Material is perfectly elastic until it undergoes brittle fracture when applied stress reaches $\sigma_f$.

Material undergoes plastic deformation when stress exceeds yield stress $\sigma_0$.

Permanent strain results from plastic deformation when stress is raised to $\sigma_0$ and released.
AT LOW PRESSURES ROCKS ARE BRITTLE, BUT AT HIGH PRESSURES THEY BEHAVE DUCTILY, OR FLOW

Consider rock subjected to compressive stress that exceeds a confining pressure

For confining pressures less than about 4 Kb material behaves brittlely - it reaches the yield strength and then fails

For higher confining pressures material flows ductilely. These pressures occur not far below the earth's surface - 3 km depth corresponds to a kilobar pressure - so 8 Kb is reached at about 24 km

This experimental result is consistent with the idea of strong lithosphere underlain by the weaker asthenosphere

Note: 1 MPa = 10 b; 100 MPa = 1 Kb
Note: compressive stress convention

Positive in rock mechanics

Negative in seismology (outward normal vector)

Normal stress:
\[ \sigma = \sigma_{11} = \sigma_1 \cos^2 \theta + \sigma_2 \sin^2 \theta = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta \]

Shear stress:
\[ \tau = \sigma_{12} = (\sigma_2 - \sigma_1) \sin \theta \cos \theta = \frac{\sigma_2 - \sigma_1}{2} \sin 2\theta. \]

Mohr’s circle shows the values of \( \sigma \) and \( \tau \) as functions of \( \theta \) (the angle between the normal to a plane and the principal stress direction, \( \sigma_1 \)).
Laboratory results for sliding on existing faults of various rock types find relation between normal stress on fault and shear stress required for sliding.

Byerlee's Law: \[ |\tau| = \tau_0 - n \sigma \]

- \[ \tau \approx -0.85\bar{\sigma} \] when \( \bar{\sigma} \leq 200 \text{ MPa} \)
- \[ \tau \approx 50 - 0.6\bar{\sigma} \] when \( \bar{\sigma} > 200 \text{ MPa} \)

Byerlee, 1978

Need confining pressure
Laboratory experiments on rocks under compression show that fracture occurs when a critical combination of the absolute value of shear stress and the normal stress is exceeded. Higher normal stress requires higher shear stress for fracture.

This relation, the Coulomb-Mohr failure criterion, is:

$$|\tau| = \tau_o - n\sigma$$

where $\tau_o$ and $n$ are material properties known as the cohesive strength and coefficient of internal friction.

**INCREASED STRESS BREAKS ROCK WHEN MOHR’S CIRCLE REACHES COULOMB-MOHRR FAILURE CRITERION**

Often: for given normal stress, rock breaks when shear stress high enough.
Laboratory experiments on rocks under compression show that fracture occurs when a critical combination of the absolute value of shear stress and the normal stress is exceeded.

Coulomb-Mohr failure criterion: \[ |\tau| = \tau_o - n\sigma \]

\( \tau_o \) = cohesive strength; \( n \) = coefficient of internal friction

\[ |\tau| = \tau_o - \sigma \tan \phi \]

where \( n = \tan \phi \) and \( \phi \)

\( \phi = 2\theta - 90^\circ \) so \( \theta = \phi/2 + 45^\circ \)
With no internal friction, fracture occurs at 45°.

With internal friction, e.g. $n=1$, fracture angle is 67.5°, and fault plane is closer (22.5°) to the maximum compression ($\sigma_1$) direction.

Figure 5.7-8: Failure with and without internal friction.
Anderson’s theory of faulting

orientation of $\sigma_1$ relative to Earth’s surface dictates type of fault…
…three possibilities yield three types of faults…

from: http://earth.leeds.ac.uk/learnstructure/index.htm
New fracture would form at an angle $\theta_f$ given by fracture line.

However, slip will occur on any preexisting fault with angle between $\theta_{S1}$ and $\theta_{S2}$ given by intersection of the circle with frictional sliding line.

If so, as stress increases, sliding favored over new fault formation.
PORE PRESSURE EFFECTS

The fluid pressure, known as pore pressure, reduces the effect of the normal stress and allows sliding to take place at lower shear stresses. The effective normal stress \( \sigma = \sigma - P_f \), where \( P_f \) is the pore fluid pressure.

Similarly, taking into account pore pressure, effective principal stresses

\[
\sigma_1 = \sigma_1 - P_f \quad \sigma_2 = \sigma_2 - P_f
\]

are used in the fracture theory (not the deviatoric stress).
USING BYERLEE’S LAW AND
MOHR’S CIRCLE
RELATES PRINCIPAL
STRESSES AS A
FUNCTION OF
PRESSURE AND HENCE
DEPTH

1. Assume faults of all orientations exist in the crust
2. Byerlee’s Law gives failure envelope (criteria)
3. \( A (= \text{pressure}) \) from depth
4. Derive eqn for circle
5. Gives \( \sigma_1 \) and \( \sigma_2 \)

\[
\tan \phi = -\cot 2\theta = \frac{-1}{\tan 2\theta} = \frac{\tan^2 \theta - 1}{2 \tan \theta}
\]

\[
\sigma_1 = -2\tau_o \tan \theta + \sigma_2 \tan^2 \theta
\]

(Used to estimate the maximum stresses in the crust)

Using the trigonometry of Mohr’s circle gives:

\[
\bar{\sigma}_1 \approx 5 \bar{\sigma}_3, \quad |\bar{\sigma}| < 200 \text{ MPa}
\]

\[
\bar{\sigma}_1 \approx -175 + 3.1 \bar{\sigma}_3, \quad |\bar{\sigma}| > 200 \text{ MPa}
\]
Laboratory experiments on minerals find ductile flow to be:

\[ \frac{de}{dt} = \dot{e} = f(\sigma) \ A \exp[-(E^* + PV^*)/RT] \]

\( T \) = temperature

\( R \) = the gas constant

\( P \) is pressure

\( f(\sigma) \) = function of the stress difference \( |\sigma_1 - \sigma_3| \)

\( A \) = a constant

\( E^*, V^* \) = activation energy and volume (effects of \( T \) and \( P \))
In terms of the principal stresses,
\[ f(\sigma) = |\sigma_1 - \sigma_3|^n \]
\[ \dot{\epsilon} = |\sigma_1 - \sigma_3|^n A \exp[-(E^* + PV^*)/RT] \]

The rheology of such fluids is characterized by a power law. If \( n = 1 \) the material is called Newtonian, whereas a non-Newtonian fluid with \( n = 3 \) is often used to represent the mantle.

The viscosity depends on both temperature and pressure
\[ \eta = (1/2A) \exp[(E^* + PV^*)/RT] \]

The viscosity decreases exponentially with temperature, and increases exponentially with pressure!

**Viscosity decrease with temperature is assumed to give rise to strong lithosphere overlying weaker asthenosphere, and the restriction of earthquakes to shallow depths**
STRENGTH OF THE LITHOSPHERE

- The strength of the lithosphere as a function of depth depends upon the deformation mechanism.

- At shallow depths rocks fail either by brittle fracture or frictional sliding on preexisting faults. Both processes depend in a similar way on the normal stress, with rock strength increasing with depth.

- At greater depths the ductile flow strength of rocks is less than the brittle or frictional strength, so the strength is given by the flow laws and decreases with depth as the temperatures increase.

- This temperature-dependent strength is the reason that the cold lithosphere forms the planet's strong outer layer.

- To calculate the strength, a strain rate and a geothermal gradient giving temperature as a function of depth are assumed.
Strength increases with depth in the brittle region due to the increasing normal stress, and then decreases with depth in the ductile region due to increasing temperature. Hence strength is highest at the brittle-ductile transition. Strength decreases rapidly below this transition, so the lithosphere should have little strength at depths > ~25 km in the continents and 50 km in the oceans.
Higher pore pressures reduce strength

Lithosphere is stronger for compression than for tension in the brittle regime, but symmetric in the ductile regime.
Cooling of oceanic lithosphere with age also increases rock strength and seismic velocity. Thus

elastic thickness of the lithosphere inferred from the deflection caused by loads such as seamounts,

maximum depth of intraplate earthquakes within the oceanic lithosphere,

& depth to the low velocity zone determined from surface wave dispersion

all increase with age.

Stein and Stein, 1992