho_{hs}(\theta) + \rho_{hd}(\theta) \]  

(49)

which S-R divide into specular and diffuse components, \( \rho_{hs}(\theta) \) and \( \rho_{hd}(\theta) \).

The diffuse hemispherical reflectance is represented by

\[ \rho_{hd}(\theta_i) = \rho_d \frac{g(\theta_i, b)}{G(b)} \]  

(50)

where \( \rho_d \) is one of the model parameters controlling the strength of the diffuse reflectance. The \( g(\theta_i, b) \) function approximates Fresnel reflectance behavior or the grazing angle reflectance dependence and is given by

\[ g(\theta_i, b) = \frac{1}{1 + b^2 \tan^2 \theta_i} \]  

(51)

The \( G(b) \) function in Eq (50) is a normalization factor for the angular distribution of the energy and is given by

\[ G(b) = \frac{1}{1 - b} \left[ 1 + \frac{b^2}{1 - b^2} \log(b^2) \right] \]  

(52)
Based upon these expressions, the S-R diffuse BRDF component, \( f_d \), is given by

\[
 f_d(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{\rho_d g(\theta_i, b) g(\theta_r, b)}{\pi G^2(b)}
\] (53)

The specular portion of the BRDF is constrained by energy conservation of the hemispherical reflectance according to Eq (48), such that \( \rho_{hs}(\theta_i) = \rho_h(\theta_i) - \rho_{hd}(\theta_i) \). The specular BRDF component, \( f_s \), is given as

\[
 f_s(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{\rho_{hs}(\theta_i)}{4\pi} \frac{h(\beta)}{H(\theta_i, e) \cos \theta_r}
\] (54)

where \( \beta \) is the angle between the local surface normal and the incident angle (the same as that for the Maxwell-Beard model). The function \( h(\beta) \) describes the distribution of a surface of ellipsoids with eccentricity, \( e \) (another R-S model parameter) and is given by

\[
 h(\beta) = \left( e^2 \cos^2 \beta + \sin^2 \beta \right)^{-2}
\] (55)

The \( H(\theta_i) \) function provides the energy conservation normalization, such that the integral of the BRDF is equal to the hemispherical specular reflectance. It is represented as

\[
 H(\theta_i, e) = \frac{1}{2e^2} \left\{ (1 - e^2) \cos \theta + \frac{2e^2 + (1 - e^2)^2 \cos^2 \theta}{\sqrt{(1 - e^2)^2 \cos^2 \theta + 4e^2}} \right\}
\] (56)

The full expression for the S-R BRDF is therefore

\[
 f_r(\theta_i, \phi_i; \theta_r, \phi_r) = f_s(\theta_i, \phi_i; \theta_r, \phi_r) + f_s(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{\rho_{hs}(\theta_i)}{4\pi} \frac{h(\beta)}{H(\theta_i, e) \cos \theta_r} + \frac{\rho_d g(\theta_i, b) g(\theta_r, b)}{\pi G^2(b)}
\] (58)

4.3.4 Phong

Phong’s development of a reflection model was inspired from early (1970’s) advances in computer graphics, and realistic rendering of 3-D objects. His proposed “shading” technique provided interpolation between polygon surface normals used to render 3-D objects [63]. The Phong model is perhaps the most common BRDF function used in computer graphics, due to its simplicity and speed. Phong’s model uses a linear combination of specular and
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diffusely reflected components and is given by

\[ L = \rho(\lambda) [\cos(\theta_i) (1 - d) + d] + W(\theta_i) \cos^n(\theta_{sp}) \]  

(59)

where \( d \) is the environmental diffuse reflection component, \( W(\theta_i) \) is the ratio of specularly reflected light to that incident, \( \theta_{sp} \) is the angle between the specular direction and the observer and \( n \) is a power for modelling the specular reflected light. The values of \( W(\theta_i) \) and \( n \) are adjusted as free parameters to fit the data, without any physical basis for the adjustments. Typical values are \( 0.1 < W(\theta_i) < 0.8 \) and \( 1 < n < 10 \). When used strictly as a BRDF representation, the diffuse reflection component is zero (\( d = 0 \)).

4.3.5 Cook-Torrance

The Cook-Torrance (C-T) BRDF model has its heritage in computer graphics [64]. As with other models, the reflected radiance is a sum of the diffuse and specular components. The C-T model includes an additional ambient term, which results from large extended light sources (such as the downwelled radiance from the skydome, or equivalently to Phong’s environmental diffuse component, \( d \)). The ambient contribution is simply another linear combination of the diffuse and specular terms. Allowance is made for a shape factor in which part of the ambient illumination is blocked. A similar approach is used in remote sensing for nearby objects which occlude the skylight [5]. However, it should be noted that the addition of the ambient term is helpful for constructing realistic appearance in computer graphics, but not for interpolating BRDF values. Superposition is used to include multiple light sources in their model—which is of little utility for remote sensing (unless you happen to reside in a solar system with a binary star).

The model is similar to the Torrance-Sparrow BRDF model and includes the same geometric attenuation factor for shadowing and masking. Provision is made for alternative distributions of microfacets—one is not limited to the Gaussian distribution used in the T-S model. Additional spectral considerations are included in the C-T model. As the specular reflectance becomes more prominent at increasing incident angles, the spectral content of the specular lobe becomes increasingly similar to the source. When \( \hat{n} \) is not known for a given material, a near normal incidence measurement is made on a polished sample, from which an “effective” \( \hat{n} \) may be derived [Eqs (6, 8 and 9)].
4.4 Physical Models

A physical BRDF model based upon first-principles is now reviewed. Many physical models have their origin in physical optics and is manifested in the Kirchoff integral of scalar diffraction theory. Discussion of scalar diffraction is beyond the scope of this document, but a thorough treatment of the topic as applied to rough surfaces is available from Beckmann [65]. Discussion of physical BRDF models will be limited to the popular He model [66]. Like many other models, the He model has its roots in computer graphics.

The He BRDF model is a physical optics treatment of surface scatter which uses many of the same concepts as the semi-empirical models such as shadowing and masking. The model is developed from Beckmann’s general formulation [65], with the following inclusions: 1) the vector form of Kirchoff diffraction, thereby enabling polarization; 2) a surface roughness averaging scheme using a joint probability distribution function which includes height, slope, and two spatial points; 3) incorporation of an effective roughness due to illumination angle and 4) a geometrical shadowing factor.

He’s BRDF model is presented as a summation of three different components: specular-diffuse \( (sp) \), directional-diffuse \( (dd) \) and uniform-diffuse \( (ud) \).

\[
f_r = f_{r_{sp}} + f_{r_{dd}} + f_{r_{ud}} \tag{60}
\]

The \( sp \) component is due to the specular reflection by the mean surface orientation. The \( dd \) component is from diffraction by the surface roughness, analogous to the geometric optical treatment of the microfacet distributions. These first two contributions result from first surface (Fresnel) reflectance. Finally, the \( ud \) component is a constant which results from multiple scattering internal to the material.

Each of these components is represented as follows using previously defined variables where possible.

\[
f_{r_{sp}} = R_F \cdot e^{-g} \cdot S \cdot \delta(\theta_i - \theta_r) \tag{61}
\]

\[
f_{r_{dd}} = \mathcal{F}(\vec{n}_b, \vec{n}_b, \vec{p}) \cdot S \cdot \frac{\tau^2}{\cos \theta_i \cdot \cos \theta_r} \cdot \frac{1}{16\pi} \cdot \sum_{m=1}^{\infty} \frac{g^m \cdot e^{-g}}{m! \cdot m} \cdot \exp \left[ -\frac{(k v_{xy} \tau)^2}{4m} \right] \tag{62}
\]

\[
f_{r_{ud}} = \alpha(\lambda) \tag{63}
\]

where \( S \) is a shadowing function, \( g \) is a function of the effective surface roughness \( (\sigma) \) given
by
\[ g = \left( \frac{2\pi \sigma}{\lambda} \right) (\cos \theta_i + \cos \theta_r)^2 \] (64)

The delta function ensures the \( sp \) component is zero outside the specular reflection angle for the mean surface (per Snell’s law). \( F \) is a function dependent upon \( R_F \) (from Eq 9) and \( \mathbf{p} \), the incident polarization state. \( k \) is the wavenumber \( \left( \frac{2\pi}{\lambda} \right) \). \( v_{xy} \) is a function dependent upon the illumination and reflection angles and finally \( \tau \) is the surface roughness autocorrelation length. The explicit representations of \( S, F, \tau \) and \( v_{xy} \) are not given here.

As a surface becomes perfectly smooth, \( g \to 0 \) and \( S \to 1 \), so there is no attenuation of the \( sp \) component. Likewise, it is seen that the \( dd \) component contribution is zero as \( g \to 0 \), as expected. Finally, the free parameter \( \alpha(\lambda) \) representing the isotropic (Lambertian) component of scatter may be set out of energy conservation considerations, by using measurements of \( \rho_{DHR} \) for instance.

The fundamental parameters required for the He BRDF model are therefore the index of refraction \( \hat{n}(\lambda) \), surface roughness \( \sigma \), surface roughness autocorrelation length \( \tau \) and for practical purposes \( \rho_{DHR} \). So even for a highly physical BRDF model, the empirical quantity \( \rho_{DHR} \) is used.

He’s model has been criticized as being slow due to the series in Eq (62) which may require many terms to converge under some conditions. To address this, He later provided a means of pre-computing a look up table with minimal accuracy sacrificed [67].

4.5 Polarimetric Models

Polarimetric BRDF models are required for predicting the reflected polarized radiance given incident irradiance having arbitrary polarization. Polarized versions of some BRDF models have been created by incorporating the Fresnel reflectance contributions from the micro-facet surface representation. The Mueller matrix for each micro-facet is considered and related back to the global geometry, and a superposition of their polarized radiance contributions are made.

The Torrance-Sparrow BRDF model has been “polarized” by Priest [68], and the Sandford-Robertson BRDF model by Conant [60]. He’s BRDF model provides an intrinsic means of modelling polarization. Like He’s model, a new physical optics model by Duncan also provides polarization information [69]. Polarimetric BRDF models and their implementation are being investigated in much greater detail by the author.

\[ \text{N.B. the original paper contains an error in the expression for } f_{dd} \text{ (Eqs 7 and 78 in [66]) and does not include } k \text{ in the exponent.} \]
4.6 Model Performance

Earlier it was observed that the proliferation of BRDF models was in part due to the dissatisfaction of researchers with their performance. That being said, how well do the models perform? Some examples are present in the literature which provide some insight, though they are perhaps negatively biased in that most focus on a new model which provides an improved fit relative to “classical” models. The manner in which data is fit to a model is also a topic of discussion unto itself. Typical approaches are to derive model parameters which minimize the least square error of the fit, but algorithms which do so are subject to localized minimums. Some have used visual inspection of the fit, which may be appropriate when accuracy around one particular scattering geometry is desired. Note that discussion on the performance of heterogenous material BRDF models, which are suitable for most remote sensing scenarios, is postponed until § 5.5.

The performance of the Phong (§ 4.3.4), Cook-Torrance (§ 4.3.5) and Harvey BRDF models were investigated by Culpepper [70] for glossy white and black paints and mill finished aluminum. These materials are relatively specular with the BRDF spanning 3–4 orders of magnitude. Culpepper limited the measurements to the plane of incidence and desired a model to fit all the measured data to within 10%. However, differences from a factor of 2 to a factor of 10 were present with all the models at some point over the $-90^\circ \leq \theta_r \leq 90^\circ$ scan. Culpepper resorted to empirical interpolation to obtain the required accuracy. Perhaps it should have been anticipated that models would be challenged in achieving the desired accuracy given the dynamic BRDF range of specular materials.

Other publications present measured data which is fit to the Oren-Nayer model [71] and the Sandford-Robertson model [72].

4.7 Synthetic Image Generation

Synthetic image generation (SIG) programs have enabled the theoretical analysis of remote sensing algorithms and payload performance. This is completed at a minimal cost compared to field campaigns, and has proven beneficial for exercising many parameters which would not be feasible with experiments. SIG programs, unlike computer graphics programs and animations, place a premium on maintaining high radiometric accuracy. Essential to SIG programs is the inclusion of a radiometrically correct atmosphere, which is often calculated by well-established atmospheric radiation propagation programs such as MODTRAN. The Digital Imaging and Remote Sensing Image Generation (DIRSIG) model\textsuperscript{15} at the Rochester

\textsuperscript{15}http://dirsig.cis.rit.edu/
Institute of Technology’s Center for Imaging Science is one such example and renders scenes with a spectral coverage from 0.3 to 14 \( \mu m \) [73, 74].

Incorporating realistic background spatial and spectral signatures (i.e., non-target materials) into SIG models is challenging. Early efforts resulted in background materials being unrealistically spectrally pristine, thus artificially enhancing the performance of spectral detection algorithms. An array of “mapping” approaches have been made which provide varying levels of realism to individual material facets. These approaches include texture, mixture and reflectance maps ([75], [76, Ch. 4]). These mapping functions are analogous to the BTF (§ 2.5), and result from the natural variability within a material “class.”

Separate from the material maps, BRDF models may also be implemented for facets in the synthetic scene. The use of polarimetric BRDF models with the Mueller matrix formalism is also being pursued, and enables the analysis and comparison of polarization information content versus spectral information. DIRSIG was recently upgraded to include a polarimetric BRDF model based upon a modified Torrance-Sparrow BRDF model [68] which is detailed by Meyers [77]. Other SIG programs such as IRMA and SPIRITS (Spectral and Inband Radiometric Imaging of Targets and Scenes) have also incorporated polarimetric signatures.

IRMA calculates the Mueller matrix based upon the Fresnel reflectance given the complex index of refraction, and simply adds the diffuse reflectance to the \( m_{00} \) element (equivalent to adding the scalar BRDF). As an alternative, IRMA allows the manual entry of Mueller matrix data for \( \sim 10 \) different scattering angles [78].

The SPIRITS SIG code is owned by the U.S. Government and a polarized version, called “POLAR”, was first produced in 1986. When first developed, POLAR implemented polarized BRDF based upon the Maxwell-Beard model, distinct from the Sandford-Robertson BRDF model which is used with the unpolarized, SPIRITS, code [79]. However, the POLAR BRDF model failed to couple the specular and diffuse BRDF components, and was limited to dielectric materials. In the late 1990’s, the POLAR code was supplanted by a new version of SPIRITS which incorporated a polarized Sandford-Robertson BRDF model [60, 78].

The availability of polarimetric BRDF data continues to be a bottleneck in simulating polarimetric imaging through SIG. While there is some pBRDF data on homogeneous materials, appropriate background polarimetric signatures is lacking. The background materials are typically natural, heterogeneous materials such as vegetation and soils.

\[^{16}\text{http://dirsig.cis.rit.edu/doc/maps.pdf}\]
5 Remote Sensing

As seen from the previous discussion, the directional reflectance or BRDF of materials results in varying radiance values at the sensor dependent upon the illumination and observation geometry. Incorporating this information into the remote sensing imaging chain is challenging and will be explored by examining the sources of radiance reaching the sensor. Determining and applying BRDF data at the relevant spatial resolution or ground sample distance (GSD) of the system is also problematic and will be addressed. BRDF models specifically developed for remote sensing will also be examined.

5.1 Governing Scalar Radiance Equation

The impact BRDF has on the radiance reaching a sensor aperture may be understood if the radiance contributions are examined in some detail. The total radiance in the visible to near infrared (VNIR) portion of the spectrum (i.e. that of solar origin) reaching a sensor aperture ($L_s$) may be approximated as the sum of radiance values which have their origin in

1. solar reflections from the target, $L_r$
2. target-reflected downwelled radiance from the skydome, $L_d$
3. upwelled atmosphere radiance resulting from solar scatter in the atmosphere along the target to sensor path, $L_u$

The order of the radiance terms above is that which is typically decreasing in magnitude, though the ground or target reflectance and atmospheric conditions greatly influence their relative values. Additional discussion on these radiance values and the sensitivity of their relative magnitude may be found in Schott (Figure 4.12, Table 4.1) [5]. Therefore, a simple expression for the radiance reaching a sensor aperture is given by

$$ L_s = L_r + L_d + L_u $$

More detailed expressions for the individual radiance components are given by Eqs (66a–66c) where the explicit functional dependencies are included. Note the wavelength dependency will not been explicitly shown, but all radiance values are spectral radiance
having units of \( \frac{W}{sr \cdot m^2 \cdot \mu m} \).

\[
L_r = f_r(\theta_i, \theta_r, \phi) \ E_s(\theta_i) \ \tau_i(\theta'_i) \ \cos \theta_i \ \tau_r(\theta'_r, \phi') \ \tag{66a}
\]

\[
L_d = \int_0^{2\pi} \int_0^{\pi/2} f_r(\theta_i, \theta_r, \phi) \ E_d(\theta_i, \phi) \ \tau_r(\theta'_r, \phi') \ \sin \theta'_i \ d\theta'_i d\phi' \ \tag{66b}
\]

\[
L_u = L_u(\theta_r, \phi) \ \tag{66c}
\]

In these expressions the primed coordinate system is the global system, or that relative to the normal of the plane of the horizon of the earth. As before, the unprimed coordinates are relative to the individual material surface normals for which the BRDF \( f_r \) is defined. \( E_s(\theta_i) \) is the incident exo-atmospheric solar irradiance, which is approximated as a point source. \( E_s \) is attenuated by the atmospheric transmission to the target, \( \tau_i(\theta'_i) \). Similarly, \( E_d(\theta_i, \phi) \) is the downwelled irradiance from the entire hemisphere (minus the solar disk) above the target and \( \tau_r(\theta'_r, \phi') \) is the transmittance along the target-to-sensor path. The limits of integration encompass the entire hemisphere in the earth-based, or primed coordinate system. (Prior to evaluating \( L_d \), the local target surface normal coordinate system must be converted to the primed coordinate system).

Note that the BRDF plays a direct role in \( L_r \) and \( L_d \). As a reminder, for a completely diffuse target, the BRDF function in Equations 66a and 66b is not directional and may be replaced with \( \rho(\lambda) \), or that of a Lambertian surface. Details of the upwelled radiance, \( L_u \), are not shown, but depend upon atmospheric parameters such as the scattering phase function. Often \( L_u \) and \( E_d \) are obtained through the use of atmospheric propagation codes, such as MODTRAN, which is also used to calculate the atmospheric transmission terms \( \tau_i \) and \( \tau_s \).

Most often, Lambertian targets and backgrounds are assumed, resulting in solving for the spectral reflectance \( \rho(\lambda) \) given \( L_r \). This approach does not require knowledge of the illumination-target-sensor geometry for calculating \( L_r \) and \( L_d \), other than the incorporation of the atmospheric attenuation, \( \tau \). However, as stated in the introduction, the failure to consider the BRDF properties results in a poor derivation of \( \rho(\lambda) \) given \( L_r \). At this point one is presented with a quandary—incorporation of the BRDF function requires an assumption on the target geometry. This is not an issue for many overhead systems, which are nadir looking and have modest GSDs of 10s of meters or more, in which case the integrated area normal vector lies near zenith (unless in very rugged terrain). However, in the case of resolved targets where the GSD is \( \sim 1 \) m or less, some assumptions must be made. The most reasonable assumption is that, again, the target normal points toward zenith, which results
in the material surface coordinates coinciding with the earth-based coordinate system (i.e., \( \theta_r = \theta'_r \)).

The objective of quantitative spectral remote sensing may be summarized as the task of obtaining as accurate an estimate as possible of the spectral BRDF of the individual image pixel. That is, solving for \( f_r \) (and hence \( \rho(\lambda) \) for the solar-target-sensor geometry) given the aperture-reaching radiance, \( L_s \). Once this is accomplished, an array of spectral algorithms may be employed \cite{80} such as matched filters based upon spectral BRDF libraries of materials. Such algorithms provide autonomous classification of land cover to potentially include species of vegetation. It has been reported that BRDF has a more important role in improving spectral classification algorithms than does hyperspectral resolution for some scenarios \cite{81}.

One general result which is typical of including the BRDF is a reduction in the target radiance relative to the background radiance. The source of this change results from many “target” materials being man-made, which typically are more specular than the natural, background materials. This is of course not always the case—when the target normal vector lies in the same plane as the sun-to-target and target-to-sensor vector, the inverse may easily occur if \( \theta_r \) is \( \sim \theta_i \), or within the specular lobe. It is suggested that in general, target detection algorithm performance decreases as the background (non-target materials) and target BRDF differences increase. If one had truly Lambertian targets and background materials, no performance degradation is incurred.

### 5.2 Governing Polarimetric Radiance Equation

At this point, only the scalar BRDF of materials has been considered. A more generalized treatment may include the polarized BRDF, where \( f_r \) in Equations \( 66a \) and \( 66b \) is replaced with the material Mueller matrix for that given geometry. Mueller matrices also replace the atmospheric transmission terms, \( \tau_i \) and \( \tau_s \). The result is each of the radiance terms in Equation \( 65 \) becomes a four component Stokes vector as discussed in Section 2 containing all the radiance information. The equivalent polarimetric radiance components are now presented as

\[
\vec{L}_r = \mathbf{M}_r(\theta'_r, \phi'_r) \mathbf{M}_r(\theta_r, \phi) \mathbf{M}_r(\theta'_i) \cos \theta_i \vec{E}_s(\theta_i) \tag{67a}
\]

\[
\vec{L}_d = \int_0^{2\pi} \int_0^\pi \mathbf{M}_r(\theta_i, \theta_r, \phi) \mathbf{M}_r(\theta'_r, \phi'_r) \vec{E}_d(\theta_i, \phi) \sin \theta'_i \, d\theta'_i \, d\phi' \tag{67b}
\]

\[
\vec{L}_u = \vec{L}_u(\theta_r, \phi) \tag{67c}
\]
where the terms in Eq 67a and 67b have been arranged in the order of the propagation of light such that the non-communicative Mueller matrix multiplication is correct.

Fortunately, further simplification of Eqs (67a–67c) is possible. The solar irradiance, \( \vec{E}_s(\theta_i) \), is unpolarized and as such keeping a simple scalar value is warranted. In addition, line-of-sight propagation of light through the atmosphere preserves the polarization. Given this, scalar values for \( \tau_i \) and \( \tau_s \) may be reverted to without having to use the generalized Mueller representation. However, atmospheric scattering, or propagation which is not line-of-sight does impart polarization and for this reason the Stokes vector must be retained for \( \vec{E}_d(\theta_i, \phi) \) and \( \vec{L}_u(\theta_r, \phi) \). These simplification are shown in Eqs (68a–68c).

\[
\vec{L}_r = \tau_r(\theta'_r, \phi') \ M_r(\theta_i, \theta_r, \phi) \ \tau_i(\theta'_i) \ \cos \theta_i \ E_s(\theta_i) \quad (68a)
\]

\[
\vec{L}_d = \int_0^{2\pi} \int_0^{\pi/2} M_r(\theta_i, \theta_r, \phi) \ \tau_r(\theta'_r, \phi') \ \vec{E}_d(\theta_i, \phi) \ \sin \theta'_i \ d\theta'_i d\phi' \quad (68b)
\]

\[
\vec{L}_u = \vec{L}_u(\theta_r, \phi) \quad (68c)
\]

As with the scalar radiance reaching the sensor aperture, these vector terms may be summed to provide the polarized radiance reaching the sensor aperture.

\[
\vec{L}_s = \vec{L}_r + \vec{L}_d + \vec{L}_u \quad (69)
\]

Now, rather than attempting to estimate the scalar BRDF, the objective is deduction of the target reflectance Mueller matrix, \( \mathbf{M}_r \) for each pixel in the scene, which may potentially be used to enhance spectral target-detection algorithms.

### 5.3 Spatial Scale

One of the first considerations made in the application of BRDF data to remote sensing is determining over what spatial extent is the data required? Recall the painted metal plate and the bidirectional texture function (BTF). Can the payload resolve the individual paint bubbles and flakes? If it can’t, then the integrated BRDF for the painted metal plate is appropriate. If individual features of the plate may be resolved, then a more appropriate BRDF should be considered. For remote sensing and imaging in general, the spatial resolution of the BRDF must be at least as small as the sensor GSD.

In general, the radiance reaching a single detector will not result from a homogeneous material, but rather from a collection of many homogeneous materials with many different geometric orientations. For example, consider a 10 × 10 m section of a soybean field. The
sensor-reaching radiance from this section will be due to multiple homogeneous materials such as leaves, dirt and rocks. Of course these “homogeneous” materials really aren’t—leaves may be decomposed into the veins and flesh areas, dirt into different organic materials and silicates, etc. This process may of course be continued *ad nauseam*.

For many overhead systems, the BRDF of multiple materials are integrated, as the GSD is on the order of 10s of m or greater. However, higher resolution systems often have GSDs of the same scale as many natural objects, such as individual trees, shrubs and leaves. For target detection applications, the BRDF measurements should be at an appropriate scale which characterizes the key identifying materials for that target. Additional fidelity may be obtained by also characterizing the background directional reflectance at the same GSD.

As will be seen, the spatial scaling of BRDF values from individual “fundamental” material measurements and geometries (e.g., a leaf canopy) is challenging and an active area of research [82]. Large area BRDF measurements which integrate multiple materials may be more effective than attempting to scale single material measurements, whose radiance contributions are often not linear. Additional discussion on this topic is provided in § 6.

### 5.4 Remote Sensing BRDF Models

Remote sensing BRDF models are specifically tailored for heterogeneous materials. Natural surfaces and landcover contain significant variability, which is difficult to capture with homogeneous material BRDF models (§ 4). The spatial resolution (GSD) of satellite-based remote sensing platforms has historically been limited to 10s of m (e.g., Landsat). For most regions of the earth, this results in a per-pixel radiance value due to the integration of many unidentified landcover features. It is from this perspective that remote sensing BRDF models have been developed. A special issue of *Remote Sensing Reviews* (Vol. 18, 2000) provides several articles of interest, which originated from papers and discussions at the International Forum on BRDF held in December 1998.

Remote sensing BRDF models serve many functions. One purpose is the normalization of radiance from multi-angle views. It is standard practice to normalize oblique angle data to that of nadir viewing. This enables improved comparisons to other data sets of the same region. BRDF models are also instrumental in deriving surface DHR, or the albedo, for input to climatology models and modelling the planet’s thermal exchange. Finally, as with homogeneous BRDF models, model parameters may be linked to extracting information from the scene such as leaf area index (LAI) or leaf area distribution. Often, remote sensing models must be used to approximate a full hemispherical BRDF representation when only
a few measurements are made, such as with a single pass of POLDER or or a few passes of an off-nadir viewing payload like MODIS (§ 3.5).

The general situation considered is a vegetative canopy region over a ground material. The canopy may have varying crown and leaf orientations, analogous to microfacet surface variations for homogeneous model. However, leafy materials have significant transmittance and multiple interactions which complicate the radiometry. The vegetative canopy may also have varying spacing or areal density. A heavily forested region may have the entire surface covered by tree crowns, with little ground visible, while other areas may have sparse trees allowing the ground to drive the signature. Note that new commercial sources of high resolution (GSD ~ 1 m) satellite imagery such as Space Imaging® and DigitalGlobe® have enabled resolved surface features, providing a priori knowledge of the land cover type such as “trees” or “grass.”

As with homogeneous material BRDF models, a range of empirical to physical models have been developed. Full empirical models are often not useful for satellite-derived observations, since the geometric sampling density is very sparse. Physical models provide radiometric modelling of the surface and enable extraction of parameters having physical significance. Finally, semi-empirical models provide a hybrid approach from which fits to empirical data provide some physical basis for the BRDF. Good reviews of remote sensing BRDF models are provided in an extensive treatment by Goel [83] which was summarized and updated in [84]. Additional overviews of BRDF models are given by Strahler [85] and [86].

The discussion will be limited to semi-empirical models, which are often composed of a summation of linear contributions expressed as

\[ f_r(\theta_i, \theta_r, \phi) = \sum_n a_n K_n(\theta_i, \theta_r, \phi) \]  

where \( n \) typically ranges from 2 to 10. Many models have \( n = 3 \) parameters with the \( K_n \) terms consisting of isotropic, surface and volumetric functions. The \( K_n \) contributions are called kernels from which these models are also called kernel-driven or kernel-based models [86]. The kernels provide different geometric expressions for a particular type of BRDF contribution. Detailed examination of a particular BRDF model by Roujean provides further details on the kernel representation.

The semi-empirical Roujean BRDF model [87] was among the first kernel-based models and has maintained popularity due to its versatility and accuracy for modelling a number of land cover types. The BRDF is quantified by a geometric and volumetric scattering
component. The geometric scattering component models randomly placed vertical, opaque “long-wall protrusions” on a flat horizontal plane [87]. Both the protrusions and the background plane are assumed to be Lambertian surfaces. Increasing zenith angle illumination produces increased shadowing of the background plane according to the protrusion height. However, the protrusions are distributed such that mutual shadowing of one protrusion onto another is not considered.

The volumetric component models randomly placed facets which absorb and scatter radiation according to a simple radiative transfer model. The facet distribution is treated as isotropic, with a specified density. Several assumptions are made in deriving the volumetric term—only single scattering is considered and the facet reflectance and transmittance are treated as being equal. This last assumption is a gross one, but was necessary to reduce the number of free parameters in the model to three.

All scattering results from the geometric and volumetric components, which are considered independent. The resulting BRDF model, is given by

\[
f_r(\theta_i, \theta_r, \phi) = k_0 + k_1 f_1(\theta_i, \theta_r, \phi) + k_2 f_2(\theta_i, \theta_r, \phi)
\]

(71)

where the geometric scattering results from \(k_0 + k_1 f_1\) and the volumetric scattering is contained in \(k_2 f_2\). Roujean used the function in terms of reflectance units, which is proportional to the BRDF \((f_r \cdot 100\pi)\).

The \(f_1\) function is derived from geometric considerations of the protrusion placement and without further derivation is given by

\[
f_1 = \frac{1}{2\pi}[(\pi - \phi) \cos \phi + \sin \phi] \tan \theta_i \tan \theta_r
- \frac{1}{\pi} \left(\tan \theta_i + \tan \theta_r + \sqrt{\tan^2 \theta_i + \tan^2 \theta_r - 2 \tan \theta_i \tan \theta_r \cos \phi}\right)
\]

(72)

The \(f_2\) function has its origin in a simple radiometry model, which includes some of the previously mentioned assumptions. It is given by

\[
f_2 = \frac{2}{3\pi} \frac{(\pi - 2\xi) \cos \xi + \sin \xi}{\cos \theta_i + \cos \theta_r} - \frac{1}{3}
\]

(73)

where \(\xi\) is the scattering angle between the incident and scattering directions (called the phase angle by some) defined by

\[
\cos \xi = \cos \theta_i \cos \theta_r + \sin \theta_i \sin \theta_r \cos \phi
\]

(74)
Figure 18: Roujean model results fit to empirical data for a wheat field in the visible for $\theta_i = 27^\circ$, where the magnitude is normalized to reflectance units which replicates the results in [87, Fig. 7, p. 20463]. The $x$-axis is $\theta_r$ and the $y$-axis is $\phi$—note the principle plane is $\phi = 0^\circ$ and $\phi = 180^\circ$. (Ideally, this data would be presented in cylindrical coordinates for easier interpretation). The peak reflectance is noted at the “hot spot” or retro-reflection location. Parameter values are $k_0 = 27.3$, $k_1 = 5.2$ and $k_2 = 26.9$.

Roujean’s convention is for $0^\circ \leq \phi \leq 180^\circ$ and $0^\circ \leq \theta_r \leq 90^\circ$, where the backscattering orientation is $\phi = 0^\circ$ (which is opposite the convention defined earlier where $\phi = 180^\circ$ is the backscattering orientation). The full derivation of these functions is lengthy, to which the reader may reference [87].

The free parameters, $k_0$, $k_1$ and $k_2$ have some physical significance. $k_0$ correlates to an isotropic Lambertian contribution and is equivalent to the reflectance when $\theta_i = \theta_r = 0^\circ$. $k_1$ is related to the protrusion height, length and width while $k_2$ is related to the facet reflectance and area (e.g., canopy leaf reflectance and coverage, or leaf area index). The model was implemented in a computer program, and some of Roujean’s results are replicated from parameters in the paper as seen in Figure 18.

5.5 Remote Sensing BRDF Model Performance

The performance of several BRDF models, most of which are remote sensing models, is reported by Boucher [30] and Hautecoeur [88]. Boucher made measurements on sand, “dry” grass, and “green” grass at 600 and 800 nm. Significant in this work is the quantification
of model performance for different azimuth sampling densities. The Roujean model [87] provided one of the best performances, and had minimal errors for all the materials at both wavelengths when at least 25 hemispherical samples were acquired (for a single $\theta_i$ value). The Roujean model also provided good performance when measurements were limited to only three azimuth planes: $\phi = 0^\circ$ (or $180^\circ$), $60^\circ$ and $120^\circ$. Roujean’s model was also commended by Hautecoeur [88], having the best performance of the semi-empirical models surveyed.

5.6 Old vs. new “yellow school buses”

Sad to say, but once BRDF data and models are in place for employment in spectral algorithms, the job is only partially complete (though what a job it was). More often than not, the BRDF characterization for material X does not represent material X in the field. Weathering of the material alters the optical properties via UV “washing,” oxidation, surface abrasions, etc. Also, the material may be covered by dust and other particulates—or perhaps mud for tactical military vehicles. A cursory examination of these effects has been made by Thomas [89].

It’s worth noting that since material weathering and/or degradation impacts the BRDF, it also impacts the general “reflectance” of a material [Eq (13)]. This is readily apparent by visual inspection of materials. The result is that algorithms using simple spectral reflectance are also impacted.
6 Proposed Measurement Technique

A recommendation for a spectral BRDF measurement technique is now made after having reviewed BRDF theory, measurements, models and the unique application to remote sensing. The technique is motivated in part by the advent of higher spatial resolution systems, which resolve many of the natural features traditional remote sensing BRDF models described as a mixed pixel (§ 5.3 & 5.4). Another impetus is the inadequacy of traditional measurement techniques to capture the BRDF variability (or BTF, § 2.5) which is present in all natural, heterogeneous targets.

The system implicitly enables the generation of spectral BTF or texture statistics at varying spatial resolutions. In this manner, appropriate BTF for the sensor of interest may be considered. The technique is developed with outdoor measurements in mind, as this is the most challenging; however, it is equally applicable for laboratory measurements.

As seen in Section 3.4, many of the so-called “portable” BRDF measurement devices for outdoor use are rather massive and require extensive transport and set-up time, or they require large, homogeneous areas of the material of interest to enable a measurement. The recommended system is extremely portable and easy to use, but makes some sacrifices in accuracy, as the geometric angular resolution and hemisphere sampling density is sparse compared to other techniques. However, these deficiencies are quite acceptable when one considers other uncertainties in the imaging chain, such as the previously discussed background and target surface orientation.

6.1 System Description and Use

In Section 3, imaging systems were shown to provide a valuable means for the simultaneous acquisition of multiple view angles. However, an alternative use of an imaging technique is proposed. The proposed system is essentially the inverse of that used by Dana as discussed in Section 3.3. Rather than using imaging techniques to acquire multiple, simultaneous geometries, it may be used to acquire the BRDF at one approximate geometry, from which the BTF or texture metrics may be generated from the BRDF image. This system may also be viewed as a traditional BRDF measurement, but replacing a single element detector with a focal plane.
6.1.1 FOV and standoff distance determination

Imaging with a narrow field of view (FOV) camera produces the equivalent of a single BRDF measurement. The BRDF of a sample imaged in this manner is directly proportional to the average irradiance across the focal plane, or the average digital count (DC) value for a spectral band across the focal plane. Positioning and imaging with the camera at locations in the scattering hemisphere provides geometric coverage in the same manner as a single detector. The high frequency or texture variation is manifested in the details captured at levels less than the full frame of the camera down to the individual pixel or instantaneous field of view (IFOV, more on this below). The angular resolution of the BRDF measurement, $\Delta \theta_r$ and $\Delta \phi_r$, is therefore limited by and equivalent to the camera FOV.

The selection criteria for an appropriate FOV is a function of three interrelated criteria:

1. minimizing the angular resolution, $\Delta \theta_r$ and $\Delta \phi_r$,

2. maximizing the spatial extent or ground field of view (GFOV) of sample measurement and

3. a reasonable camera to sample distance for ease of repositioning and adjustments.

The importance of criteria 1 is determined by the magnitude of the derivative of the BRDF with respect to $\theta_r$ and $\phi_r$. Significant changes in the BRDF are characteristic of specular materials near the specular lobe. Fortunately, most natural materials in the VNIR are not appreciably specular.

The sample size (criteria 2) is determined by the variation or texture scale of that material. For instance, a painted automobile surface may have the texture sufficiently averaged out over a 1 in $\times$ 1 in area, while grass may require 1 ft $\times$ 1 ft area and shrubs a 10 ft $\times$ 10 ft.

Criteria 3, the camera to sample distance, will mostly drive the ease of repositioning the camera in the hemisphere above the sample. While an increased standoff distance may result in decreasing $\Delta \theta_r$ and $\Delta \phi_r$ and/or potential increases in the spatial area measured, it may become increasingly difficult to transport and quickly set up the system. Easy access to the camera at all hemispherical positions would limit this distance to $\sim 6$ ft.

A qualitative estimate of a good compromise of these variables is a camera having a $10^\circ$ (full angle) FOV. For nadir viewing ($\theta_r = 0^\circ$) of a sample from a distance of 6 ft, this corresponds to a GFOV $\approx 1$ ft $\times$ 1 ft. This configuration is estimated to cover most measurements of interest. If this measurement area is not adequate for a material
6.1.2 Radiance calibrations

Calibration of radiance levels will be ensured by characterizing the camera response for nonlinearities and imaging a Spectralon® or pressed halon calibration surface prior to and after each measurement. Being a perfectly (or nearly so) Lambertian surface with \( \rho(\lambda) \lesssim 1 \), the BRDF of the halon will be \( \approx \frac{1}{\pi} \text{sr}^{-1} \). However, measurement of the calibration target will also contain contributions from the scattered downwelled skydome radiance.

The sky radiance may not be eliminated, but the direct solar irradiance may. Imaging the calibration target when in shadow from the sun will provide a measure of the downwelled radiance contributions. Therefore, the radiance received in full sun and when illuminated by the sky, minus the radiance with only the sky results in reflection due to the direct solar radiance. In terms of digital counts, deriving the Lambertian BRDF in this manner is expressed as

\[
DC_{\text{Lamb}} = DC_{\text{sky+sun}} - DC_{\text{sky-sun}} = \pi^{-1} \text{ sr}^{-1}
\]

(75)

The initial measurement and camera exposure adjustments should be made at the geometry where the signature is greatest in order to ensure the dynamic range of the camera is not exceeded on subsequent measurements, as changes in the camera response/exposure settings are not desired.

A sample image of grass acquired using this measurement technique is now discussed and used as an example (Figure 19). An arbitrary GSD unit is also shown in the figure and will be used for subsequent discussion on texture and the adequacy of the image to capture the BRDF. The sample or image is \( \sim 1 \text{ ft} \) across making the representative GSD unit approximately 3 in.

6.1.3 Hemispherical sampling strategy

Any strategy devised for sampling the hemisphere above the material must consider the total time required to complete all measurements. Depending on the latitude and time of year, the sun has an apparent angular rate of up to 15°/hr. Changes in the incident solar angle should be kept less than or commensurate with the reflection angle resolution (FOV) or \( \Delta \theta_r \approx \Delta \phi \approx 10^\circ \). For materials with azimuthal symmetry, only changes in
θ_i need be considered. The rate of change in θ_i is a function of the time of day and reaches a minimum at “solar noon” or the time of day corresponding to the maximum sun height. If measurement campaigns are limited to ±3 hrs relative to solar noon, then a total acquisition time of ≤1 hr is required for mid-latitude sites. Early morning or late afternoon measurements will require a shorter measurement duration to maintain an equivalent Δθ_i.

Camera positioning in the hemisphere without a goniometer may be accomplished by simple predetermined trigonometric measurements such that a constant radius is maintained. (The mechanics of appropriate structure(s) necessary for accomplishing this task will not be addressed here). It is anticipated that a minimum of 5 zenith positions over 4 azimuth planes be measured. For a material with azimuthal symmetry, an appropriate sampling density would be θ_r = 0°, 15°, 30°, 45° and 60° at φ_r = 0°, 60°, 120° and 180°, or a minimum of 17 measurements (considering the redundancy at θ_r = 0°). There is not much need for BRDF data for θ_r > 60° for target detection algorithms.

In addition, the use of real-time feedback on the changes in BRDF with hemispherical repositioning may be used to acquire additional measurement positions where there is significant change, such as in the retro-reflection (hot-spot) position or forward scattering lobe location for specular materials. Including these few additional measurements, the total number of hemispherical samples is ∼20. This sampling density results in an average measurement time which must be less than ∼3 min per hemisphere location, to maintain a Δθ_i ≤ 10°.
6.1.4 Downwelled radiance compensation

When making outdoor measurements, the diffuse downwelled skydome radiance, $L_d$, becomes stray light for the purposes of the BRDF measurement. Various techniques may be considered for eliminating contributions from $L_d$ to the signal received by the camera. An optically thick and high absorption cover could be used over the entire measurement area to block all but the solar disk, however, this is not a practical solution. An alternative may be to use PVC pipes with a flat black coating on the interior. The target area would be shadowed by the pipe from everything but the solar disk, and a similar pipe would be required between the target and camera. Again, this is also not practical and internal reflections from the pipe due to the high angles of incidence may cause additional problems.

An easier means of stray light correction is possible. Analogous to the manner in which the calibration target was imaged, the BRDF image should also be made while occluding the sun. As before, the image made while blocking the sun is the radiance scattered by the material/target of interest by the downwelled skydome. A simple subtraction of this value from the image with the sun and skydome may be made to yield the radiance reaching the camera from direct solar ($L_r$) contributions only, assuming no changes in the skydome radiance distribution between the measurements.

It is recommended that the measurements first acquire an image while blocking the sun, then image under full sun, followed by another image while blocking the sun. Acquiring two images of the downwelled radiance will provide a measure of downwelled radiance changes which may have occurred during the solar reflectance measurement. This is particularly important for specular materials where a cloud may have entered the specular bi-static angle during a measurement, which could significantly bias the measurement. Others have noted similar techniques to compensate for the incident skydome irradiance [24, 25] and other potential error sources [26], such as wearing absorbing (dark) clothing during the measurements.

The sequence of events for a measurement using this technique is shown in Figure 20. A simple wide FOV RGB image of the skydome may also be made periodically to provide a visual reference of the sky conditions.

6.1.5 Measurements with an overcast sky?

Finally, the use of this stray light correction technique potentially lends itself to outdoor measurements under cloudy conditions. However, the signal strength of the source must be appreciable. Also, the spectral irradiance of the source relative to that of the diffuse
Figure 20: Step-by-step process for BRDF measurements using the recommended system.
skydome needs to be considered. The incident irradiance on the target area should be
greater than or equal to the solar insolation, which is approximately 1 kW/m\(^2\). For the
notional 10° FOV setup previously described, the incident VNIR power in the target area
should be greater than 100 W. Higher spectral irradiance values than that delivered by the
sun will improve the measurements relative to those that may be made with the sun.

Often cloud cover is not dense, which results in a temporally varying magnitude of
direct solar irradiance incident upon the target. In these circumstances, alignment of the
illumination source adjacent to the solar incidence angle is recommended.

A better solution to competing with the diffuse sky radiance is to make the measure-
ments at night. Calibration of stray light may still be required if nearby illuminants are
present and the source signal is modest. Even at night, irradiance uniformity over the tar-
get and divergence of the irradiance source from a planar field are additional complications
which must be managed.

6.2 GSD Sample Statistics Generation

The measurement technique described above readily lends itself to derivation of the texture
or BTF statistics which may be used in algorithms and rendering synthetic imagery. For
example, consider the GSD unit in Figure 19, (which in reality is approximately 3 in). The
variability or texture of this grass material for a system having a GSD=1 may be derived
by examining the moving average statistics of the area underneath this GSD=1 square.
This is mathematically equivalent to convolution which is given as

\[
g[x, y] = f[x, y] * h[x, y] = \int \int_{-\infty}^{+\infty} f[\alpha, \beta] \ h[x - \alpha, y - \beta] \ d\alpha \ d\beta
\]

(76)

where \(f[x, y]\) is the image and \(h[x, y]\) is the convolution kernel. For the case of a digital
image with pixels (i.e, non-continuous), the discrete convolution is

\[
g[x, y] = \frac{1}{X^2} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} f[x, y] \ h[x - i, y - j]
\]

(77)

For simple local averaging, the convolution kernel is a square of size \(X \times X\) pixels corre-
sponding to the fractional GSD of interest. The convolution kernel has a magnitude of “1”
everywhere inside the GSD box and zero outside (i.e., RECT function). The \(\frac{1}{X^2}\) factor in
Eq. (77) compensates for the summed values of the kernel and hence enables the equation
Figure 21: Local averaging with the image used for BRDF statistics generation. At left is the original $400 \times 400$ (GSD=0.01) image followed by images averaged to $40 \times 40$ (GSD=0.1), $10 \times 10$ (GSD=0.4), $4 \times 4$ (GSD=1.0) and $1 \times 1$ (GSD=4.0).

to return the DC value of the area under the kernel. Note that a simple RECT function is used here, but the actual point spread function (PSF) of the imaging system in question may be used to provide a sensor-specific BTF.

Using this approach, BTF statistics may be generated from a BRDF image. Consider the $400 \times 400$ pixel image of grass shown in Figure 19. If a GSD of 1 is defined as a $100 \times 100$ area of the image, a convolution kernel of this GSD size may be used to generate an image, $g[x, y]$, of local averaged values. The size of $g[x, y]$ is determined by the size of the convolution kernel and original image. For a convolution kernel of $100 \times 100$ as previously discussed, $g[x, y]$ will be $400-100+1$ or $301 \times 301$. Increasing the GSD of interest will result in more averaging, while decreasing will result in more variability. For instance, if the convolution kernel were the same size as the original image, then a single value (pixel) corresponding to the global image average would be returned. At the other extreme, a convolution kernel of $1 \times 1$ would return the original image. By this means, the texture or BTF information (§2.5) may be extracted directly from the image.

Figure 21 provides a direct representation of this process. The original image used to determine “grass” BRDF is shown at left. The other images correspond to increased averaging by selection of different GSD scales. Note that the averaging shown here is for non-overlapping, contiguous GSD regions, whereas the statistics generated by the convolution are done for overlapping GSD regions. The far right of Figure 21 is the average for the entire image, and is proportional to the BRDF value for this measurement. In this case, the average digital count value for the R, G and B channels is 78.86, 94.40 and 33.31, respectively.

Using the convolution approach for texture statistics, histograms or equivalently the BRDF probability distribution may be generated as a function of GSD for each acquired image which corresponds to a unique solar zenith angle ($\theta_i$) and viewing geometry ($\theta_r, \phi$).
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Figure 22: The histograms of the three color bands (RGB) are shown for fractional GSD resolutions, with GSD=1 defined by the box size in Figure 19. The histogram for the original image is included, along with fractional GSD values of 0.1, 1.0 and 1.8. The corresponding standard deviations are shown in Figure 23.

Figure 22 illustrates these distributions as the GSD fraction is changed. These distributions may be used as a basis for “texture mapping” in synthetic image generation.

The standard deviation, \( \sigma \), of these distributions is shown in Figure 23 where it is seen for a GSD=1, \( \sigma = \pm \sim 5 \) digital counts. The convergence of \( \sigma \) may be used as a metric to determine when a sufficient sample size has been imaged to integrate out high frequency inhomogeneities. Note that \( \sigma \to 0 \) as GSD \( \to 4 \), since GSD=4 corresponds to the entire image (Figure 19) for this example.

6.2.1 GSD tangent error effect

Caution is required when applying this technique for increasing zenith reflectance angles. The so called “tangent error” effect results in a changing GSD within the image caused by the ground in the “bottom” of the image being closer to the imaging system than the “top” of the image. The top-to-bottom orientation of the image is subsequently referred to the image \( y \)-axis orientation. This effect on GSD is given by

\[
GSD = R \cdot \cos \theta_r \left[ \tan \left( \theta_r + \frac{FOV}{2} \right) - \tan \left( \theta_r - \frac{FOV}{2} \right) \right] \tag{78}
\]
This is the same equation that is applied to the tangent error effects of overhead systems for off-nadir viewing [5, p. 153], but rather than having a constant altitude, the height of the BRDF imaging system changes according to the radius, $R$ multiplied by the cosine of the reflectance angle.

For the proposed system where $R = 6$ ft and FOV = 10°, the effect at the extreme measurement angle, $\theta_r = 60$°, is considered. At this position, the $y$-axis resolution at the center of the image is modified by a factor of $\frac{1}{\cos 60°} = 2$. The “top” of the image is effectively at $\theta_r = 60° + \frac{\text{FOV}}{2}$ or 65°, and the “bottom” of the image is at $\theta_r = 60° - \frac{\text{FOV}}{2}$ or 55°. This results in a $y$-axis GSD which varies from 1.78 to 2.46 of that of the normalized GSD with a nadir view. Therefore, for high $\theta_r$ values, one may consider using only a central horizontal “band” or a narrower FOV in the $y$-axis of the BRDF image to derive the BTF statistics. The effects of the tangent error on GSD may be seen in Figure 24 which is for a system with a FOV of 10°. (The magnitude of the effect is proportional to the FOV).

There is also some minor, but not insignificant $x$-axis distortion in the image. Again for a FOV = 10° and $\theta_r = 60$°, the effective distance along the $x$-axis at the bottom of the image is $\approx 87\%$ of that in the center, and the $x$-axis distance at the top of the image is $\approx 118\%$ of the center distance. This variation is insignificant compared to the $y$-axis effects, and the $x$-axis errors also decrease if the $y$-axis FOV is narrowed for BTF statistics generation, as the $x$-axis errors are a function of $y$-axis position.
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Figure 24: GSD tangent error effects with increasing $\theta_r$ for a system with an FOV of $10^\circ$. The results are normalized to the nadir view where the solid (red) line corresponds to the $y$-axis GSD at the center of the image, and the dashed (blue) line is the difference between the top and bottom of the image or $y$-axis extremes.

6.2.2 Image mis-registration errors

Another area of concern is registration of the two images made to correct for the downwelled sky radiance: 1) the image made with sun and sky illuminating the target and 2) that with only the sky radiance illuminating the target. Depending on the stability of the camera mount and the weather and target conditions (e.g., tall grass in wind), significant BTF statistic errors may be present from the subtraction of the mis-registered calibration images. Note that these errors are limited to BTF metrics, and do not affect the BRDF determination. The errors may become appreciable as the GSD size (or convolution kernel) decreases to the fidelity of the image registration. With a reasonably sized focal plane ($> 256 \times 256$), this should only become an issue for GSD sizes less than $\frac{1}{100}$ of the image size, unless wind is appreciably moving the target, which may become an issue for tall grass.

6.2.3 Spectral correlation

To faithfully reproduce the spectral BTF using the aforementioned technique, another level of fidelity is required. The spectral values for each pixel are not statistically independent, but are correlated. Therefore, simply generating a spectral BRDF based upon the individual band probability distributions shown in Figure 22 is not sufficient. Spectral errors are
Figure 25: Synthetically-generated grass images using the original image (Figure 19) statistics with no spectral correlation. Spectral errors are evident (with a color copy). The left image is generated from a GSD equivalent to 1 pixel in the original image, while the right image is generated from a GSD=1 unit as presented in Figure 19. The images are only a 20 × 20 section so that individual pixels are discernable.

evident when band-to-band correlation is not considered, as illustrated by Figure 25. The occurrence of the predominantly blue pixels in the figure is easily understood by inspection of the histogram of the original image (Figure 22), where it is seen that random selection of a high blue value may be accompanied by lower red and green values. To first order, most of the correlation is simply a change in the magnitude or a DC shift across all spectral values; however, the spectral shape may also change depending on other unique conditions.

The covariance matrix between bands may be used to capture the band-to-band spectral dependencies. However, when several random variables are considered, the process is not easily managed as the dimensions of the covariance matrix increases to the number of random variables. For hyperspectral images, generation of a ∼100+ dimensional covariance hypercube is not feasible.

An alternate technique of maintaining the spectral correlation is to begin with one spectral band (band 1) and generate a random value per that band’s probability density function, \( f_i(DC) \). Then, the distribution of the next band (band 2) will be altered by the conditional probability based upon the value selected for band 1. The new probability density for band 2 will then be used to generate that band’s value. This process continues until all bands have been selected. This technique should provide an improvement over the Single-Bandpass (SBP) Z-score algorithm used in DIRSIG, which essentially selects an appropriate spectral vector given the average value over a narrow spectral band [90].

A potential pitfall of this method is having too many spectral bands for the spatial dimension of the image. Each band selection narrows the population available for the next
band’s selection. If a sparse population (or no population) is left for selection of the last
band, then the bin size of the bands may be increased which will enable a larger surviving
population. This technique becomes crucial as one considers high dynamic range sensor
having 10 or more bits. Consider a hypothetical case of a 1000 \times 1000 10-bit spatial image
with 10 spectral bands. Assume a uniform distribution for each band. Given 1024 values
available, the first band selected would each have \( \frac{1000^2}{1024} \) or approximately 1000 pixels per
digital count value. Selection of a single digital count value would only leave 1000 pixels
left to determine the subsequent band’s distribution. One quickly exhausts the available
population of pixels from which statistical draws may be made. Increasing the bin size
around the selected digital count provides a means of alleviating this problem, with minimal
accuracy sacrificed.

This technique was implemented in a computer algorithm and found to be effective, but
very slow. Even with more efficient coding, it is doubtful this technique will be fast enough
for hardware in the near future. An alternative approach is to provide a much coarser
approximation which is still effective. Rather than maintaining the spectral probability
density function at the resolution of single digital count values, the PDF may be divided
into much more coarse bins across the dynamic range of the system. For instance, five
divisions per distribution would allow bands having 20% of the population in each group.

Using such a division, the condition probabilities may be computed once and stored
in two multi-dimensional arrays: one having the mean values and the other having the
variance. The number of dimensions will equal the number of bands, \( i \), which results in the
total number of elements in each matrix equal to \( 5^i \). For three bands such as used in this
example, this is manageable. However, as with the first technique, once a realistic number
of bands are considered, an unrealistic magnitude results. Principal components analysis
may then be used to reduce the dimensionality of the data to a manageable number (\(< 10\)).
For producing the remaining bands, it is suggested to use their mean and variance without
consideration of conditional probability.

After having accounted for the spectral correlation, it should be noted that spatial
correlation or texturing effects at a level greater than a single pixel have not been considered.
This becomes important if one uses hyperspectral algorithms which consider the spatial
extent of features, such as the RX algorithm \([91]\). Realistic spatial texture generation has
been and will continue to be an active area of research \([90]\).
6.3 Hemispherical interpolation

Having seen the significant research invested in deriving BRDF models, one feels compelled to apply an appropriate model(s) to the acquired data. However, simple empirical interpolation of the measured values is recommended as performed by Sandmeier (§ 4.2). This is driven by two considerations: 1) the density of the hemispherical sampling using the recommended technique is sufficient to provide a reasonable estimate of BRDF given the other uncertainties in the imaging chain and 2) application of models is highly dependent upon the particular material type. One model is not likely to provide accurate results for all ranges of material types measured.

Some means of extrapolating to other solar incident angles, \( \theta_i \), must be used. A semi-empirical heterogeneous model, such as Roujean’s model (§ 5.4), is recommended for natural targets. For highly homogeneous (typically man-made) targets, a homogeneous BRDF model may provide improvements to other \( \theta_i \) positions. However, these materials are often suitable for lab measurements which are not constrained by the sun position for various \( \theta_i \) measurements.

The volume of data is limited to \( \sim 20 \) images per \( \theta_i \), and the number of \( \theta_i \) values will likely be less than three. So the raw data notionally consists of 20–60 images. For a 256 × 256 focal plane having a 16 bit dynamic range, each spectral band for each image will be \( \sim 130 \) KB. Considering a range of 3 to 100 for the number of spectral bands, along with the total number of images for each material, the entire data set for a material would therefore consist of 7.8–780 MB.

6.4 Spectral Interpolation

Ideally, one would like to employ the described BRDF imaging system with a hyperspectral imaging system having contiguous spectral coverage with resolution of \( < \Delta \lambda = 10 \) nm. If such a system is used, spectral interpolation may not be required; however, it is likely that such a robust system is not available or impractical due to signal to noise considerations.

In the event one is left with a spectrally modest system, an efficient and accurate means of providing detailed spectral coverage is required. One candidate technique is to use a non-imaging spectrometer which has a FOV approximately the same as the imaging system. Other techniques, such as that used by the NEF database, make the reasonable assumption that the spectral-dependent variance in the BRDF is proportional to the variance in the DHR for that wavelength.
6.5 Polarized Measurements

Determination of the Mueller matrix, $\mathbf{M}_r$, is possible using this system according to the measurement protocol described in § 3.6. Lab measurements using such a technique are relatively straightforward, but outdoor measurements using this technique are problematic. Polarization filters would have to be positioned between the sun and target for the measurements, and the number of required measurements would likely not be completed in a time reasonable to maintain reasonable accuracy in the solar zenith position, $\theta_i$.

However, the use of linear filters over the camera enables derivation of the Stokes parameters, which enables derivation of the first column of the BRDF Mueller matrix. This is an adequate descriptor when illumination sources are randomly polarized. However, the downwelled radiance of the skydome is polarized by atmospheric scatter. There is still some utility in acquiring Stokes BRDF images, since the direct solar illumination often exceeds 80% of the incident irradiance onto materials, and of the remaining 20% skydome diffuse irradiance, less than 5% (or one quarter) in a weighted-average sense is estimated to be comprised of linearly polarized radiance. A much more significant contributor to the polarized signature may be the upwelled radiance ($\vec{L}_u$, Eq 68c) along the target to sensor path. These sensitivities are a topic for further study by the author.
7 Conclusion

The bidirectional reflectance distribution function (BRDF) has been examined in detail. Electromagnetic theory fundamentally describes the interaction of light with matter, and the BRDF may be used to characterize the reflected radiance for real materials. An array of measurement techniques have been examined. Significant advances in BRDF measurement have been enabled by the use of focal plane arrays. Models are still required to effectively represent BRDF, as there are an infinite number of measurements to fully quantify BRDF. Physical parameters of interest may also be derived by fitting empirical data to BRDF models. Remote sensing requirements for BRDF may be understood by examining the individual radiance contribution components reaching a sensor aperture. BRDF data provides for the normalization of multiple views, and improves quantitative remote sensing algorithm performance.

A BRDF measurement technique has been recommended that provides the ability to quantify the BRDF variance or texture within natural materials. This is particularly important for high-resolution systems. The measurement technique inherently provides a means to derive texture statistics, which may be tailored to a particular sensor’s point spread function.
References


REFERENCES

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