FUNDAMENTAL MODES OF ATMOSPHERIC CFC-11 FROM EMPirical MODE DECOMPOSITION

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Received 27 July 2012
Revised 30 September 2012
Accepted 12 November 2012
Published 26 March 2013

Following an initial growth, the concentrations of chlorofluorocarbon-11 (CFC-11) in the atmosphere started to decline in the 1990's due to world-wide legislative control on emissions. The amplitude of the annual cycle of CFC-11 was much larger in the earlier period compared with that in the later period. We apply here the Ensemble Empirical Mode Decomposition (EEMD) analysis to the CFC-11 data obtained by the U.S. National Oceanic and Atmospheric Administration. The sum of the second and third intrinsic mode functions (IMFs) represents the annual cycle, which shows that the annual cycle of CFC-11 has varied by a factor of 2–3 from the mid-1970’s to the present over polar regions. The results provide an illustration of the power of the EEMD method in extracting a variable annual cycle from data dominated by increasing and decreasing trends. Finally, we compare the annual cycle obtained by the EEMD analysis to that obtained using conventional methods such as Fourier transforms and running averages.

Keywords: Chlorofluorocarbon; time series filtering; annual cycle; amplitude modulation.

1. Introduction

Before the world-wide ban on its production, trichlorofluoromethane (CFCl₃; Chlorofluorocarbon-11), or CFC-11, was one of the most widely used chlorofluorocarbons in industrial applications [McCulloch et al. (1994); McCulloch et al. (2001)]. It was used as a blowing agent in foam production, an aerosol propellant, a...
refrigerant, degreasing agent, solvent, fire extinguishing agent, and a chemical inter-
mediate. Among all chlorofluorocarbons, CFC-11 has the greatest chlorine content
and is most sensitive to ultraviolet photodissociation. It therefore has the highest
ozone depleting potential [WMO (1992)]. Emissions of CFC-11 have been declining
as a result of the production ban worldwide since 1996 [Newman et al. (2007)]. The
profile of the CFC-11 emission from 1960 to 2005 is given in Fig. 1(b) of Liang
et al. [2008]. In summary, CFC-11 production dropped 74% in 1994 due mainly
to its replacement by hydrochlorofluorocarbons, such as HCFC-141b, which have
much lower ozone-depleting potentials. Following a rapid initial rise from 1960 to
about 1974, the emission remained between 0.25 to 0.35 Mtons/yr between 1974
and 1988, followed by a rapid decline after the regulation went into effect. The total
emission over the entire period is 8.831 Mtons. All CFC-11 that is produced will
eventually be released to the environment as emissions because there is no known
sink for it in the troposphere. The primary loss mechanism is photolysis in the
stratosphere,
\[ \text{CFCl}_3 + h\nu \rightarrow \cdot \text{CFCl}_2 + \text{Cl}, \tag{1} \]
which gives it a residence time of \( \sim 65 \) years.

Figure 3(b) of Liang et al. [2008] shows the observed evolution of CFC-11
between 1978 and 2005, averaged at ground stations. In both hemispheres the CFC-
11 concentrations increased from \( \sim 150 \) pptv in 1978 almost linearly to \( \sim 270 \) pptv
in 1990, which then leveled off until 1994. Due to the ban of CFC-11 use, the
CFC-11 concentrations started decreasing after 1994 and dropped to \( \sim 240 \) pptv
in 2005. On top of this long-term variability, there is also an annual cycle on the
order of a few pptv. The cause of the annual cycle is mainly due to its destruction
in the stratosphere [Liang et al. (2008)]. Advection of air that is poor in CFC-11
from the stratosphere by the Brewer–Dobson circulation results in lower concen-
trations of CFC-11 at the surface. Through a series of simulations, Liang et al.
[2008] showed that stratosphere–troposphere exchange (STE) alone would result in
an intra-annual variation of less than 1.6 pptv in the northern hemisphere (NH) and
1.0 pptv in the southern hemisphere (SH). The fact that an intra-annual variation
of 4–6 pptv was observed over some ground stations in the NH before the CFC
regulation clearly indicated the effects on fresh CFC-11 emissions. As a result, the
annual cycle was strong during the growing phase of emission and became weaker
in the declining phase, showing an “amplitude modulation” in the evolution.

The annual cycle of CFC-11 described above is very interesting because it marks
a regime change before and after the onset of CFC-11 regulation. Together with the
long-term trend, these two modes represent fundamental features of the atmospheric
CFC-11 concentration over the past three decades that we would like to quantify
numerically. A number of ways for extracting these fundamental modes from the
CFC-11 time series are available. These include parametric techniques such as fast
Fourier transform (FFT) filtering [Press et al. (1992)] and non-parametric methods
such as empirical mode decomposition (EMD). Most of the parametric methods
are linear, from which the results can be easily interpreted in terms of the physical meanings of the chosen parameters. However, the use of parametric methods requires at least a minimal prior knowledge about the system being studied before a set of parameters can be chosen for use in the data extraction. On the other hand, non-parametric methods can be applied to any time series without prior knowledge about the underlying system.

The EMD is an adaptive method which decomposes a time series in the temporal domain [Huang et al. (1998)]. It has become popular since its introduction because of its simple implementation. Properties, such as frequency, of the time series are determined a posteriori from each of the decomposed components [Wu and Huang (2004); Huang and Wu (2008)]. The EMD has been extensively applied in various scientific fields such as atmospheric, geological and medical studies, and engineering [see for, e.g. Coughlin and Tung (2004); Huang and Wu (2008); Fine et al. (2010); Jackson and Mound (2010); Vecchio and Carbone (2010) and references therein], although a theoretical basis is still lacking [Hou et al. (2009); Hou and Shi (2011)] and caution is usually required when interpreting the resolved frequencies and the variance of the individual components [Rilling et al. (2007); Rilling and Flandrin (2008); Roy and Doherty (2008)].

EMD is useful for decomposing a non-linear time series because no prior information is required. Indeed, Wu et al. [2008] showed that EMD can be used to extract a nonstationary annual variation of an atmospheric time series, in which the extracted “annual cycle” time series is a combination of an “annually repeating” cycle modulated by irregular inter-annual variability such as the El Niño/Southern oscillation (ENSO). Their work is applicable to extracting the regime change in the CFC-11 concentration discussed above. Our goal is to make use of the EMD to characterize the amplitude-modulated annual cycle in the CFC-11 time series resulting from the CFC regulations. In contrast to the work of Wu et al. [2008], we will not be interested in how sporadic or irregular events like ENSO interact with the annual variation. Rather, our work is complementary to Wu et al.’s in the sense that the annual variability is only perturbed once during the regime change, and the annual cycles before and after the regime change are themselves quasi-steady states in the respective regimes. In this paper, we are interested in (1) How well the EMD can extract the two different annual variability of CFC-11 before and after the regime change in a single time series, and (2) How well this extraction is compared to other linear methods. The rest of this paper is divided in four parts, as follows: Sections 2 and 3 show the data and their analysis using the EMD method. Section 4 presents the results or our analysis of synthetic data in the presence of trends, and Sec. 5 presents our discussion and conclusions.

2. Data

We use the CFC-11 data from the National Oceanic and Atmospheric Administration (NOAA)/Earth System Research Laboratory (ESRL)/Global Monitoring Division (GMD) halocarbons program [Thompson et al. (2004)]. This is the same
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Fig. 1. (a) Locations of the twelve ground-stations of CFC-11 measurements. Solid triangles are stations with CFC-11 measurements covering both pre- and post-1995 era; CFC-11 data from stations represented by open circles are excluded in this paper. (b) Latitudinal distribution of the available stations and the temporal coverage. Five stations (in brackets) are excluded from the analysis in this work. The filled region in pale yellow represents the pre-1995 era.

dataset that was used by Liang et al. [2008] with extension to 2010. It is combined from flask and in situ measurements conducted by the Halocarbons and other Atmospheric Trace Species (HATS) group since 1977. Data are available at 12 stations: Alert, Nunavut (ALT; 82.5°N, 62.3°W), Pt. Barrow, Alaska (BRW; 71.3°N, 156.6°W), WLEF TV tower, Wisconsin (LEF; 45.6°N, 90.2°W), Niwot Ridge, Colorado (NWR; 40.052°N, 105.585°W), Cape Kumukahi, Hawaii (KUM; 19.5°N, 154.8°W), Mauna Loa, Hawaii (MLO; 19.5°N, 155.6°W), Cape Matatula, American Samoa (SMO; 14.3°S, 170.6°W), Cape Grim, Tasmania (CGO; 40.7°S, 144.8°E), Palmer Station, Antarctica (PSA; 64.6°S, 64.0°W), South Pole (SPO; 90°S), Mace Head, Ireland (MHD; 53°N, 10°W), Trinidad Head, California (THD; 41°N, 124°W; 120 m), Summit, Greenland (SUM; 72.6°N, 38.4°W; 3,210 m) [Thompson et al. (2004)]. The latitudinal locations and data coverage are summarized in Fig. 1. To study the secular change in the annual cycle, only seven of them which have measurement across 1995 are included in the analysis below.

3. Methods

3.1. Ensemble EMD

EMD will be used to extract the annual cycles in the CFC-11 time series. In an EMD, all the local extrema in a raw time series are identified first. Then the local maxima (minima) are joined through a cubic spline to form an upper (lower) envelope. Then the mean of the upper and lower envelopes are subtracted from the time series to remove the slow-oscillating components. This sifting process may be repeated to ensure that the mean of the envelopes of residual time series is zero at any data point. The resulting time series is the first intrinsic mode function (IMF) where the number of extrema and the number of zero crossings must either be equal or differ at most by one. Higher order IMFs are obtained by subtracting
the previous orders of IMFs from the raw time series and performing the sifting process again. A complete description of the sifting process is described in Huang et al. [1998]. Variants of EMD developed by different groups may employ different spline fitting functions, boundary conditions for defining the splitting and fitting and/or stoppage criteria for the sifting process [see, Rilling et al. (2003); Coughlin and Tung (2004)]. The EMD algorithm used in this study is adopted from Wu and Huang [2004], where (1) The time series is assumed to be uniformly sampled and (2) A fixed number (10 in the present study) of sifting is applied to get an IMF, and no stoppage criterion is applied. With this algorithm, the EMD is shown to be an effective dyadic filter [Wu and Huang (2004)]. The maximum number of IMFs is directly proportional to the logarithm (base 2) of the number of sampling ordinates, \( \log_2 N(t) \). The current algorithm repeats the sifting process until exactly \( \log_2 N(t) \) IMFs are obtained. The residual is defined as the raw time series minus the sum of the \( \log_2 N(t) \) IMFs.

In some cases, the actually number of intrinsic modes may be less than \( \log_2 N(t) \). However, the current algorithm forces EMD to decompose any time series to into \( \log_2 N(t) \) IMFs, potentially yielding some IMFs that are statistically insignificant compared to noise. Note that these insignificant IMFs are not redundant because they must also be considered in order that the decomposition is complete, i.e. the sum of all IMFs reproduces the original time series. One way to reduce the number of insignificant components is to combine them with other IMFs, provided that the IMFs to be combined represent the same/similar physical processes. For the CFC data, we will show below that the sum of IMFs 2 and 3 represents the annual cycle and the sum of IMFs 5–7 and the residual represents the long-term trend; the actually number of IMFs is no more than four.

Edge effects of the spline fits are the major source of uncertainties in the IMFs. To minimize these problems, Wu and Huang [2009] introduced a modification termed the ensemble EMD (EEMD), which is a noise-assisted version of EMD incorporating a Monte–Carlo simulation. Given the same raw time series, an artificial time series of white noise with an amplitude \( \pm 1 \) pptv is added, and the EMD is performed to obtain \( \log_2 N(t) \) IMFs. This noise addition and the application of EMD are repeated many times (\( \geq 1000 \)) to create a statistical ensemble of IMFs. The final set of IMFs is the average of the ensemble. The uncertainty of each IMF is then defined as the standard deviation of the ensemble at each ordinate.

3.2. **Power spectra**

For raw time series, the mean of the data is first removed and then the power spectrum is obtained using FFT.

For IMFs obtained using EEMD, the spectral properties are studied through the normalized Huang transform [Huang et al. (2009)]. In this transform, the upper (lower) envelope of an IMF is the spline fit of local maxima (minima). The IMF is divided by the absolute difference between the upper and lower envelopes, and the
resultant time series, $F(t)$, is normalized such that the standard deviation is equal to unity. The instantaneous phase function $\phi(t)$ is defined as

$$\phi(t) = \tan^{-1} \frac{F(t)}{\sqrt{1 - F^2(t)}}$$

and the instantaneous frequency $\omega(t)$ is then given by the time derivative of $\phi(t)$:

$$\omega(t) = \frac{d\phi(t)}{dt} = \frac{1}{\sqrt{1 - F^2(t)}} \frac{dF(t)}{dt}$$

The power spectrum, known as the marginal Hilbert spectrum (MHS) [Li et al. (2009)], is then defined as the probability distribution function of $\omega(t)$, which may be obtained by binning $\omega(t)$ over a finite number of intervals between $[1/T, 1/(2\Delta t)]$, where $T$ is the length of the observation (e.g. ~20 years over BRW) and $\Delta t$ is the time step of the time series (e.g. 1 month in the CFC data). The power spectrum is normalized such that the area under the curve equals the variance of the corresponding IMF.

4. Results

Results are presented here from applying the EEMD technique to CFC-11 time-series data from the seven ground-station sites. The amplitude modulation in CFC-11 is largest over the subarctic region. For simplicity, we will only show the IMFs over Barrow, Alaska (BRW; Sec. 4.1) and South Pole Observatory (SPO; Sec. 4.2) as examples, with the remaining data in the on-line supplement. A global picture of the extracted annual cycle will be discussed in Sec. 4.3.

4.1. Pt. Barrow, Alaska

The raw BRW data is dominated by an upward trend before 1995 and a downward trend after 1995 that are related to the CFC-11 regulation described earlier (Fig. 2, upper left panel). Also shown on the upper right panel is the FFT of the BRW time series. The dotted, dashed, dash-dotted, and dash-triply-dotted lines are the 99%, 95%, 90%, and 50% confidence levels (C.L.) of the spectral power, respectively. It is evident that only the annual cycle signal has a peak above the 99% C.L. There is also a broad feature around 20 years (see MHS of IMFs 5 and 6). Although this broad band may represent the regulation effect (IMF 5 and 6 show most of the change occur near the time of regulation), the resultant period is comparable to the time span of the data and longer observations are required to establish its statistical significance.

Decomposition of the BRW data results in a total of seven IMFs and a residual mode. The second row of Fig. 2 shows the first EEMD mode and the corresponding MHS. The blue shade represents the standard deviation of the ensemble of the first IMF of the Monte Carlo realizations. The MHS is dominated by a semi-annual cycle. However, the ensemble mean is much smaller than the ensemble standard deviation,
Fig. 2. EEMD analysis for the in situ CFC-11 data at Barrow, Alaska (BRW). (a) The decomposed IMFs of the raw time series. The filled cyan represents the uncertainty of the IMFs (see text). (b) Marginal Hilbert spectra (MHS) of the IMFs. The MHS have been normalized such that the shaded area equals the variance of the corresponding IMF. (c) Blue line: Raw time series; red dashed line: The sum of IMFs 5–7 and the residual; green line: The sum of IMFs 1–4; black line: The sum of IMFs 2 and 3.
implying that the first mode may be just an average of the artificial noise and may not represent any physically meaningful modes [Wu and Huang (2009)].

IMF 2 is characterized by an oscillation with period slightly less than 1 year. The amplitude of the oscillation before 1995 was ±2 pptv and it became significantly less than ±1 pptv after 1995. This may represent the CFC-11 transport modulated by the surface emission before the regulation. The MHS is dominated by periods between 0.7 and 1 year.

IMF 3 is also dominated by a near-annual oscillation. The difference between IMF 2 and IMF 3 is that the amplitude of IMF 3 is more uniform at ±1 pptv throughout the whole time span. This mode is dominated by the STE after 1995; before 1995, non-linear interactions between STE and the surface source emissions may play an important role in this mode. The corresponding MHS is dominated by a sharp peak at 1 year.

IMF 4 is dominated by a triennial oscillation of amplitude less than ±2 pptv. The spread of the ensemble amplitudes are much larger than the averaged IMF. The rest of the IMFs (5–7) are characterized by decadal variations. Among these, only IMF 5 has a mean amplitude ±10 pptv that is larger than the ensemble standard deviation. IMF 6 and 7 have mean amplitudes ±5 pptv and ±0.5 pptv, respectively, which are both much less than the ensemble standard deviation.

The residual mode goes from 150 pptv in 1978, reaching a peak of 280 pptv in 1999 and dropping to 190 pptv in 2010. Its peak does not coincide with that of the raw time series. To reconcile this, we sum the IMFs 5–7 and the residual (Fig. 2(c), upper panel). The resultant time series agrees very well with the trend of the raw series. Therefore, it is the sum of the IMFs 5–7 that represents the long-term variations of CFC-11 due to the regulation. The lower panel of Fig. 2(c) also shows the filtered annual cycle represented by the sum of IMFs 2 and 3, which is compared with the CFC-11 time series after the trend is removed (which is the sum of IMFs 1–4).

4.2. South Pole

The upper left panel of Fig. 3 shows the raw CFC-11 time series over SPO. It started from 140 pptv in 1978, increased to 270 pptv in 1993, and decreased to 240 pptv in 2010. Unlike that over BRW, the annual cycle is not well defined as the FFT spectral power at 1 year is weak (upper right).

Application of EEMD to the SPO data yields an IMF 1 that is dominated by a semi-annual cycle. The oscillation was stronger (≈1 pptv) before 1985. After 1985, IMF 1 shows amplitude much less than 1 pptv. IMF 2 is also dominated by periods 0.5–0.7 year. However, there were significant oscillations (≥1 pptv) only before 1995; it contained some of the annual cycle signals. There was a quiescent period during 1985–1987 when the oscillation was almost insignificant. It is not clear whether this period was due to measurement uncertainty or generic processes; a detailed modeling is required for investigating the underlying reasons. IMF 3 is
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Fig. 3. Same as Fig. 2 except for the South pole station, Antarctica (SPO).
dominated by an annual variation, with stronger amplitudes (≥ 1 pptv) before 1995 and weaker (< 0.5 pptv) afterwards; the quiescent period during 1985–1987 is also present in IMF 3. IMF 4 is dominated by a biennial oscillation with amplitude ~1 pptv before 1995 and much less than 1 pptv afterwards. Similar to those over BRW, the trend over SPO is represented by the sum of IMFs 5–7 and the residual, shown in Fig. 3(c).

4.3. Global pattern of the annual variations

We next examine the latitudinal dependence of the CFC-11 annual variations by applying EEMD on data from all seven of our selected stations, complimenting the detailed analyses presented above. We found that IMFs 2 and 3 have periods close to the annual cycle; we therefore combined IMFs 2 and 3 to represent the annual cycle for all stations. The resultant time series are shown in Fig. 4(a). The amplitude of the annual cycle is the largest over BRW in the subarctic region. There was also an obvious reduction in the amplitudes after 1995. For example, the annual variations over BRW were about ±2 pptv before 1995 and ±1 pptv after 1995. At other stations, the amplitudes were generally less than ±1 pptv before 1995 and were further reduced after 1995.

To better illustrate the changes in the annual cycles before and after 1995, we averaged the annual cycles in the respective eras. Figure 4(b) shows the averaged annual cycle before 1995. Except for BRW, where the amplitude was ±2 pptv, the annual cycles over the other six stations were ±(0.25–0.75) pptv. Similarly, Fig. 4(c) shows the averaged annual cycle after 1995. Again, except for BRW, the annual cycles over the other 6 stations were less than ±0.25 pptv.

For both eras, the minima of the annual cycle generally occurred in summertime during May–August in the NH and January–April in the SH. There was apparently a northward propagation of the annual cycle from the South Pole to the North.
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Pole at the surface. However, the modeling work of Liang et al. [2008] suggests that there is a rapid southward movement between 0°S–30°S during April–July; see their Fig. 9(c). Unfortunately, there is not enough surface data in the tropics to confirm the model results. More data are needed to test this theoretical prediction.

It has been suggested that the annual minima at high latitudes were caused by the down-welling of aged CFC-depleted air from the polar stratosphere during wintertime [Nevison et al. (2004)]. The amplitude of the annual cycle in the NH is stronger than in the SH because of the stronger wave driving mixing rates and downward Brewer–Dobson stratospheric circulation [Holton et al. (1995)]. The descent from the lower stratosphere to the lower troposphere takes about 4–5 months [Liang et al. (2008)]. Therefore, the annual minima are seen at the surface during the summertime.

4.4. Global trends of CFC-11

For the filtered trends (defined as the sum of IMFs 5–7 and the residual mode), we further calculate the rates of change of CFC-11 in the pre- and post-1995 eras. To avoid the turning point, we calculate the pre-1995 trend using data before December 1993 and the post-1995 trend using data after January 1997. The trends at the selected stations are shown in Fig. 5. ALT and CGO have been excluded from the calculation for the pre-1995 era because of the short observational period before 1993; see Fig. 1. The linear trends, in units of pptv/yr, are defined as the slopes of the linear fits in the two eras. The error bars shown in Fig. 5 are the 2-σ uncertainty of the linear fits.

Before 1995, the atmospheric CFC-11 concentrations increased by ∼8.8 pptv/yr uniformly over the globe. Note that the increasing trend over the SPO was weaker but it was statistically consistent with those at the other stations within the fitting uncertainty. After 1995, the CFC-11 concentration has been decreasing at a rate of −2.2 pptv/yr almost uniformly over the globe, except at ALT near the North

![Fig. 5. Linear trends of CFC-11 concentrations in pre-1995 (blue dots) and post-1995 (red triangles) eras in the unit of pptv/yr. Error bars I are ±2-σ uncertainty of the linear fits.](1250024-11)
Fig. 6. Comparisons between the sum of IMFs 2–3 over BRW and boxcar-filtered time series of bandpass windows $[W_1, W_2] = [7 \text{ months, } 13 \text{ months}], [5 \text{ months, } 17 \text{ months}]$ and $[3 \text{ months, } 23 \text{ months}]$. 

Fig. 7. (a) Comparisons between the sum of IMFs 2–3 over BRW and bandpass-filtered time series of windows $[W_1, W_2] = [7 \text{ months, } 13 \text{ months}], [5 \text{ months, } 17 \text{ months}]$ and $[3 \text{ months, } 23 \text{ months}].$ (b) Similar to Fig. 6 except that the FFT-filtered time series is shown instead of the boxcar-filtered time series.
Pole, where the decreasing rate was stronger at $-2.4 \text{pptv/yr}$. Detailed modeling is required to explain these global trends.

5. Discussion and Conclusions

The above results demonstrate that EEMD, which minimizes mode mixing through a noise-assisted Monte Carlo simulation and produces meaningful MHS, is a good filter of amplitude-modulated signals. To further elucidate the power of EEMD, we compare these results with those obtained from conventional linear filtering methods.

A great advantage of linear filtering methods is that their statistical properties are usually derivable and predictable. However, these methods are usually parametric and require a priori knowledge for setting up the parameters. Consequently, there are always concerns of whether the a priori is appropriate and how sensitive the result be if the a priori parameters are perturbed.

We shall compare with two linear methods. The first and the simplest linear filter is the running average, also known as the boxcar average. Given a raw time series $\{X(i), i = 1, 2, \ldots, N\}$, the boxcar average $\{X^b(i) = 1, 2, \ldots, N\}$ of width $W$ (an odd number) for elements not close to the edge is defined as

$$X^b(i) = \frac{1}{W} \sum_{k=-(W-1)/2}^{(W-1)/2} X(i + k), \quad \frac{W-1}{2} \leq i \leq N - \frac{W-1}{2}. \quad (4)$$

Two common ways are applied for elements near the edges, i.e. when $i < (W-1)/2$ or $i > N - (W-1)/2$. The first way is to simply truncate the time steps at the edges:

$$X^b_W(i) = \begin{cases} \left(\frac{W-1}{2} + i\right)^{-1} \sum_{k=-i+1}^{(W-1)/2} X(i + k), & i < \frac{W-1}{2}, \\ \left(\frac{W+1}{2} - N + i\right)^{-1} \sum_{k=-(W-1)/2}^{N-i} X(i + k), & i > N - \frac{W-1}{2}. \end{cases} \quad (5)$$

The second way is to copy the rightmost endpoints $(W/2 - i + 1)$ times and the leftmost endpoints $(W/2 - N + i)$ times as though they are extended beyond the edges, and then do the boxcar averaging using the extend time series:

$$X^b_W(i) = \begin{cases} \frac{1}{W} \left[ \left(\frac{W+1}{2} - i\right) X(1) + \sum_{k=-i+1}^{W/2} X(i + k) \right], & i < \frac{W-1}{2}, \\ \frac{1}{W} \left[ \left(\frac{W-1}{2} - N + i\right) X(N) + \sum_{k=-W/2}^{N-i} X(i + k) \right], & i > N - \frac{W-1}{2}. \end{cases} \quad (6)$$
Equation (6) normally produces smoother edges and we shall adopt this method in the following discussion. A boxcar-filtered time series \( \{ X_{W_1, W_2}^i(i), i = 1, 2, \ldots, N \} \) with a bandpass window between \( [W_1, W_2] \) using boxcar averaging can then be obtained by applying Eqs. (4) and (6) to the raw time series:

\[
X_{W_1, W_2}^i(i) = X_{W_1}^b(i) - X_{W_2}^b(i).
\]  

(7)

Figure 6 shows the boxcar-filtered CFC-11 time series with three slightly different bandpass windows, all of which aim to capture the annual and semi-annual cycles. For example, the blue solid line corresponds to the bandpass window \( [W_1, W_2] = [7 \text{ months}, 13 \text{ months}] \). The phases of the oscillation from the two methods are consistent with each other. But the amplitude of oscillation is moderately smaller compared with that of the sum of IMFs 2 and 3, probably because there is spectral leakage outside the bandpass window. As a sensitivity test, we relax the window size to \( [5 \text{ months}, 17 \text{ months}] \) and \( [3 \text{ months}, 23 \text{ months}] \). There is almost no effect in the phase relation yet the amplitudes after the relaxation evidently grow with the window size, especially for the pre-1995 domain but are still slightly smaller than that of EEMD. This suggests that EEMD has a better performance in retaining the variability of the fundamental modes. The second commonly used linear filter we shall discuss is the Fast-Fourier Transform (FFT) [Press et al. (1992)].

Given a raw time series \( \{ X(i), i = 1, 2, \ldots, N \} \), let \( F_X(\nu) \) be the FFT of \( X \) in the spectral domain and let \( F^{-1} \) denotes the inverse FFT operator. Given a spectral window \( \Psi(\nu) \), the filtered time series \( \{ X^\Psi(i), i = 1, 2, \ldots, N \} \) can be obtained by the inverse transform of the product \( F_X(\nu) \) and \( \Psi(\nu) \):

\[
X^\Psi(t) = F^{-1}[F_X(\nu)\Psi(\nu)].
\]  

(8)

The simplest example of \( \Psi(\nu) \) is the square window function:

\[
\Psi(\nu) = \begin{cases} 
1, & \nu_1 \leq \nu \leq \nu_2, \\
0, & \text{otherwise}
\end{cases}
\]  

(9)

In other words, the square window function is used to isolate the frequencies exactly between \( \nu_1 \) and \( \nu_2 \) and to ignore all other irrelevant frequencies. However, the discontinuity at the edges may cause unrealistic oscillatory behavior in the filtered signal, known as the aliasing effect. Various window functions with modified edges have been proposed to minimize the Gibbs phenomenon. For example, the Tukey window function aims to minimize the aliasing effect by introducing the smooth boundaries. It is derived from a square window but with the shape edge replaced by the Hann’s function:

\[
\Psi(\nu) = \begin{cases} 
\frac{1}{2} \left[ 1 - \cos \frac{\pi (\nu - \nu_1)}{\nu_2 - \nu_1} \right], & \nu_1 \leq \nu \leq \nu_2, \\
1, & \nu_2 \leq \nu \leq \nu_3, \\
\frac{1}{2} \left[ 1 - \cos \frac{\pi (\nu - \nu_3)}{\nu_4 - \nu_3} \right], & \nu_3 \leq \nu \leq \nu_4, \\
0, & \text{otherwise}
\end{cases}
\]  

(10)
Thus, frequencies between $\nu_2$ and $\nu_3$ pass through the filter directly, whereas frequencies outside the region bounded by $\nu_1$ and $\nu_4$ will be stopped completely [Camp et al. (2003)].

Before applying the FFT, any trends or constant components have to be removed from the time series to avoid large errors in high frequencies due to the spectral leakage of low frequency components. Figure 7(a) shows the FFT spectrum of the time series after removing a least-square second-order fit. While the annual cycle clearly stands out, the semiannual cycle is barely seen and is swamped in the spectral background that leaks out from the low frequency components (i.e. the broad 20-year peak). The Tukey window function with boundaries at $(\nu_1^{-1}, \nu_2^{-1}, \nu_3^{-1}, \nu_4^{-1}) = (15 \text{ months, 12 months, 6 months, 5 months})$ is applied to the FFT spectrum (red dashed line). The FFT-filtered time series is compared with the EEMD result in Fig. 7(b). Sensitivity tests show that the FFT-filtered time series remains similar if the window size is relaxed by 1 month. Away from the edges of the time domain, the results from these two methods are very similar. The amplitude of the FFT-filtered time series in the post-1995 region is also comparable with that from EEMD, implying an improvement over the boxcar averaging method described above. However, there is a huge edge effect at the right hand boundary, which EEMD manages to avoid this kind of effect by the ensemble averaging. These results illustrate the power of EEMD as a non-parametric and empirical filtering tool.

Acknowledgments

KFL performed the linear filtering analysis using the computer facilities at the Atomic and Molecular Physics Laboratories, The Australian National University provided by Dr. Franklin P. Mills. This research was supported by NSF grant ATM-9903790 to the California Institute of Technology. The CFC-11 data measured by the Halocarbons and other Atmospheric Trace Species (HATS) group were obtained from http://www.esrl.noaa.gov/gmd/hats/combined/CFC11.html.

References


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