Measurement of Elastic Velocities of MgO under Shock Compression to 500 Kilobars

GEORGE F. DAVIES1 AND THOMAS J. ADKINS

Neuhaus Laboratory, California Institute of Technology, Pasadena, California 91125

The velocities of rarefaction waves in shock-compressed MgO have been measured by observing the reduction of the shock front velocity near the sample edges due to the rarefaction waves propagating from the edges. The effect of this edge effect is difficult to determine accurately because of its convergent nature. Arrangements sensitive to differences in shock front velocity yielded rarefaction wave velocities close to predicted longitudinal velocities in the high-pressure shock state. Velocities close to the hydrodynamic sound speed in the shock state were obtained from less sensitive arrangements. These results can be interpreted in terms of a two-stage elastodynamic model of the decompression. The longitudinal velocities measured in shock states up to 528 kbar imply strong pressure derivatives of the elastic moduli $v_p^0$, given by $K/v_p^0 = 1.21$ ± 0.15, where $K$ is the bulk modulus.

The measurement of the velocities of rarefaction waves that propagate into shock-compressed materials provides a method for directly determining the elastic properties of the material in the high-pressure shock state. Direct measurement of the elastic moduli of solids, using isodynamic techniques, have to date been made only to pressures of about 20 kbar. At higher pressures, information about elasticity is usually obtained from pressure-density curves determined by static compression X-ray, piston cylinder measurements, or from shock waves in Hgmonot or Hg. Those determinations yield only one elastic modulus, the bulk modulus, and considerable accuracy is lost because the derivative of the pressure-density data has to be taken. Alternatively, many direct methods of determining the elastic moduli are therefore potentially very useful. Initial results are reported here of measurements of rarefaction wave velocities in MgO shock-compressed to over 500 kbar.

EXPERIMENTAL TECHNIQUE

The method used in this study consists of measuring the extent of the rarefaction that propagates into the shock region from the sides of the sample. Such a method was applied by Atkinson et al. (1963) in the study of some anodes. The configuration of the sample and the waves is shown in Figure 1. The shock wave is generated at the lower surface of the sample as a result of the impact of a projectile whose velocity is measured just prior to impact (Adkins et al., 1971). As the shock wave propagates through the sample, rarefaction propagates inward from the unconstrained (zero pressure) side of the sample, reducing the shock pressure and hence reducing the velocity of the shock front near the edge of the sample. The combined lower shock velocity and lower pressure give rise to a lower free surface velocity near the sample edges when the shock front reaches the top sample surface. A mirror is placed a small distance above the sample with the silvered surface facing the sample, and the shock of the sample onto the mirror is rec- orded by viewing the mirror through a slit, the image of which is streak-recorded by an image converter camera (Figure 2) (Adkins et al., 1971). Typical records are shown in Figure 3. The cutoff of the film streak, schematically illustrated in Figure 2, occurs first in the central part, corresponding to that part of the shock front unaffected by lateral rarefactions. The cutoff then moves progressively further toward the edges of the sample as the slower moving free surface near the edges reaches the mirror. The extent of the edge effect is determined by measuring the central linear portion of the cutoff.

1 Now at the Holford Laboratories, Harvard University, Cambridge, Massachusetts 02138.

Copyright © 1973 by the American Geophysical Union.
Fig. 1. Configuration of the shock and rarefaction waves produced by passage of a shock wave from the lower surface of the sample. Arrows show the geometrical relationship between the rarefaction wave velocity \( V \), the particle velocity \( u_a \), and the shock front velocity \( U_s \).

The first rarefaction signal to reach an interior point is that propagating from the lower corners of the crystal. Simple geometrical relations (Figure 2) then give the rarefaction velocity \( V \) as

\[
V = U_s \tan^2 \alpha + \left( U_s - u_a \right) \tan \alpha,
\]

where \( U_s \) is the shock front velocity, \( u_a \) is the particle velocity, and \( \alpha \) is the angle between the side of the sample and the locus of intersection of the shock and rarefaction waves (Figure 1).

The sample is mounted on a tungsten 'driver plate' that is struck by a tungsten 'driver plate' mounted at the tip of the projectile. The pressure \( P \) and particle velocity in the sample are determined by impedance matching (e.g., Rice et al., 1958), using the measured projectile velocities and \( P-u_a \) curves for tungsten (McQueen et al., 1970) and MgO (Carter et al., 1971). The shock velocity is then calculated from the Rankine-Hugoniot relation

\[
U_s = \frac{P}{\rho_a u_a},
\]

where \( \rho_a \) is the zero-pressure density of the sample.

The MgO samples used in this study were five polycrystalline samples supplied by T. Vasile of the Aven Corporation and three single crystals with (100) cleavage faces purchased from the Norton Research Corporation. The polycrystalline samples described and measured ultrasonically by Schochelber and Anderson [1964] and Spetzler [1970] were obtained from the same source.

**Results**

A basic difficulty of this method is the emergent nature of the edge effect (Figure 3). Initial experiments were therefore performed with the mirror moved a small distance (0.25 mm) from the sample surface. This spacing allows the contrast in free surface velocities across the sample to simply the contrast in the total transit time, which is the sum of the shock wave transit time through the sample and the free surface transit time to the mirror. Sample specifications and results for four such shots are...
given in Table 1. Figure 3a (shot 232) is a
typical record. Since there was not a really well-
defined flat-crystal section of the cutoff in these
records (variations of light extinction over 10
msec existed over the central 2 mm of the
samples), it was thought that some deformation
of the sample free surface might be occurring
during the transit. To test this idea, a shot
(2328, Figure 3b and Table 1) was fired with the
mirror-sample separation reduced to 0.11
mm. (The spacing was not reduced to zero to
avoid any complications from the elastic pre-
cursor to the shock wave [e.g., Abravanel, 1965].)
A well-defined flat central section of the cutoff
was obtained (Figure 3b), but the measured
refraction velocity was considerably lower than
that in the previous shots (Table 1). The
remaining three shots were fired on the single-
crystal samples with intermediate mirror-sample
spacings (0.25 mm). Results are given in Table 1
and Figure 3c; and exhibit refraction veloci-
ties intermediate between those of the previous
shots. (The origin of the low-angle irregularities
in shot 236, shown in Figure 3c, which were
also observed in the other single-crystal shots,
is unknown. Some uncertainty in the extent of
the edge effect results from their presence.)

**Discussion**

An interpretation of these observations is
suggested by the observations of other workers.
Hatherley et al. [1969] observed that the edge
effect had a much sharper beginning for shocked
liquids than for shocked solids and that the
measured refractive velocities corresponded
closely to the hydrodynamic (bulk) sound
speeds for the liquids but that for the solids
they were higher and corresponded more closely
to the expected longitudinal elastic velocities of
the solids. Vasilenko and von Thüls [1969] ob-
served the decompression of 6061-T4 aluminum
directly using magnum pressure gauges and
found that the decompression occurred in two
stages, the first stage preceding at about the
velocity of longitudinal elastic waves and the
second stage, identified by an increase in the rate
of decompression, propagating at about the
bulk sound speed of aluminum. These observa-
tions were interpreted in terms of a two-stage
elastoplastic decompression, in which the
decompression is one dimensional and elastic until
a critical deviatoric stress is reached, after

---

**Table 1. High-refraction velocity data**

<table>
<thead>
<tr>
<th>Sample Separation</th>
<th>Refractive Velocity</th>
<th>Shock Velocity</th>
<th>Pressure Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.29 mm</td>
<td>3.42</td>
<td>4.46</td>
<td>48,000 psi</td>
</tr>
<tr>
<td>3.19 mm</td>
<td>3.40</td>
<td>4.48</td>
<td>48,000 psi</td>
</tr>
<tr>
<td>2.79 mm</td>
<td>3.34</td>
<td>4.25</td>
<td>48,000 psi</td>
</tr>
<tr>
<td>2.39 mm</td>
<td>3.30</td>
<td>4.25</td>
<td>48,000 psi</td>
</tr>
<tr>
<td>1.99 mm</td>
<td>3.26</td>
<td>4.15</td>
<td>48,000 psi</td>
</tr>
<tr>
<td>1.59 mm</td>
<td>3.23</td>
<td>4.10</td>
<td>48,000 psi</td>
</tr>
<tr>
<td>1.19 mm</td>
<td>3.15</td>
<td>4.05</td>
<td>48,000 psi</td>
</tr>
<tr>
<td>0.79 mm</td>
<td>3.00</td>
<td>3.95</td>
<td>48,000 psi</td>
</tr>
</tbody>
</table>
which it is approximately plastic. The reverse
process has been observed in the shock compres-
sion of MgO (Ahrens, 1966) and other
materials.

The interpretation of the present observations is
aided by comparisons between the measured veloci-
ties and various extrapolations of the longitudinal
elastic velocity $V_L$ and the bulk sound speed $C$ (which are discussed in detail
below). The measured and extrapolated veloci-
ties are plotted against pressure in Figure 4.
The shots with the greatest mirror-sample separa-
tion $z$, which were the most sensitive to con-
tact in free surface velocities, yielded the
highest measured refraction velocities. These
velocities are close to the extrapolations of $V_L$.
The shots with smaller $z$ yielded considerably
lower measured refraction velocities closer to
$C$ for the smallest value of $z$ (2258). This find-
ing suggests that the arrangement in the latter
shots was not sensitive enough to detect the
initial elastic decompression observed in the
previous shots, so that the observed edge effect
was produced mainly by the hydrodynamic (or
plastic) decompression. (The half error bars
drawn in Figure 4 were obtained by picking
points on the records where edge effects defi-
nitely existed. The best estimates of the extent
of the edge effect and the lower bounds are
indicated by the inner and outer parts of tick
marks in Figure 3, respectively. Obviously,
upper bounds cannot be estimated.)

The major uncertainty in the above inter-
pretation of the data is whether the profile of the
free surface when it strikes the witness
mirror accurately reflects the conditions at the
free surface when the shock wave reached it or
whether the free surface undergoes some deforma-
tion during transit to the witness mirror. Sup-
notions of the latter effect were in fact the original
reason for varying the mirror-sample spacing.
Finite difference computations are being carried
out in an attempt to verify the source of the
observed effects.

The extrapolations of $V_L$ and $C$ in Figure 4
are based on the elastic moduli and their pres-
sum and temperature derivatives determined
ultrasonically by Spetzler (1970). They are third-
and fourth-order finite strain extrapolations
in terms of the Virial and Lagrangian strain
parameters $e$ and $\eta$ and include the thermal
effects in the ‘quasi-harmonic’ approxi-

ation (Davis, 1972; unpublished manuscript,
1973). In terms of $e$, the fourth-order finite strain formula for the effective elastic moduli
c_{ij} (Voigt notation) from which the velocities are calculated has the form (Davis, 1972;
unpublished manuscript, 1973)

$$c_{ij} = \alpha_i (1 - 2\epsilon) \epsilon_j$$
$$+ \alpha_{ij} \epsilon_i \epsilon_j + \beta_{iklj} \epsilon_i \epsilon_j$$
$$+ \gamma_{ijkl} \epsilon_i \epsilon_j$$

(3)

where the $\epsilon$, $\alpha$, $\beta$, $\gamma$ are parameters, $P$ is the pressure, and, in the case of cubic symmetry,

$$\Delta_1 = -3 \quad \Delta_2 = -1 \quad \Delta_4 = -1$$

(4)

The corresponding expression for the pressure in (Davis, 1972, 1973)

$$P = -\beta_{ijkl} (1 - 2\epsilon)^{\beta_{ijkl} (1 - 2\epsilon)}$$

(5)

The strain parameter $\epsilon$ is related to density by

$$\epsilon = \left(1 - \rho / \rho_0 \right)^{1/3}$$

(6)

The parameters $\epsilon$, $\alpha$, $\beta$, and $\eta$ in (3) and (5) are related to the $c_{ij}$ and their pressure derivatives
evaluated at $\rho$. These relations are given in the
appendix. Analogous equations in terms of the
Lagrangian strain $\eta$ can also be derived (Davis,
MgO remains in the solid state to at least 500 kb and 600 K under shock compression. Our results yield the approximate bounds $\zeta = -1$ to 15 as determined from Edlund's finite strain equations. Further refinement of this technique should allow more accurate determinations of the high-pressure, high-temperature elastic properties of materials of geophysical interest.

**Appendix**

The parameters $c_i$ in (5) are related to the $c_i'$ in (3):

$$c_i = 6(c_i' + 2w_i')$$  \hspace{1cm} (A1)  

$$c_2 = 2(c_1' + 2w_1')$$  \hspace{1cm} (A2)  

$$c_3 = c_1' + 2w_1'$$  \hspace{1cm} (A3)  

By evaluating (5) at $\phi$, we see that

$$\zeta = -3p_0/\rho_0$$  \hspace{1cm} (A4)  

where $P_0$ is the pressure at $\rho_0$ and $P_0$ depends on the temperature $T$.

By differentiating (3), the parameters $c_i''$ can be related to $P_0$ and the $c_i$, and their pressure derivatives $c_i'$, $c_i''$, etc., evaluated at $P_0$:

$$c_i'' = (c_i + P_0 v_i)/\rho_0$$  \hspace{1cm} (A5)  

$$c_2'' = -3K(c_2' + \Delta_k)/\rho_0 + 7v_i$$  \hspace{1cm} (A6)  

$$c_3'' = 9K(c_3' + \Delta_k)/\rho_0 - 3Kv_i(\rho_0 - \rho_i)$$  

$$+ 10v_i' + 49v_i$$  \hspace{1cm} (A7)  

where $K = 2/3(\varepsilon_0 + 2\varepsilon_2)$ is the bulk modulus.

Thermal effects in the finite strain equations 3 and 5 can be described through the parameters $c_i''$ and $c_i'$. Expressions for the temperature dependence of these parameters have been derived [Davis, 1972, 1973, unpublished manuscript, 1973] from the quasi-harmonic approximation of lattice dynamics [Luttinger and Kohn, 1962]. These expressions require for their evaluation the thermal expansion coefficient, the temperature derivatives of the elastic modulus, and the specific heat of the material.

**Acknowledgments.** We appreciate the help of D. Edlund and H. Richardson in performing the experiments. We thank T. Vadas of the AVO Corporation for providing the polycrystalline samples.
This research was supported by National Science Foundation grant GA-23306. Contribution 2727, Division of Geological and Planetary Sciences, California Institute of Technology, Pasadena, California. 91109.

REFERENCES


(Received January 10, 1972; revised July 20, 1972.)