Polarization of the Radiation Reflected and Transmitted by the Earth's Atmosphere

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The polarization of the reflected and transmitted radiation is calculated for a realistic model of the earth's atmosphere at five wavelengths ranging from 0.27 μ to 1.67 μ. The single scattering matrix is calculated from the Mie theory for an aerosol size distribution appropriate for our atmosphere. The solar photons are followed through multiple collisions with the aerosols and the Rayleigh scattering centers in the atmosphere by a Monte Carlo method. The aerosol number density as well as the ratio of aerosol to Rayleigh scattering varies with height. The proportion of aerosol to Rayleigh scattering is adjusted for each wavelength; ozone absorption is included where appropriate. The polarization is presented as a function of the zenith and azimuthal angle for six values of the earth's albedo, two values of the solar zenith angle, and four values of the total aerosol concentration. In general the polarization decreases as the wavelength increases and as the total aerosol concentration increases (because of the increasing importance of aerosol scattering). In most situations the polarization is much more sensitive than the radiance to changes in the parameters which specify the atmosphere.

Introduction

Photons undergo multiple scattering by the aerosols and molecules in the earth's atmosphere as well as reflection from the ground back into the atmosphere. The resulting radiation field depends in a complex manner on the various parameters that are involved. The radiance and polarization for an atmosphere with nothing but Rayleigh scattering centers have been given in a valuable set of tables. However, aerosol scattering with the aerosols inhomogeneously distributed with height is more important at many wavelengths than molecular scattering in determining the radiance and polarization. The present calculations appear to be the first that use a realistic model of the earth's atmosphere including: (1) Rayleigh and aerosol scattering with the proportion varying with height; (2) aerosol phase function with strong forward scattering; (3) aerosol number varying with height; (4) ozone absorption.

The advantage of the Monte Carlo method is that all the relevant physical parameters may be included in the calculation. The accurate three-dimensional path of the photon is followed as it undergoes multiple scattering. In particular, the exact angular aerosol scattering function with a strong forward peak, as calculated from Mie theory, is used to select the scattering angles. Aerosol concentrations may vary in any desired manner with height. The results include all orders of scattering and any number of reflections from the ground surface that make any contribution to the radiance. The ground is assumed to be represented by a Lambert surface. The Monte Carlo method that we use here has been described in detail in the literature. A previous paper has studied the reflected and transmitted radiance for the earth's atmosphere. This work is extended here to include the polarization of the radiation and the effects of changes in the aerosol concentration.

Method

The aerosols are represented by particles with a real index of refraction of 1.55 and with radii distributed according to the Haze C model proposed by Deirmendjian. In this model the number density is constant for 0.03 < r < 0.1 μ and is proportional to r^{-4} for r > 0.1 μ, where r is the aerosol radius. The scattering matrix for this distribution of spherical particles was calculated exactly from the Mie theory for the wavelengths used in this calculation. The scattering function is shown in Fig. 1 of Ref. 3.

The polarization from a single scattering event is shown in Fig. 1. The polarization varies much more rapidly with wavelength than does the scattered intensity. Particularly at the wavelength λ = 0.4 μ there are several regions where the polarization alternates between positive and negative values. The Rubenson definition of the degree of polarization $P$, $P = (I_r - I_l)/(I_r + I_l)$, is used here, where $I_r$ and $I_l$ are the scattered intensities with the two directions of polarization. When it is positive, the plane of polarization of the scattered light
is perpendicular to the scattering plane. All four of the independent components of the scattering matrix were calculated as input data for the Monte Carlo calculations.

The Rayleigh attenuation coefficient, ozone absorption coefficient, and the aerosol number density as a function of height and wavelength were taken from the tables compiled by Elterman. The total optical thickness of the atmosphere was calculated from the Rayleigh and aerosol attenuation coefficients and the ozone absorption coefficient. The atmosphere was divided into a number of layers, and the ratio of the Rayleigh extinction to total extinction coefficient and the scattering to extinction coefficients for both the Mie and Rayleigh particles was established for each layer. The probability of ozone absorption was also calculated from the ozone absorption coefficient for each layer. All calculations were done with the optical depth as the parameter.

The Monte Carlo code which includes polarization has been described by Kattawar and Platt. Briefly, the four-component Stokes' vector is obtained after each scattering event from the vector before scattering from an appropriate matrix transformation. This matrix involves the four independent components of the scattering matrix previously calculated from Mie theory. Both of the polar angles which describe the angle of scattering are chosen from approximate distributions. However, the weight associated with each photon is multiplied by an appropriate factor, so that the final result is exactly the same as though the angles had been chosen from the correct bivariate distribution.

Collisions are forced so that the photon never leaves the atmosphere, and again the weight associated with the photon is corrected so that the correct result is obtained. The photons are followed until their weight is so small that they make a negligible contribution to any of the detectors. Reflection from the planetary surface is taken into account by a special method which is equivalent to following a photon through an infinite number of collisions and reflections from the assumed Lambert surface. All of the methods used in the Monte Carlo program have been thoroughly checked against calculations made from the exact radiative transfer equations. It should be emphasized that the exact three-dimensional path of the photon is followed in the Monte Carlo method, and that the only averaging is done at the detectors, where the radianc is averaged over finite intervals of solid angle.

A compromise must always be made in a Monte Carlo method in the choice of the angular intervals over which the results are averaged. If the intervals are too large, the angular variation of the function is not obtained. On the other hand, if they are too small, the statistical fluctuations may become large due to insufficient counts. For the present calculations we chose intervals of 0.1 for the cosine of the zenith or nadir angle and 30° intervals for the azimuthal angle. In all cases the calculated value is a true average over the interval. All four components of the Stokes vector are calculated for each interval. A true average value is obtained for each of these quantities.

There is strictly no average value for the polarization, since it is not a scalar quantity, but depends on the orientation of the plane containing the zenith direction and that of the scattered photon. We have computed the average polarization from the average intensity components. In the limit of small angular intervals this value approaches the actual value of the polarization at the center of the interval. Our intervals should be sufficiently small so that the average polarization still has a physical meaning. Figures 3 and 4 of Ref. 7 indicate that this is so, since they indicate excellent agreement with polarization values calculated from Monte Carlo techniques as compared to those obtained from the table of Coulson et al.

Our program calculates in each case both the Ruben-son polarization as obtained from Eq. (1) and the polarization obtained from the expression

$$P = (Q^2 + U^2 + V^2)^{1/2}/I,$$  \hspace{1cm} (2)

where $I$, $Q$, $U$, $V$ are the four components of the Stokes vector. Since $U$ and $V$ are usually much smaller than $Q$, there is little difference in the polarization values calculated from these two expressions, except when $\phi$ is in the neighborhood of 45° or 135°. The Rubenson polarization is given here, since the sign gives additional information as to the orientation of the various components of the radiation. With this definition the polarization near the zenith or nadir depends on the azimuthal angle which, in turn, defines the reference plane.

Fig. 1. Single scattering polarization of aerosols as a function of cos $\theta$, the cosine of the scattering angle, at wavelengths $\lambda = 0.4 \mu, 0.7 \mu$, and $1.67 \mu$. The polarization for Rayleigh scattering is shown for comparison.
For example, the polarization at the zenith or nadir with an azimuthal angle of 90° is the negative of the polarization for an azimuthal angle of 0°.

**Polarization at 0.27 μ and 0.30 μ**

At wavelengths of 0.27 μ and 0.30 μ the ozone absorption coefficient is larger than either the Rayleigh or aerosol attenuation coefficient at all altitudes. In turn the Rayleigh attenuation coefficient is larger than the aerosol coefficient at all altitudes above 1 km. The optical thickness of the atmosphere is 73.25 at λ = 0.27 μ, and is 4.97 at λ = 0.30 μ.

The polarization of the reflected radiation is shown in Fig. 2 when μo = -1 (μo is the cosine of the zenith angle of the incident beam). The results at the detector at the top of the atmosphere have been averaged over intervals of 0.1 in μ. For comparison purposes the polarization that would be observed from pure Rayleigh scattering is also shown; this result is also averaged over the same μ intervals for easier comparison. It is immediately obvious that the polarization of the reflected beam is the same as that for pure Rayleigh scattering to within 1%. This result indicated that the fluctuations in our Monte Carlo results are less than 1% in this particular case.

The polarization of the reflected radiation when μo = -0.15 is shown in Fig. 3. The solar horizon is on the left, the nadir is at the center, and the antisolar horizon is on the right of this figure. The values shown in Fig. 3 have been averaged over a range of the azimuthal angles measured from the incident plane. The upper set of curves has been averaged over the range 0° < φ < 180°, i.e., over all azimuthal angles within 30° of the incident plane. The middle set of curves has been averaged over the range 30° < φ < 60° and 120° < φ < 150°. The lower set of curves has been averaged over the range 60° < φ < 90° and 90° < φ ≤ 120°.

Near the incident plane the polarization is slightly negative near both the solar and antisolar horizons and reaches a maximum value of almost +0.8 near the nadir. There is a moderate negative value for the polarization for the second set of azimuthal angles shown in Fig. 3. On the other hand, when 60° < φ < 120°, the polarization is almost independent of μ; it shows only small oscillations around the value -0.8. This occurs because the scattering angle is always near 90° for any single scattering event within this range of φ angles. Higher order scattering is small in this case because of the high ozone absorption. The polarization is always recorded as it would be observed in the meridian plane.

**Polarization at 0.4 μ**

At a wavelength of 0.4 μ the Rayleigh extinction length is greater than the aerosol extinction length at all altitudes above 3 km. The ozone absorption is negligible at this wavelength. The optical depth of the atmosphere is 0.577 when the Elterman aerosol distribution is used.

The polarization of both the reflected and transmitted radiation is shown in Fig. 4 when μo = -1. The detector for the reflected radiation is at the top of the atmosphere and that for the transmitted radiation is at the earth’s surface. The earth’s surface is assumed to reflect radiation as a Lambert surface. Curves are given for surface albedo A = 0, 0.2, 0.4, 0.6, 0.8, and 1.

When A = 0 the polarization of the reflected beam is only slightly modified from the values of pure Rayleigh scattering. Since the aerosols are concentrated in the lowest layers of the atmosphere, they have relatively little influence on the polarization of the reflected radiation.
radiation. On the other hand, the polarization of the transmitted radiation is very different from the values calculated for pure Rayleigh scattering. The maximum value is 0.43 near the horizon and the polarization decreases toward the zenith. The concentration of aerosols is relatively much greater near the ground. The photons that reach the earth’s surface have a greater probability of colliding with an aerosol than do those reflected back to space. Thus the polarization values for the transmitted radiation are materially reduced from the values for pure Rayleigh scattering.

The polarization for other values of the surface albedo is also shown in Fig. 4. Even when $A = 1$, the polarization of the reflected radiation has the surprisingly large value of 0.37 near the horizon and an appreciable value at most other angles. The polarization of the transmitted radiation reaches a maximum value when $0.2 < \mu < 0.3$ and the surface albedo has any value other than zero. The radiation scattered from the earth’s surface is assumed to be unpolarized in agreement with the properties of a Lambert surface. Near the horizon there is a greater proportion in the transmitted radiation of the photons which have been scattered from the earth’s surface. Because of the large probability of small angle scattering from the aerosols, the photons, which are scattered from the ground and thus unpolarized, have a relatively large probability of being scattered by an aerosol through a small angle, while changing their direction from upward to downward.

The polarization when $\mu_0 = -0.15$ is shown in Fig. 5 for $A = 0, 0.2,$ and 1. When these results for the reflected polarization are compared with those for essentially pure Rayleigh scattering shown in Fig. 3, the differences caused by the aerosols are readily apparent. For $0^\circ \leq \phi \leq 30^\circ$ and $150^\circ < \phi < 180^\circ$, the major difference between the two is the lower value at the maximum and the larger region of negative polarization near both the solar and antisolar horizons when $\lambda = 0.4 \mu$. Note also the variation with albedo. For $60^\circ < \phi < 120^\circ$, the polarization has been reduced from $-0.8$ for essentially pure Rayleigh scattering to around $-0.6$ at most angles for $\lambda = 0.4 \mu$. The curve when $A = 1$ is especially interesting, since there is still appreciable polarization at most angles. There is some fluctuation in our Monte Carlo results from this long and complicated calculation, but the fluctuations in the value of the polarization in a typical case are of the order of 0.03.

The results already discussed are based on the Elterman aerosol distribution. Actually there is considerable evidence that the aerosol concentration varies with time over rather wide limits. In order to study the effects of changes of aerosol concentration, both the radiance and polarization at $\lambda = 0.4 \mu$ have been computed for the following cases: (1) pure Rayleigh scattering; (2) one-third of the normal aerosol amount; (3) the normal aerosol amount; (4) three times the normal aerosol amount. In all cases the same relative variation of aerosol concentration with height as given by Elterman is used, the same number of Rayleigh scattering centers at each altitude is used in each case.

The reflected and transmitted radiance are given in Fig. 6. These results show that the reflected radiance increases at angles near the nadir and the curves become flatter as the aerosol amount increases, when $A = 0$. The transmitted radiance shows more variation with aerosol amount. When $A = 0$ at all angles except those near the horizon, the transmitted radiance increases appreciably, in some cases by a factor of six, as the aerosol amount increases; near the horizon the transmitted radiance decreases as the aerosol amount increases. The strong increase of the radiance near the zenith is caused by the many small angle scattering events from the aerosol particles. Even when $A = 1$, there is still a strong increase of the transmitted radiance with aerosol amount at most angles.

The polarization of the reflected and transmitted radiation is shown in Fig. 7 when $A = 0$. The curve for pure Rayleigh scattering is shown for comparison; the number of Rayleigh scattering centers is kept fixed in all the calculations at this wavelength, and is equal at each altitude to the number given in the Elterman tables. As the aerosol amount increases, the polarization decreases in each case; the effect is more pronounced for the transmitted radiation than for the reflected. One reason for this is the relative concentration of the aerosol particles near the ground; it is more probable for the transmitted photon than for the reflected one to have collided with an aerosol than a Rayleigh particle. Another reason is that single scattered photons from an aerosol in the transmitted radiation have a negative polarization over most angles; this negative polarization is most effective in reducing the large positive polarization from photons that have made a similar collision with a Rayleigh scattering center.

The polarization of the reflected and transmitted
Fig. 5. Polarization of the reflected and transmitted radiation for $\lambda = 0.4\mu$, $\mu_0 = -0.15$, and $\Lambda = 0, 0.2,$ and 1. See caption to Fig. 3.

Fig. 6. Reflected and transmitted radiance for $\lambda = 0.4\mu$, $\mu_0 = -1$, and $\Lambda = 0$ and 1. Curves are shown for pure Rayleigh scattering plus ozone absorption, one-third of the normal aerosol amount, the normal aerosol amount, and three times the normal aerosol amount. The same number of Rayleigh scattering centers is also included in the last three cases as in the first.

Fig. 7. Polarization of the reflected and transmitted radiation for $\lambda = 0.4\mu$, $\mu_0 = -1$, and $\Lambda = 0$. See caption to Fig. 6.

Fig. 8. Polarization of the reflected and transmitted radiation for $\lambda = 0.4\mu$, $\mu_0 = -1$, and $\Lambda = 1$. See caption to Fig. 6.
radiation is shown in Fig. 8 when \( \lambda = 1 \). In the case of the reflected radiation, the polarization near the horizon is reduced by about a factor of two compared to the \( \lambda = 0 \) case. The different aerosol amounts also have a smaller influence on the polarization when \( \lambda = 1 \). The polarization of the transmitted beam is sensitive both to the surface albedo and the aerosol amount.

The calculated polarization is given in Fig. 9 for six different values of the surface albedo when \( \mu_0 = -1 \). The polarization curves for \( \lambda = 0.4 \mu \) and 0.7 \( \mu \), as shown in Figs. 4 and 9, respectively, are rather similar in shape, but with consistently lower absolute values for the polarization at the higher wavelength. This occurs because of the greater proportion of aerosol over Rayleigh scattering at the higher wavelength.

The calculated polarization is given in Figs. 10 and 11 for three values of the surface albedo when \( \mu_0 = -0.15 \). These polarization curves have a similar shape but consistently show lower values for the magnitude of the polarization than do the corresponding curves for \( \lambda = 0.4 \mu \) as given in Fig. 5.

The reflected and transmitted radiance is shown in Fig. 12 for \( \lambda = 0.7 \mu, \mu_0 = -1 \), and for the following cases: (1) a pure Rayleigh atmosphere, with ozone absorption; (2) one-third the normal aerosol amount; (3) the normal aerosol amount; (4) three times the nor-

**Polarization at 0.7 \( \mu \)**

At a wavelength of 0.7 \( \mu \) the aerosol attenuation coefficient is larger than the Rayleigh attenuation coefficient up to an altitude of 5 km, while the opposite is true at higher altitudes. Ozone absorption is important at this wavelength; the ozone absorption coefficient is larger than the Rayleigh attenuation coefficient at all altitudes above 20 km which are considered in this calculation. Rayleigh and aerosol scattering as well as ozone absorption is taken into account at all altitudes and the coefficients are varied with altitude as given in Elterman's tables.6

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Fig. 9. Polarization of the reflected and transmitted radiation for \( \lambda = 0.7 \mu, \mu_0 = -1 \), and \( \lambda = 0, 0.2, 0.4, 0.6, 0.8, \) and 1.

Fig. 10. Polarization of the reflected and transmitted radiation for \( \lambda = 0.7 \mu, \mu_0 = -0.15 \), and \( \lambda = 0 \) and 0.2. See caption to Fig. 3.

Fig. 11. Polarization of the reflected and transmitted radiation for \( \lambda = 0.7 \mu, \mu_0 = -0.15 \), and \( \lambda = 1 \). See caption to Fig. 3.
mal aerosol amount. The number of Rayleigh scattering centers is held fixed while the aerosol amount is changed. In each case the number of aerosols at each height is determined by multiplying the number in the Elterman table by the indicated factor. When \( A = 0 \), the reflected radiance increases appreciably at all angles as the aerosol amount increases. The variation is rather small when \( A = 1 \).

The transmitted radiance shows an even stronger dependence on the aerosol amount; the curve for three times the normal aerosol amount is fifty times greater at some angles than the curve for pure Rayleigh scattering plus ozone absorption. Furthermore, the shape of the curve changes from one with a maximum at the horizon and a minimum at the zenith for pure Rayleigh scattering to one with a maximum at the zenith and a minimum at an intermediate angle when there are an appreciable number of aerosol particles. The variation with aerosol number is much more pronounced at \( \lambda = 0.7 \mu \) than it is at \( \lambda = 0.4 \mu \), as is shown by a comparison of Figs. 6 and 12. The aerosol scattering is much more important compared to Rayleigh scattering at \( \lambda = 0.7 \mu \) than at \( \lambda = 0.4 \mu \).

The polarization of the reflected and transmitted radiation is shown in Fig. 13 for \( \lambda = 0.7 \mu \) and \( A = 0 \). As the aerosol amount varies from zero to three times the normal amount as given in Elterman’s tables, the polarization shows considerable variation. In all cases the polarization decreases as the aerosol amount increases. The variation of the polarization with aerosol amount is considerably more pronounced at \( \lambda = 0.7 \mu \) than at \( \lambda = 0.4 \mu \), because of the greater relative importance of aerosol scattering compared to Rayleigh scattering at the former wavelength.

The polarization is given in Fig. 14 when \( A = 1 \). In this case there is little variation of the polarization of the reflected radiation with the aerosol amount. On the other hand, the polarization of the transmitted radiation does depend appreciably on the aerosol amount.

### Polarization at \( \lambda = 1.67 \mu \)

The Rayleigh scattering is only of minor importance at 1.67 \( \mu \). The aerosol attenuation coefficient is larger than the Rayleigh below 8 km and above 15 km when the Elterman aerosol distribution is used. There is no ozone absorption. The optical thickness of the atmosphere is 0.123.

The polarization of the reflected and transmitted radiation is shown in Fig. 15 for \( A = 0, 0.2, 0.4, 0.6, 0.8, \) and \( 1 \). The polarization values are in general the smallest of those encountered at any of the wavelengths considered here. The polarization curve of the reflected radiation becomes appreciably negative when \( A = 0 \) near the nadir. This occurs because of the strong negative single scattering polarization for scattering through angles of nearly 180° from the aerosols and the relatively small contribution of Rayleigh scattering at this wavelength. The polarization of the transmitted radiation never rises above 0.2. In both cases the polarization decreases rapidly as the surface albedo increases.

The polarization when \( \mu_0 = -0.15 \) and \( A = 0 \) is shown in Fig. 16. The general trend of the curves is on the whole similar to that at \( \lambda = 0.7 \mu \), but in general the magnitude of the polarization is smaller at the longer wavelength. In the incident plane the polarization...
continues to reach its maximum value near the zenith and to be relatively constant in the plane that is perpendicular to the incident plane.

The same curves are shown in Fig. 17 for the case when \( A = 1 \). As expected, the polarization of the reflected beam changes more rapidly with \( A \) than does that of the transmitted beam. The polarization of the reflected beam is never greater in magnitude than 0.1 and at most angles is quite close to zero. The polarization of the reflected radiation shows more variation with azimuthal angle.

**Conclusions**

At the uv wavelengths of 0.27 \( \mu \) and 0.30 \( \mu \) the polarization is almost entirely determined by multiple Rayleigh scattering. When the sun is at the zenith, the polarization of the reflected radiation decreases monotonically from the horizon to the nadir. When the sun is near the horizon, the polarization of the radiation near the incident plane reaches a maximum of +0.8 near the nadir, while the radiation at azimuthal angles near 90° from the incident plane is nearly independent of \( \mu \) with a value of −0.8.

At \( \lambda = 0.4 \mu \) the polarization of the reflected radiation is as large as 0.8 (\( A = 0 \)) and 0.37 (\( A = 1 \)), since the Rayleigh scattering is still more important than the aerosol scattering at most levels in the atmosphere. The polarization of the transmitted radiation is very sensitive to the aerosol amount in the atmosphere.

At \( \lambda = 0.7 \mu \) both the radiance and the polarization of the reflected and transmitted radiation depend strongly on the aerosol amount when \( A = 0 \), and to a lesser degree as \( A \) increases. The numerical values of the polarization decrease in general as the wavelength increases until at \( \lambda = 1.67 \mu \) no polarizations larger than 0.24 are calculated, even for \( A = 0 \).
References


Photos by Dreyfus

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