Why does Differentiation Happen?

We saw in our discussion of Rayleigh-Taylor instabilities that there will always be a tendency to overturn a system in which heavy material sits on top of light material. This can be expressed more generally: A system will seek to minimize gravitational energy. (Of course, gravitational energy is negative, so this means the system aspires to have an even more negative gravitational energy.) There is also a tendency for a system with a lateral density gradient to settle into the same final state (assuming no mixing):

(It could also evolve to a state of uniform intermediate density but note that only the LHS case is then higher energy, available to do the stirring). Notice that the LHS case is an instability that develops from infinitesimal perturbations. The RHS is a situation that predicts finite acceleration from the beginning. For that reason, the RHS case has larger motions (at least initially) despite having less total energy change.

Thermal convection is a special case in which the system is kept out of equilibrium by the continued creation of new buoyant material (negatively buoyant material near the surface and/or positively buoyant material at depth.) Moreover, the potential for overturn is mediated by the elimination of density differences because of thermal diffusion. When we speak of differentiation, we usually mean a one-way transport of light material up or heavy material down. It’s “one way” to the extent that compositional
differences (unlike thermal differences) do not usually have a continuous source. It may not be “one way” if the planet can continually recycle (as Earth does, driven by the thermal convection engine) or if partial melting (for example) persists throughout geological time. Of course, this only happens when the thermal convective heat engine can do the work required to keep the compositional density differences continuously recycled.

A good example of “one way” differentiation is the formation of the core of a terrestrial planet. Formation of the lunar crust or terrestrial continents are also thought to be mostly “one way”. Basaltic oceanic crust on Earth is recycled (though it’s fate is controversial; we don’t really know what fraction of it finds its way “back up”). Another complicated case is Earth’s water cycle (water can outgas from Earth but also be carried back down at subduction zones). Of course, the total amount of water is small (as a mass) compared to the rock mass.

It is important to realize that although the “potential” for differentiation may (and usually does) exist, this does not mean it will happen in a geological timescale. For example, Earth’s mantle is a mixture of mineral grains of different density that would like to separate under the action of gravity, but they will not do so (if everything stays solid) because the flow of one grain relative to another is extremely slow (essentially negligible). So we need to look at process. To take advantage of the Rayleigh-Taylor process, for example, you need to aggregate a large amount of material of one phase, and this is not readily done except by partial or total melting. One way or another, the process of differentiation is contingent on the existence of an easily flowing medium ... a melt or fluid phase.

**How does Differentiation Occur?**

Four processes are important:

**A. Separation of Grains (or Droplets) from a Fluid**

Consider a convecting “box” or ocean of fluid. Grains or droplets that form a separate phase in this fluid will generally be different in density from the fluid and will tend to rise or sink. The velocity of motion relative to the fluid is the Stokes velocity:
where the grain has radius R, the density difference is $\Delta \rho$, and $\eta$ is the fluid viscosity. (This formula can be derived to order of magnitude from the approximate equation we used to describe Rayleigh Taylor instabilities. The vorticity generation is $\sim g \Delta \rho / R$ and the viscous term is $\sim \eta v / R^3$; setting these two terms equal leads to the result). This will typically give velocities of order cm/sec for grains of order 1cm in size if the viscosity is low (but of course, a wide range of answers are possible). If this is less than the convective velocity then the grains are entrained, but if there is a top or bottom boundary layer, where the vertical flow must be small, then the particles can settle out and aggregate. It is generally not correct to imagine that particles fail to aggregate when convective velocities exceed Stokes settling velocities. The volume flux of grains or droplets out of the fluid is accordingly $fv$ where $f$ is the fraction of the volume occupied by the material that is settling. This is presumably how cumulates form.

Planetary Examples:

(1) Formation of the lunar highland crust by crystal fractionation from a magma ocean.
(2) Partial differentiation in the early Earth mantle.
(3) Helium rainout in the giant planets.

B. Melt Percolation
This is an extremely important process, probably the most important, since it is the essential first stage in going from a homogeneous to a differentiated system when there is only partial melting.

Consider, for example, an element of mantle material (peridotite) rising up until it reaches and crosses the solidus. As it starts to melt, thin films and pockets of the melt between the solid grains are believed to form an interconnected network. Since the melt is generally less dense than the solid, it will rise relative to the solid (which also rises) presumably to some "catchment zone" (magma chamber) where it accumulates prior to the rapid, sometimes explosive release along conduits (e.g., cracks) to the surface.

We want to derive an approximate formula for this percolative flow. The first step is very simple. The formula for channel flow (often called Pouseille flow) is very similar to that for Stokes flow of a grain, only now we are talking about the melt flowing relative to a “fixed” solid that provides the cylindrical path of radius a. The flow along any channel has an average velocity

\[ u_z = \frac{a^2 g \Delta \rho}{8 \eta} \]  

(19.2)

provided the channel is vertical and can be treated as cylindrical. As with 19.1 you can immediately recognize this as the balance between the buoyancy force in the liquid and the viscous drag on the channel walls. The density difference between solid and liquid is assumed to be \( \Delta \rho \) and it is assumed that the conduit walls do not sustain a substantial normal stress difference, so that the pressure gradient is close to \( g \Delta \rho \) in magnitude. Now in a real porous medium, the channels are likely to be distributed randomly in direction so the mean fluid velocity is considerably less, presumably by at least a factor of three (probably more, since the tubes are likely to be irregular in cross-section.) To accommodate these numerical factors, we define a parameter called permeability, which has dimensions of area and scales as \( a^2 \) but with a small proportionality constant. We also choose to speak of the flux of fluid rather than velocity. This is smaller than the velocity by a factor of \( f \), the melt fraction, since flux is defined as melt flow per unit area and only a fraction \( f \) of a randomly chosen area perpendicular to the flow is melt. Note that melt fraction is the same as porosity since all pore space will be filled with melt at the pressures of interest (nature cannot
tolerate empty pores at pressures exceeding about a kilobar). Thus, we rewrite the equation in the form

\[ w = k(f)g\Delta\rho/\eta \]  \hspace{1cm} (19.3)

where \( k(f) \) is the permeability function and \( w \) is approximately \( fu_z \) for small \( f \). This is Darcy’s law, written for the special case where the pressure gradient is derived solely from buoyancy. Since the melt fraction is roughly the area covered by melt divided by the area covered by solid in any slice of the partial melt (cf. a Swiss cheese), \( f \approx a^2/R^2 \), where \( R \) is the grain size and hence also the mean spacing between channels. See the cartoon. It turns out (after taking care of all the numerical factors) that a plausible estimate is \( k(f) \sim 10^{-3} R^2 f^2 \). However, nobody knows for sure. For an estimate, suppose we have a melt fraction of say 0.03 and a grain size of 1mm. Then \( k \) is about \( 10^{-8} \) cm\(^2\). We then get \( w \sim 1\) cm/yr, not too different from plate tectonic motions. It is suspected that melt moves faster than this, probably because wider channels form.

Planetary Examples:

(1) Migration and aggregation of basaltic melt beneath mid-ocean ridges.
(2) Initial stage of melt segregation for plume volcanism and all forms of basaltic volcanism in terrestrial planets.
(3) Migration of fluids (e.g. from a slab into the neighboring mantle at subduction zones).
(4) Migration of fluids (e.g. water-ammonia liquid, methane) through water ice in icy satellites.
C. Diapirism

In this process, macroscopic “blobs” of stuff rise by buoyantly deforming the surrounding solid medium. This is generally a rather slow process. It is described by the same formula (Stokes velocity formula) as for grains:

\[ v = \frac{2g\Delta \rho R^2}{9\eta} \]  

(19.4)

where the blob has radius R, the density difference is \( \Delta \rho \), and \( \eta \) is the now the solid matrix viscosity. It is the natural end-point in the development of a Rayleigh-Taylor instability. For example, if \( R \sim 1 \text{km} \), \( \Delta \rho \sim 1 \text{g/cc} \), and \( \eta \sim 10^{21} \text{Poise} \), then \( v \sim 0.01 \) to 0.1 cm/year, less than plate tectonic velocities. Diapirism is a very popular “picture” for geologic processes (you see it often portrayed in geophysical and geological textbooks) but one has to suspect that it an overused concept because diapirs are very sluggish, especially as they encounter increasingly more viscous material upon approaching the planetary surface.

Possible Planetary Applications:
1. Core formation
2. Part of the flow beneath mid-ocean ridges.
3. Grooved terrain formation on icy satellites (e.g. Ganymede)?
4. Domes on Europa?
5. South polar region of Enceladus?

D. Cracks and Magma Fracturing
The final stages of ascent of melt to a planetary surface must be a very fast process, since otherwise the melt would freeze by contact with the cold surrounding material. The process involves wide conduits (cracks) that form in the brittle lithosphere and crust under the action of both the stresses arising from fluid buoyant and the regional tectonic stresses. Using the same channel flow formula as above

\[ u_z = a^2 g \Delta \rho / 6 \eta \]

but now with \( a \) of order meters, we can easily get velocities of many meters /sec, and ascent times through 10km of crust that are measured in hours rather than geologic timescales. (An example of this is provided by the shallowing of small earthquakes associated with the Long Valley Caldera near Mammoth Village in Owens Valley.)

**Planetary Examples:** All eruptive volcanic events, including

1. \( \text{SO}_2 \) “geysers” on Io,
2. methane plumes on Triton,
3. possible water/gas eruptions on Europa (apparently observed by HST; published in 2013)
4. plumes on Enceladus
5. Delivery of water to the current surface of Mars? (Or possible past delivery to the surface in hydrothermal systems.)
Terrestrial Planet Temperature Structures Revisited.

One of the consequences of differentiation could be the development of a layered structure, with each layer convecting separately. We saw earlier that because the heat flow is far too large to be carried by conduction alone, a convective state develops in which the system has boundary layers top and bottom of a given convective layer, and the state is neutral (i.e. adiabatic) in between. We do not know how many convective layers are present, although seismic tomography for Earth suggests that the mantle may convect from top to bottom. Above is a sketch of that situation, and the alternative two layered case. The core will be adiabatic provided enough energy is released to keep it in that state (we will discuss this further next chapter). On non-plate-tectonic planets, the lithosphere is not recycled, so only a sub-layer participates, as discussed earlier. The bottom boundary layer carries a small amount of heat relative to the internal heat generation and thus plays a small role in the large scale circulation.

Relation to Basaltic Volcanism
Basaltic volcanism is overwhelmingly the most important means of generating melt in terrestrial planets. The reason is elementary: Basalt is the primary melt (most easily formed melt) in the material out of which terrestrial mantles are made. How do we make this melt? We can do it by having material that is anomalously hot or we can do it by pressure release (i.e. cooling but depressurizing on an adiabatic path). The latter overwhelmingly dominates on Earth. It is unclear how important it is on other planets. Here are cartoons for the process in each case.

![PASSIVE (PRESSURE RELEASE) MELTING](image1)

Plumes are important in proportion to the amount of bottom heating. It is not known how much heat comes from Earth’s core, but estimates suggest only
~10%. If this is correct then plumes are relatively unimportant in the overall convective circulation of Earth’s mantle.

**Implications of Thermal Histories**

According to our models, planets were once much hotter. This implies much higher production of basalt in the past than now. Certainly, the volcanic history of Mars is qualitatively supportive of this, and lunar mare volcanism also died off long ago, but neither case provides strong quantitative support for this picture. One of the problems is that models tend to predict that Mars should still be generating lots of basalt... maybe it is not getting to the surface? Venus is predicted to produce lots of melt (not just in hot upwellings but wherever the lithosphere is around 150km or less in thickness) if it has Earthlike heatflow- presumably the heatflow is lower. There is no strong evidence that basaltic volcanism was enormously higher two or three billion years ago relative to now. Crudely speaking, we expect the following kind of scaling:

Volcanism ~ (fraction of melt reaching surface)x(Melt generation)
Melt generation~(fraction of upwelling material crossing solidus) x(Mass flux across solidus)
Mass flux across solidus~ (Mantle Heat flux) x(factor that depends on mean mantle temperature) + (heat flux from core)x(factor that depends on core-mantle boundary temp.)

It is remarkable that current Earth just happens to reside at a temperature so close to the solidus at shallow mantle depths. Perhaps this is not a coincidence but part of how plate tectonics functions. **This is not understood!**

**Problems**

19.1 (a) Consider vertical flow of a fluid in a vertical crack of width a, driven by a pressure gradient dp/dz, assumed constant. (This is the pressure gradient that can drive flow, i.e. the hydrostatic head has already been subtracted.) By definition, a crack has one small dimension (the width) and two very large dimensions (the vertical extent and the other horizontal dimension). The equation of motion is accordingly well approximated by:
\[ 0 = -\frac{dp}{dz} + \eta \frac{d^2 u}{dx^2}; \quad u(x = -a/2) = u(x = a/2) = 0. \]

The walls of the crack are the planes \( x = \pm a/2 \) and \( \eta \) is the dynamic viscosity. Make sure you understand where all this comes from. Solve for \( u(x) \) in terms of \( dp/dz \) and then find the mean velocity of the flow \( \bar{u} \equiv \frac{1}{a} \int_{-a/2}^{a/2} u(x) \, dx \). This is a standard problem (closely related to Poiseuille flow, which is for a cylindrical pipe) and the result is summarized in this chapter.

(b) Suppose we inject a fluid into a crack. (Indeed, we should think of the fluid “opening” the crack.) We envisage that the crack has vertical length \( H \) (think of it as lithospheric thickness for a planet). As fluid rises, it can partially freeze because the neighboring solid material is much colder and can accept heat from the freezing of the fluid. Show that the fraction of the fluid that freezes during ascent to the surface is small provided:

\[ C_p \Delta T \left( \frac{\kappa H \bar{u}}{L} \right)^{1/2} < La \]

where \( C_p \) is the specific heat of the crack wall solid, \( \Delta T \) is the temperature difference between the neighboring cold material and the freezing point of the fluid, \( L \) is the latent heat of freezing for the fluid, and \( \kappa \) is the thermal diffusivity of the solid material or fluid (they’re similar). [Note: This is an oversimplification; one normally wants to have the liquid have some temperature excess relative to the freezing point (which we ignore) and the temperature difference varies along the ascent; but the analysis here is close enough.] Hence derive an estimate for the thinnest crack that can allow (most of) the liquid to escape.

(c) Evaluate the thinnest crack (dike thickness) for Mars, where the pressure gradient is due to buoyancy; i.e., \( dp/dz = -\Delta \rho g \), where \( \Delta \rho \) is the density difference between solid and liquid; assume 0.1 g/cc. Use a viscosity appropriate to basalt (~100 Poise), and assume \( L \approx C_p \Delta T \) (typically true to a factor of a few), \( H \approx 100 \text{km}, \kappa \approx 10^{-2} \text{ cm}^2/\text{sec} \).

(d) Repeat this, still for Mars, but now for the problem of a “spring”: water rising through a crack from a subsurface reservoir. Now, the viscosity is 0.01 Poise, the density difference is ~1g/cc, \( H \approx 3 \text{km} \), other assumptions similar to (c). The situation of relevance is discussed in Malin, M. C. and Edgett, K. S. Evidence for recent groundwater seepage and surface runoff on Mars, *Science* **288**, 2330-2335 (June 30), 2000.

(e) We can also assess the ascent of highly viscous warm ice through a “crack” (now better thought of as a rift) using exactly the same analysis. (Think of this as toothpaste being squeezed out of a tube, except that it is a linear instead of cylindrical orifice). The only difference is that latent heat is irrelevant, but we still cannot afford to lose much heat to the neighboring colder ice since that will increase the viscosity of the rising material and gum up the rift. Estimate the thinnest rift for the ascent of ice on Europa.
assuming buoyant rise (dp/dz = -Δρg, where Δρ=0.01 g/cc, appropriate to warm ice rising through cold ice). Ice viscosity ~10^{15} Poise.