Atmospheric correction of ocean-color sensors: effects of the Earth’s curvature

Kuiyuan Ding and Howard R. Gordon

We investigate the influence of the curvature of the Earth on a proposed atmospheric-correction scheme for the Sea-Viewing Wide-Field-of-View Sensor (SeaWiFS) by simulating the radiance exiting the top of a spherical-shell atmosphere and inserting the result into the proposed correction algorithm. The error in the derived water-leaving reflectance suggests that the effects of the curvature are negligible for solar zenith angles ($\theta_0$) ≤ 70°. Furthermore, for $\theta_0 > 70°$ the error in atmospheric correction can usually be reduced if the molecular-scattering component of the top of the atmosphere reflectance ($ρ$) is computed with a spherical-shell atmosphere radiative transfer code. Also, for $\theta_0 > 70°$ the error in atmospheric correction in a spherical-shell atmosphere, when $ρ$ is computed with a spherical-shell model, can be predicted reasonably well from computations made with plane-parallel atmosphere radiative transfer codes. This implies that studies aimed at improving atmospheric correction can be made assuming plane-parallel geometry and that the investigator can be confident when $\theta_0 > 70°$ that any improvements will still be valid for a spherical-shell atmosphere as long as $ρ$ is computed in spherical-shell geometry. Finally, a scheme for computing $ρ$ in a spherical-shell atmosphere in a relatively simple manner is developed.

1. Introduction
Ocean color contains information about the constituents of the ocean, because it can be related to their absorption and scattering properties. The most important constituents influencing ocean color in the open ocean are the phytoplankton, microscopic plant organisms that photosynthesize the marine light field. During photosynthesis, phytoplankton take in carbon dioxide and produce carbohydrates and thus form the primary link in the marine food chain. This production of carbohydrates is termed primary production. A portion of the carbon that they take up during the process will eventually reach the ocean floor, so understanding the spatial–temporal variability in the concentration of phytoplankton in the ocean will aid in the understanding of the ocean’s role in the global carbon cycle. In 1978 NASA launched the Coastal Zone Color Scanner (CZCS) on the satellite Nimbus-7 to study the feasibility of measuring the phytoplankton concentration by the use of space-based ocean-color sensors. The CZCS mission demonstrated that measurement of phytoplankton light-harvesting pigments, and possibly even primary productivity, could be made on a global scale.

Of the total signal received by an ocean-color sensor at satellite altitudes, typically in the blue over 80% is from the contribution of scattering by molecules and particles (aerosols) in the atmosphere. The ocean-color signal (the water-leaving radiance $L_w$), from which we derive the phytoplankton concentration, is buried in the total radiance ($L_t$) reaching the sensor. The process of retrieving $L_w$ from $L_t$ is usually referred to as atmospheric correction. Gordon developed an atmospheric-correction algorithm for processing the CZCS data. In most circumstances it performed reasonably well, considering the radiometric sensitivity of the CZCS instrument, and the entire global data set has been processed with it. However, future ocean-color sensors, such as the Sea-Viewing Wide-Field-of-View Sensor (SeaWiFS) and the Moderate Resolution Imaging Spectroradiometer, which are scheduled to be launched by NASA in 1994 and 1998, respectively, will possess sensitivities superior to CZCS and will require a more accurate atmospheric correction. To meet the needs of these instruments, a systematic investigation of processes that were ignored in the CZCS algorithm—the influence of multiple scattering, the influence of polarization on the computation of the molecular-scattering component, and the influence of wind-induced sea-
surface roughness on the molecular-scattering component\textsuperscript{13} and on atmospheric correction in general\textsuperscript{14}—was undertaken. These studies have led to a preliminary atmospheric-correction algorithm for SeaWiFS\textsuperscript{15}; however, several aspects remain to be considered. For example, in the CZCS and the proposed SeaWiFS algorithms, it is assumed that the atmosphere is a plane-parallel medium rather than the more appropriate spherical-shell medium. Adams and Kattawar\textsuperscript{16} showed that, for a simple one-layer Rayleigh-scattering medium with a totally absorbing lower boundary, there could be significant differences between the radiance reflected from plane-parallel and spherical-shell media for large solar zenith angles. In this paper we examine the influence of the Earth's curvature on the proposed SeaWiFS algorithm.

We begin with the computational techniques for working out the radiative transfer problem for a spherical-shell atmosphere. Next, we apply the simulated radiances from such an atmosphere to the SeaWiFS atmospheric correction algorithm to look at the Earth-curvature effects on the algorithm. Finally, some practical considerations for including the Earth-curvature effects in the proposed SeaWiFS correction method are discussed.

2. Computational Procedure

The distribution and propagation of the light field in an optical medium are governed by the radiative transfer equation (RTE). In general the RTE takes the following form:

$$\xi \cdot \nabla L(r, \xi) = -c(r)L(r, \xi) + \int \beta(r, \xi') d\Omega(\xi') + J(r, \xi),$$

where $L$ is the radiance to be determined, $c$ is the beam attenuation coefficient of the medium, $r$ is the position vector where the radiance $L$ is measured, $\xi$ is a unit vector in the direction in which the radiance is traveling, $\beta$ is the volume-scattering function (differential scattering cross section per unit volume) of the medium, $d\Omega(\xi')$ is the differential of the solid angle around the direction $\xi'$, and $J$ represents the total contribution from any internal sources. The beam attenuation coefficient is the sum of the scattering coefficient

$$b(r) = \int \beta(r, \xi') d\Omega(\xi'),$$

and the absorption coefficient $a(r)$.

In our study we need to solve the RTE in two geometries. The first is a plane-parallel atmosphere (PPA) in which $\beta$, $c$, $L$, and $J$ are all invariant under translations parallel to the boundaries. In this case the RTE simplifies considerably. The second geometry is for a spherical-shell atmosphere (SSA) in which $\beta$ and $c$ are functions of the radius of the shell only, i.e., functions of $r = |r|$. However, because the incident illumination does not have spherical symmetry, $L$ and $J$ still depend on $r$.

We have chosen a Monte Carlo procedure to solve the RTE in spherical geometry. There are basically four major steps in the Monte Carlo procedure. First, a photon is sent into the medium in the same direction as the solar beam; second, the distance the photon will travel before being absorbed or scattered in the medium is determined from random sampling based on the beam attenuation coefficient of the medium; third, at the point of interaction, the contribution of the photon to the radiance (the estimator) at a particular point that is due to this interaction is calculated and collected; and fourth, the new direction that the photon will travel in the medium is generated by sampling the phase function ($P = \beta/b$). This process is repeated until the photon exits the medium, and then a new photon is initiated. Because $L$ depends on the absolute position of the detector in the medium, we have chosen to solve the RTE with a backward Monte Carlo (BMC) technique as used by Collins et al.,\textsuperscript{17} Adams and Kattawar,\textsuperscript{16} and Gordon.\textsuperscript{18} This is more efficient for determining radiometric quantities at a point. The BMC works in the same way as the normal Monte Carlo except that the positions of the source (the Sun) and the receiver (the sensor) are interchanged; i.e., photons are started from the detector and traced through the medium. At the $i$th interaction point in the medium the probability that the photon will be scattered in the direction of the Sun is recorded. This probability is $(\omega_0 PT)_i$, where $\omega_0 = b/c$, $P$ is the phase function for scattering from the photon's direction of propagation to the Sun, and $T$ is the atmospheric transmittance from the interaction point to the Sun. The transmittance is given by $T = \exp[-\int c(l)dl]$, where $c(l)$ is the extinction coefficient as a function of the distance $l$ along the path $p$ that would be taken by a photon propagating from the interaction point to the top of the atmosphere in a direction toward the Sun. The photon is then permitted to scatter, and the process continues until the photon leaves the medium. The radiance at the detector is estimated by

$$L = \frac{\sum_i (\omega_0 PT)_i}{N},$$

where $F_0$ is the incident extraterrestrial solar irradiance, $N$ is the total number of photons used in the simulation, and the sum is the accumulation of the estimator collected at each interaction.

Figure 1 describes the geometry of the RTE problem of the SSA model. The $z$ axis of the coordinate system points to the sensor position from the origin (the center of the Earth), the $x$ axis is the projection of the solar beam direction on the plane perpendicular to the $z$ axis, and the $y$ axis is then determined by the right-hand rule. The atmosphere is assumed to be a two-layer medium: the Rayleigh-scattering molecules are in the upper layer (18 km thick); and the
aerosols, small particles suspended in the air, which
typically scatter strongly in the forward direction, are
in the lower layer (2 km thick). The Fresnel-
reflecting ocean surface is the inner boundary of the
atmosphere. As in an actual remote-sensing situation,
the satellite sensor is located at a height of 705
km above the Earth's surface. Of course all the
geometrical quantities are adjustable parameters in
the code for the computation. As the Monte Carlo
procedure is backward, the photon will be injected
into the medium from the sensor position in the
viewing direction (θ, φ), where φ is not shown in Fig.
(see Appendix A).

When the photon hits the sea surface, it will be
reflected. For a smooth ocean surface, the new
direction after the reflection can be decided simply
from Fresnel's laws of reflection. For the contribu-
tion to the radiance, when the Fresnel-reflecting
surface is included, there are two paths that the
photon can take toward the Sun at each point of
interaction in the atmosphere. The photon can be
scattered directly in the Sun's direction, or it can also
be first scattered toward the ocean surface and then
reflected toward the Sun (Fig. 2). A difficulty for the
evaluation of the contribution of the second part is
the determination of the proper position on the ocean
surface at which the photon has to reflect to be
directed toward the Sun. For a smooth ocean surface
this turns out to be governed by a nonlinear
equation involving the related angles and distances
(see Appendix B). This equation was solved by the
use of iteration, with convergence usually achieved in
two or three steps.

The code that executes the BMC procedure has
been tested extensively. For a one-layer Rayleigh-
scattering medium with a totally absorbing ocean
surface, the radiances generated were found to agree
very well (differences ≤ 0.3%) with computations of
Adams and Kattawar16 for the same conditions.

Also, comparisons were made with the PPA results
calculated with the successive-order-of-scattering
method13,14,20 for several situations. It was found
that the SSA computations satisfactorily approached
the PPA radiances as the radius of the Earth was
increased to very large values, e.g., 10⁹ km. We have
also included a wind-roughened Fresnel-reflecting
ocean surface obeying the Cox and Munk21 slope
distribution in the SSA model; however, the special
techniques developed to deal with roughness will not
be described here, because only a smooth surface is
considered.

3. Curvature Effects on SeaWiFS Atmospheric
Correction

The radiometric specifications of the SeaWiFS instru-
ment are provided and compared with the CZCS in
Table 1. The various quantities are given in reflec-
tance units, where a reflectance ρ associated with a
radiance L is defined to be \( \pi L / F_0 \cos \theta_0 \). ρ is the
saturation reflectance, ρ is a typical value for the
reflectance at the top of the atmosphere, and ρ is the
water-leaving reflectance (at the sea surface) for very
clear ocean water, e.g., the Sargasso Sea in summer.
NEAp is the noise-equivalent reflectance, and λ is the
wavelength. Because the SeaWiFS radiometric sen-

Table 1. Comparison of the Radiometric Performance of SeaWiFS and
CZCS for \( \theta_0 = 60° \) Near the Scan Edge

<table>
<thead>
<tr>
<th>Band</th>
<th>λ (nm)</th>
<th>ρ_{max}</th>
<th>ρ_{t}</th>
<th>ρ_{w}</th>
<th>NEAp SeaWiFS</th>
<th>NEAp CZCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>402-422</td>
<td>0.50</td>
<td>0.34</td>
<td>0.040</td>
<td>0.00068</td>
<td>—</td>
</tr>
<tr>
<td>2</td>
<td>433-453</td>
<td>0.46</td>
<td>0.29</td>
<td>0.038</td>
<td>0.00043</td>
<td>0.0011</td>
</tr>
<tr>
<td>3</td>
<td>480-500</td>
<td>0.36</td>
<td>0.23</td>
<td>0.024</td>
<td>0.00034</td>
<td>—</td>
</tr>
<tr>
<td>4</td>
<td>500-520</td>
<td>0.30</td>
<td>0.19</td>
<td>0.009</td>
<td>0.00031</td>
<td>0.00058</td>
</tr>
<tr>
<td>5</td>
<td>545-565</td>
<td>0.25</td>
<td>0.154</td>
<td>0.004</td>
<td>0.00027</td>
<td>0.00064</td>
</tr>
<tr>
<td>6</td>
<td>660-680</td>
<td>0.17</td>
<td>0.105</td>
<td>0.0004</td>
<td>0.00023</td>
<td>0.00051</td>
</tr>
<tr>
<td>7</td>
<td>745-785</td>
<td>0.15</td>
<td>0.081</td>
<td>—</td>
<td>0.00018</td>
<td>—</td>
</tr>
<tr>
<td>8</td>
<td>845-885</td>
<td>0.13</td>
<td>0.069</td>
<td>—</td>
<td>0.00015</td>
<td>—</td>
</tr>
</tbody>
</table>
sitivity will be superior to that of the CZCS, through a lower NE\(\Delta \rho\) (Table 1) and the adoption of a smaller quantization increment (10-bit as opposed to 8-bit), the requirement for the atmospheric-correction accuracy accordingly will be higher. The goal of the SeaWiFS correction is to recover the water-leaving reflectance \(\rho_w\) with no more than 5% error in the blue. At 443 nm, \(\rho_w\) is approximately 0.02–0.04 for clear water, which implies that the error \(\Delta \rho_w(443)\) should be \(\leq 0.001–0.002\).

The total signal received by a satellite ocean sensor is composed of several parts, i.e.,

\[
\rho(\lambda) = \rho_r(\lambda) + \rho_a(\lambda) + \rho_{ra}(\lambda) + \tau_p(\lambda),
\]

where \(\rho_r\) is the contribution from Rayleigh scattering, \(\rho_a\) is the contribution from aerosol scattering, \(\rho_{ra}\) represents the interaction of Rayleigh and aerosol scattering, and \(\tau_p\) is the desired water-leaving radiance (\(t\) is the diffuse transmittance from the ocean surface to the sensor). In our study of the Earth-curve effects we compute the reflectances by solving the RTE with the SSA model. This provides simulated values of \(\rho_t\) that include the effects of the Earth’s curvature. These simulated radiances are then inserted into the atmospheric-correction algorithm, which assumes plane-parallel geometry, and \(\tau_p\) is derived. The error \(\Delta \rho = t \Delta \rho_w\) was then computed.

We examined the atmosphere-correction algorithm proposed for SeaWiFS by Gordon and Wang. Briefly, the increased sensitivity of SeaWiFS over CZCS requires the consideration of multiple scattering in the atmosphere. Through simulations that used the PPA approximation, they found that the effects of multiple scattering are dependent on the model used to describe the aerosol scattering. Thus, in their scheme, aerosol models are used to include the multiple-scattering effects. The appropriate model is chosen from several candidates based on the variation of \(\rho_r - \rho_a\) between bands 7 and 8, in which \(\rho_w\) can be taken to be zero. The candidate aerosol models were taken from those developed by Shettle and Penn for LOWTRAN-6. In particular, Gordon and Wang used the Maritime and Tropospheric models, and introduced a Coastal model containing half the fraction of the sea-salt aerosol that was in the Maritime model. The Coastal model simulates situations that may be expected to occur near the coast (larger continental influence). With the resulting size distributions and refractive indices, Mie theory was used to compute the aerosol optical properties for the SeaWiFS bands as a function of the relative humidity (RH). Three values of RH (70, 90, and 98%) were used for each of the three models (Maritime, Coastal, and Tropospheric) for a total of nine candidate models. When they conducted tests using \(\rho_t\) computed with the Maritime, Coastal, and Tropospheric models with a RH of 80% (not among the candidate models), Gordon and Wang found that the resulting \(\Delta \rho\) was usually in the right range, i.e., \(\leq 0.001–0.002\).

Because the Gordon and Wang algorithm was developed for a PPA, it is important to understand its performance in the more realistic SSA. Thus we used the SSA code to simulate \(\rho_t\) for each aerosol model that Gordon and Wang used to test the algorithm. The aerosol optical thickness at 865 nm, \(\tau_a(865)\), was taken to be 0.2. This is approximately twice the value of \(\tau_a\) at 800 nm over the North Atlantic in situations in which air-mass trajectory analysis suggests a presence of only a maritime aerosol. The resulting \(\rho_t\) was then used as input into an implementation of the correction algorithm using 12 candidate models, and the error in correction \(\Delta \rho\) was computed at the center of the SeaWiFS scan (nadir viewing) and at the edge of the scan (\(\theta\) in Fig. 1 was \(\sim 45^\circ\) and in the \(y-z\) plane). The results of this exercise are presented in Fig. 3. Figures 3(a)–3(f) each contain three curves. The filled circles provide the error estimation as a function of \(\theta\), when \(\rho_t\) is simulated with the PPA, i.e., the results originally presented by Gordon and Wang in which the SSA-developed algorithm is tested with PPA-generated pseudo data. The filled squares provide the error when the SSA-generated \(\rho_t\) is used in the algorithm. In this case the PPA is used to compute \(\rho_t\). [Recall that the spectral variation of \(\rho_t - \rho_a\) in the near infrared is used in the algorithm to select an aerosol model from the candidate models and that the 443-nm \(\rho_t\) must be subtracted from \(\rho_t\) in the derivation of \(\tau_p(443)\); see Eq. (1)].

The filled triangles provide the error when \(\rho_t\) is computed with the SSA. Figure 3 shows that the PPA algorithm works well with SSA pseudo data for \(\theta \leq 70^\circ\); however, for \(\theta > 70^\circ\) there can be very large errors. In contrast, when \(\rho_t\) is computed with the SSA model (filled triangles), the algorithm provides essentially the same error pattern as when both \(\rho_t\) and \(\rho_a\) are computed with the PPA model. In other words, as long as \(\rho_t\) is computed with an SSA model, the PPA-developed algorithm should perform in an SSA in a manner quantitatively similar to its performance in a PPA. This is important because it suggests that research toward improving the algorithm can be carried out with the less costly PPA radiative transfer codes. Figures 3(e) and 3(f) emphasize this point. Clearly the algorithm developed by Gordon and Wang does not perform well for the Tropospheric model with \(\theta > 60^\circ\). This is presumably due to the fact that \(\tau_a(\lambda)\) increases rapidly with decreasing \(\lambda\), with \(\tau_a(443) = 0.5\) for this model. The results presented in Fig. 3 give us confidence that research toward improving the algorithm can be carried out with the SSA model and that any improvement will carry over to an SSA as long as \(\rho_t\) is computed with the SSA model.

We also carried out similar simulations to assess the effect of the Earth’s curvature on the performance of the original CZCS correction algorithm.
Fig. 3. Error in atmospheric correction at 443 nm with $\tau_{\text{eff}}(865) = 0.2$: (a) Maritime model at the scan center, (d) Maritime model at the scan edge, (b) Coastal model at the scan center, (e) Coastal model at the scan edge, (c) Tropospheric model at the scan center, (f) Tropospheric model the scan edge.

Owing to the fact that the original algorithm did not satisfactorily address multiple scattering, the algorithm's inherent accuracy is less than that of the SeaWiFS algorithm, and we found that neglecting the Earth's curvature often actually improved the performance of the algorithm somewhat for $\theta_0 \geq 70^\circ$. Thus there appears to be no necessity to reprocess high-latitude CZCS imagery with an algorithm in which Earth-curvature effects are incorporated into the computation of $\rho_r$. 

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4. Calculation of the Rayleigh Scattering

In Section 3 we showed that for the SeaWiFS atmospheric-correction algorithm the Rayleigh-scattering component must be calculated with the SSA model to achieve satisfactory accuracy at large solar zenith angles ($\theta_0 \geq 70^\circ$). For the results presented in Fig. 3 this calculation was done through the Monte Carlo simulation technique, which is computation intensive and, therefore, impractical for the processing of ocean-color imagery or for the preparation of lookup tables for the Rayleigh-scattering component. Thus we have tried to develop an alternative approach.

Our approach is based on the observation of Adams and Kattawar\textsuperscript{16} that the ratio of single-scattering radiance to total radiance is nearly the same for a PPA model and a SSA model under the same conditions. That is, if we write the total radiance as $T = S + M$, where $S$ and $M$ are the single- and multiple-scattering components, respectively, and use the subscripts PPA and SSA to indicate a PPA model and a SSA model, respectively, Adams and Kattawar found that

$$\frac{S_{\text{SSA}}}{T_{\text{SSA}}} \approx \frac{S_{\text{PPA}}}{T_{\text{PPA}}}$$

for the same optical thickness, viewing direction, and solar zenith angle. Based on the reciprocity principle\textsuperscript{25} we were able to calculate the single-scattering component for the case of a smooth ocean surface at the bottom of the SSA (Appendix B). Our results were extensively compared with Monte Carlo simulations under the same conditions, and excellent agreement was obtained. Adams and Kattawar tested only cases with $\tau_r = 0.25$ and 1.0, where $\tau_r$ is the optical thickness of the Rayleigh-scattering molecules. We examined the conjecture [Eq. (2)] for smaller optical thicknesses, i.e., the case of the longer-wave SeaWiFS bands, and found that the two ratios differed by $\sim$2%. This would be a significant error in the SeaWiFS correction algorithm. However, we also observed that for the same viewing direction and optical thickness, the ratio of the multiple-scattering radiance for the SSA model ($M_{\text{SSA}}$) to that for the PPA model ($M_{\text{PPA}}$) remains nearly constant for different solar zenith angles ($\theta_0$); i.e.,

$$\alpha \equiv \frac{M_{\text{SSA}}}{M_{\text{PPA}}}$$

is nearly independent of $\theta_0$. This is shown in Fig. 4.

Fig. 4. Quantity $M_{\text{SSA}}/M_{\text{PPA}}$ changes very slowly with $\theta_0$ for different SeaWiFS bands. The curves from top to bottom refer to $\lambda = 412$, 550, 670, and 865 nm, $\phi = 90^\circ$, with (a) $\theta = 5^\circ$, (b) $\theta = 20^\circ$, (c) $\theta = 40^\circ$, and (d) $\theta = 60^\circ$. 

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However, $\alpha$ is a function of the viewing angle $\theta$ for a fixed solar zenith angle. This is demonstrated in Fig. 5 for $\theta_0 = 40^\circ$ and for several SeaWiFS bands. Note that for a fixed $\theta_0$, $\alpha$ changes very smoothly with viewing angles.

From these observations we developed the following empirical scheme for deriving the total scattering radiance for the SSA model from the PPA computations. First, for different wavelengths, i.e., $T_R$, we compute the SSA total radiance exactly with Monte Carlo simulation for one solar zenith angle, e.g., $\theta_0 = 40^\circ$, and for several viewing directions to estimate the $\alpha$'s. The $\alpha$'s obtained in this manner are then used for all other solar zenith angles, and if we knew $M_{SSA}$, we could combine it with $S_{SSA}$ to obtain $T_{SSA}$. Formally, we let

$$\beta = \frac{M_{SSA}}{T_{SSA}} = \frac{\alpha M_{PPA}}{T_{PPA}},$$

and note that because multiple scattering is usually quite small throughout the visible [e.g., Gordon et al.\textsuperscript{12} show that the linearized approximation to single scattering ($\tau_r \ll 1$) is usually in error by $\sim 4\%$ compared with the exact multiple scattering (including polarization) result] a small error in $\beta$ will not be significant. Thus we approximate $\beta$ by

$$\beta \approx \frac{\alpha M_{PPA}}{T_{PPA}}.$$  \hspace{1cm} (4)

Finally, the total SSA radiance is estimated to be

$$T_{SSA} = \frac{S_{SSA}}{1 - \beta},$$  \hspace{1cm} (5)

where $S_{SSA}$ is the SSA single-scattering radiance, which can be evaluated relatively quickly.

We applied the above scheme for several SeaWiFS bands and a few selected solar and viewing angles. The results given in Tables 2 and 3 are for the 412-nm and 865-nm bands, respectively. For each band the $\alpha$ parameters are evaluated at $\theta_0 = 40^\circ$, $\phi = 90^\circ$, and $\theta = 5.01^\circ$, $20.9^\circ$, $39.9^\circ$, and $60.3^\circ$ (chosen because PPA computations are provided only at Gaussian quadrature points from a successive-order-of-scattering solution to the RTE) and are then applied to different $\theta_0$'s. The SSA total radiances were calculated with the Monte Carlo simulations and compared with the estimates by the use of the above scheme. In Tables 2 and 3 PPA is the relative error of estimation from directly replacing the SSA radiance by its PPA counterpart, A&K is the relative error from assuming that the conclusion of Adams and Kattawar about the ratio of single scattering to total scattering holds for all SeaWiFS bands, and D&G is the relative error from using the procedure described above. The D&G column also gives us the differences between the Monte Carlo simulated reflectance and the estimate with our method, in terms of the SeaWiFS digital counts (DC's), the quantization interval of the sensor. For the SeaWiFS sensor, 1 DC is equal to $\frac{P_{max}}{10^{12}}$, where $P_{max}$ is the saturation reflectance for the respective band and 1024 is the total number of quantization intervals. It can be seen that the differences were usually less than or sometimes approximately 1 DC for our method compared with several DC's in some geometries for the other two methods.

We envisage using this technique in the following manner. First, from tables for $r_T$, derived with a PPA code (including polarization), the value of $T_{PPA}$ is determined. Next, $S_{PPA}$ is computed directly, yielding $M_{PPA}$. Finally, coarse resolution tables of $\alpha(0, \lambda)$ for $\theta_0 = 40^\circ$ are used to estimate $\beta$ [Expression (4)],

\begin{table}
\centering
\begin{tabular}{cccccc}
\hline
$\theta_0$ (deg) & $\theta$ (deg) & PPA & A&K & D&G & D&G (DC) \\
\hline
60 & 5.01 & -0.11 & -0.02 & -0.02 & -0.08 & -

\end{tabular}
\end{table}

\begin{table}
\centering
\begin{tabular}{cccccc}
\hline
$\theta_0$ (deg) & $\theta$ (deg) & PPA & A&K & D&G & D&G (DC) \\
\hline
60 & 5.01 & -0.11 & -0.02 & -0.02 & -0.08 & -

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$\theta_0$ (deg) & $\theta$ (deg) & PPA & A&K & D&G & D&G (DC) \\
\hline
60 & 5.01 & -0.11 & -0.02 & -0.02 & -0.08 & -

\end{tabular}
\end{table}
and then Eq. (5) is used to generate the final result. Because the PPA tables of $\rho_r$ must be resident on any processing system designed for SeaWiFS imagery, the only additional requirements are the coarse resolution tables of $\alpha(\theta, \phi)$ and the computation of $S_{SSA}$ (Appendix B). Note that the PPA-computed values of $\rho_r$ can be used for $\theta_0 \leq 70^\circ$.

5. Conclusions

The results presented here suggest that the effects of the curvature of the Earth on the atmospheric correction of SeaWiFS (and ocean-color imagery in general) appear to be negligible for solar zenith angles $\leq 70^\circ$, even for viewing angles as large as $45^\circ$. For $\theta_0 > 70^\circ$ we can reduce the error in atmospheric correction by computing $\rho_r$ for an SSA, and the resulting error can be predicted reasonably well from computations made with PPA radiative transfer codes. Thus research aimed at improving atmospheric correction can be conducted assuming plane-parallel geometry, and the investigator can be confident that the results will still be valid at large solar zenith angles in spherical-shell geometry as long as $\rho_r$ is computed with a spherical-shell model. A scheme based on a modification of the Adams and Kattawar\textsuperscript{16} observation—that the ratio of the single-scattered radiance to the total radiance is approximately the same for SSA's and PPA's—is presented for derivation of $\rho_r$ for an SSA in a simple manner. The resulting $\rho_r$ is sufficiently accurate to apply to SeaWiFS; i.e., the error in $\rho_r$ is $\leq 1$ DC.

Tests of the CZCS atmospheric-correction algorithm\textsuperscript{7,12} with SSA-generated pseudo data suggest that correction of Earth-curvature effects is unimportant for that sensor because the error is overshadowed by the inherent CZCS algorithm inaccuracy.

Appendix A: Spherical-Shell–Plane-Parallel Coordinate Transformations

To compare the SSA results with those of the PPA model, it is necessary to transform the angles in the spherical-shell coordinate system (the absolute frame) with those in the plane-parallel coordinate system (the local frame) placed tangent to the Earth at the viewed position. In the absolute frame, the $z$ axis points from the origin at the center of the Earth to the sensor position in space, the $x$ axis points in the projection of the incident solar beam on the plane perpendicular to the $z$ axis, and the $y$ axis is defined accordingly (Fig. 6). In the local frame the $z'$ axis points in the radial direction at the point viewed by the sensor on the outer surface of the atmosphere, and the $x'$ axis is defined by the projection of the incident solar beam on the plane perpendicular to the $z'$ axis (Fig. 7).

Let $\theta_0$ be the solar angle, then the direction vector of the solar beam in the absolute frame is

$$\mathbf{s} = (\sin \theta_0, 0, \cos \theta_0). \quad (A1)$$

Let $\theta$ and $\phi$ be the viewing angles (polar and azimuthal, respectively) of the sensor, then the viewing vector in the absolute frame can be written as

$$\mathbf{v} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta). \quad (A2)$$

Given $h$ (the satellite height), $R_E$ (the radius of the Earth), $R$ (the distance from the origin at the center of the Earth to the local point $P$ on the outer surface of the atmosphere), and the viewing angle $\theta$ (the angle between the local radial direction and the $z$ axis), $\omega$ can be easily found from

$$\cos \omega = \frac{(R_E + h) + l \cos \theta}{R},$$

where $l$ is the distance from the sensor to the point $P$. The direction of the solar beam in the absolute frame can be written as

$$\mathbf{e} = (\sin \theta_0 \cos \phi, \sin \theta_0 \sin \phi, \cos \theta_0). \quad (A3)$$
where \( l \), the distance from the sensor to the local point, is given by

\[
l = -(R_E + h)\cos \theta - [R^2 + (R_E + h)^2(\cos^2 \theta - 1)]^{1/2}.
\]

The representation of the \( z' \) axis in the absolute frame is

\[
z' = (\sin \omega \cos \phi, \sin \omega \sin \phi, \cos \omega).
\]

Therefore the solar angle in the local frame, \( \theta_0' \), can be determined from

\[
\cos \theta_0' = \delta \cdot z' = \sin \theta_0 \sin \omega \cos \phi + \cos \theta_0 \cos \omega.
\]  
\text{(A3)}

The \( y' \) axis is formed by

\[
y' = \frac{z' \times \delta}{|z' \times \delta|},
\]

which in the absolute frame becomes

\[
y' = \frac{1}{\sin \theta_0'} [(\cos \theta_0 \sin \omega \sin \phi)x + (\sin \theta_0 \cos \omega - \cos \theta_0 \sin \omega \cos \phi)y + (-\sin \theta_0 \sin \omega \sin \phi)z].
\]  
\text{(A4)}

The unit direction vector of the \( x' \) axis is then obtained:

\[
x' = y' \times z' = (-\sin \theta_0 \sin \omega \sin \phi)x + (\cos \theta_0 \sin \omega \cos \phi - \cos \theta_0 \sin \omega \cos \phi)y + [\cos \theta_0 \sin^2 \omega \sin^2 \phi - \sin \omega \cos \phi \sin(\sin \theta_0 \cos \omega - \cos \theta_0 \sin \omega \cos \phi)]z.
\]  
\text{(A5)}

Let \( \theta' \) be the angle between the \( z' \) axis and the viewing direction, then we have

\[
\cos \theta' = \delta \cdot z' = \cos(\theta - \omega).
\]  
\text{(A6)}

Let \( \phi' \) be the azimuth angle of the viewing direction in the local frame, then

\[
\delta \cdot x' = \sin \theta' \cos \phi' \quad \text{and} \quad \delta \cdot y' = \sin \theta' \sin \phi'.
\]

Combining Eqs. (A2), (A4), and (A5) and the above two equations, we can write

\[
\sin \phi' = \frac{\sin \theta_0 \sin \phi \sin(\theta - \omega)}{\sin \theta_0' \sin \theta'}
\]

\[
\cos \phi' = \frac{1}{\sin \theta_0' \sin \theta'} [\sin \theta \cos(\omega \cos \theta_0 \cos \phi - \sin \omega \cos \theta_0) + \cos \theta \sin \omega \cos(\sin \theta_0 \sin \omega - \sin \theta_0 \cos \omega \cos \phi)].
\]  
\text{(A7)}

From Equations (A3), (A6) and (A7), one can convert \( \theta_0, \theta, \) and \( \phi \) to their counterparts in the local frame, i.e., \( \theta_0', \theta', \) and \( \phi' \), respectively.

Conventionally, \( \theta_0 \) and \( \theta \) (as well as \( \theta_0' \) and \( \theta' \)) are referred to by the angles from the downward vertical. Then the relevant equations become

\[
\cos \theta_0' = \cos \theta_0 \cos \omega - \sin \theta_0 \sin \omega \cos \phi
\]

\[
\cos \theta' = \cos(\theta + \omega)
\]

\[
\sin \phi' = \frac{\sin \theta_0 \sin \phi \sin(\theta + \omega)}{\sin \theta_0' \sin \theta'}
\]

\[
\cos \phi' = \frac{1}{\sin \theta_0' \sin \theta'} [\sin \theta \cos \omega \cos \theta_0 \sin \phi + \sin \omega \cos \theta_0 + \cos \theta \sin \omega \cos \theta_0 \sin \omega + \sin \theta_0 \cos \omega \cos \phi],
\]

where \( \omega \) is determined from

\[
\cos \omega = \frac{(R_E + h) - l \cos \theta}{R}
\]

and \( l \) is given by

\[
l = (R_E + h) \cos \theta - [R^2 + (R_E + h)^2(\cos^2 \theta - 1)]^{1/2}.
\]

For a special case of \( \phi = 90^\circ \), the equations are simplified to be

\[
\cos \theta_0' = \cos \theta_0 \cos \omega
\]

\[
\cos \theta' = \cos(\theta + \omega)
\]

\[
\sin \phi' = \frac{\sin \theta_0 \sin(\theta + \omega)}{\sin \theta_0' \sin \theta'}
\]

\[
\cos \phi' = \frac{\sin \omega \cos \theta_0 \sin(\theta + \omega)}{\sin \theta_0' \sin \theta'}.
\]  
\text{(A9)}

These equations were used to compute the PPA results in the preparation of Fig. 3.

**Appendix B: Spherical-Shell Single Scattering**

In Appendix B we briefly describe the computation of \( S_{SSA} \) in Eq. (2). Our approach to the calculation of the single-scattering radiance for the SSA-modeled atmosphere is similar to the BMC method used in the calculation of the total radiance described in Section 7.104 APPLIED OPTICS / Vol. 33, No. 30 / 20 October 1994
2. However, instead of accumulating the estimator through a large number of interactions, we limit our considerations to processes involving a single interaction in the atmosphere. As shown in Fig. 8, light is sent into the atmosphere from the position of the sensor in the viewing direction (θ). The light enters the atmosphere at point A. The radiance consists of three contributions: (1) scattering toward the Sun’s direction at an interaction at the point E on the line from A to B; (2) specular (Fresnel) reflection of the beam from the (smooth) sea surface at B and subsequent scattering at G into the direction of the Sun; and (3) scattering toward the sea surface by an interaction at E, followed by specular reflection into the solar direction from the smooth surface at P and propagation to the top of the atmosphere (Q); i.e.,

\[ \mathbf{L} = \mathbf{L}_1 + \mathbf{L}_2 + \mathbf{L}_3. \]

Using results from the evaluation of the Monte Carlo estimator discussed in Section 2, we can derive the components in the above equation.

\[ L_1 = F_0 P(\Theta_1) \int_{AB} \exp[-(cl_{AE})] \exp[-(cl_{EF})] cdl, \]

where \( F_0 \) is extraterrestrial solar irradiance, \( P(\Theta_1) \) is the Rayleigh phase function evaluated at the scattering angle \( \Theta_1 \) formed by the directions of the incident beam (from the sensor) and the scattered beam (toward the Sun), \( l \) is measured along the line from A to B, \( c \) is the extinction coefficient for Rayleigh scattering in the atmosphere, \( l_{AB} \) is the distance from A to E on the line, and \( l_{EF} \) is the distance from E to F, where the scattered beam exits the atmosphere. Note that as \( l \) varies along AB, the points E and F vary as well. Similarly,

\[ L_2 = F_0 P(\Theta_2) R_f(\alpha) \exp\left[- \int_{AB} cdl \right] \int_{BC} \exp[-(cl_{BG})] \exp[-(cl_{GH})] cdl, \]

where \( \Theta_2 \) is the scattering angle formed by the surface-reflected beam (from B to C) and the scattered beam (toward the Sun), \( R_f(\alpha) \) is the Fresnel reflectivity at the angle \( \alpha \) formed by the incident light beam from the sensor and the local surface normal at point B, and the \( l \)'s are the distances denoted by the respective end points shown in Fig. 8. Note that \( L_2 = 0 \) if the projection of the line AE does not intersect the surface, i.e., if \( \theta \geq \theta_1 \), where

\[ \theta_1 = \sin^{-1}\left(\frac{r_0}{r_0 + h}\right), \]

\( r_0 \) is the radius of the Earth, and \( h \) is the height of the sensor above the Earth’s surface. Finally,

\[ L_3 = F_0 \int_{AB} P(\Theta_3) R_f(\beta) \exp[-(cl_{AE})] \exp[-(cl_{EP})] \times \exp[-(cl_{PQ})] cdl, \]

where again the \( l \)'s are the distances, \( \Theta_3 \) is the scattering angle at the interaction E, and \( R_f(\beta) \) is the Fresnel reflectivity at P. In this formalism, contributions from double surface reflections, i.e., photons reflected from the surface, backscattered toward the surface, and reflected again toward the Sun, are ignored. They are always insignificant.

The contribution \( L_3 \) requires determination of the specular direction, the scattering direction such that light scattered onto the sea surface is then specularly reflected toward the Sun (the vector d in Fig. 9). The specular direction must be determined for any
point E on the line from A to B. As shown in Fig. 9, $\omega_s$ is the angle formed by the position vector $\mathbf{r}$ of the interaction point and the Sun’s direction ($\delta$). The local incident angle $\omega$ is governed by

$$\cos(\omega_s - \omega) = R \sin^2 \omega + \cos \omega(1 - R^2 \sin^2 \omega)^{1/2},$$

where $R = r_0 \mathbf{r}$. An iterative scheme is available to obtain a solution for this equation (only two or three steps of iteration are required). With $\omega_s$ and $\omega$ known, the specular direction, i.e., $\mathbf{d}$ with direction cosines $\alpha_d, \beta_d,$ and $\gamma_d$, can be readily found from the following linear system of equations:

$$\begin{align*}
\alpha_s \alpha_d + \beta_s \beta_d + \gamma_s \gamma_d &= \cos(180 - 2\omega + \omega_s), \\
\alpha_s \beta_d - \beta_s \alpha_d + \gamma_s \gamma_d &= \cos(180 - 2\omega), \\
(\beta \gamma_s - \gamma \beta_s) \alpha_d + (\gamma \alpha_s - \alpha \gamma_s) \beta_d + (\alpha \beta_s - \beta \alpha_s) \gamma_d &= 0,
\end{align*}$$

where $\alpha, \beta,$ and $\gamma$ are the direction cosines of the interaction point’s position vector $\mathbf{r}$, and $\alpha_s, \beta_s,$ and $\gamma_s$ are the direction cosines of $\delta$. These equations are easily derived with the help of Fig. 9 and from the requirement that $\mathbf{d}$ must be in the plane formed by $\mathbf{r}$ and $\delta$.

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