Impact Erosion of Planetary Atmospheres: Some Surprising Results

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Abstract: We have investigated by analytical and computational means the effect of K/T size impacts ($5 \times 10^{30}$ erg, 9 km radius bolide of $10^{19}$ gm) on terrestrial atmospheres. We have extended analytically the approximate solution due to Kompaneets (1960) for the blast wave obtained for atmospheric nuclear explosions (idealized to isothermal atmospheres) to ideal adiabatic atmospheres and to data-based models of the Earth’s atmosphere. For the first time, we have been able to obtain analytically the particle trajectories in an isothermal atmosphere. The outcome of this nonlinear analysis is that a massive impact (without the subsequent ejection of substantial mass) would only influence a column of $\approx 30$ km radius in the Earth’s atmosphere and that the shocked gas would be propelled up and against the column “wall,” but would not escape from the planet. We examined the validity of “hemispheric blowoff,” the hypothesis that all material in a hemisphere lying above a plane tangent to the point of impact radially accelerated outward and, if sufficiently energetic, would also be ejected. We adapted and used a state-of-the-art code (CAVEAT), a hybrid Los Alamos-Sandia Lagrangian-Eulerian finite difference scheme for multi-material flow problems with large distortion and internal slip. In our CAVEAT calculations, the vapor cloud produced by the impact produces a shock that is orders of magnitude stronger than any previous use of such codes. We developed new methods to test the accuracy and convergence of CAVEAT for K/T size impact events, and it proved to be a robust tool. We explored a K/T size impact where the 9 km radius bolide was vaporized and injected into the atmosphere and found no radial outflow in agreement with the analytic model but, instead, a 50 km radius vertical column formed with only a small fraction of material reaching escape velocity—no more than about 7% of the vaporized bolide plus atmospheric mass will escape the gravitation of the Earth.
Impact events have long been regarded as an accretionary source of planetary material and a major influence upon the evolution of terrestrial planetary atmospheres. Our goal is to be able to understand better the role of massive impacts that occurred on the planets during the end of the accretionary epoch as well as events such as occurred 65 million years ago which so radically, albeit temporarily, altered this planet’s climate and profoundly affected biological evolution. The discovery of the Chicxulub Crater in the Yucatan has provided strong evidence of a link between a major impact event and massive global biological extinctions (Alvarez et al., 1980; Emiliani et al., 1981; O’Keefe, and Ahrens, 1989; Hildebrand et al., 1991; Pope et al., 1994). The underlying cause of the mass extinctions at the Cretaceous-Tertiary (K/T) boundary has been hypothesized to be either a sudden and/or long-term global change in the atmosphere’s radiative properties, and/or acidification of the biosphere from NO\textsubscript{x} and/or SO\textsubscript{3} species induced by the impact of the K/T bolide. The recent observations of Comet Shoemaker-Levy 9 (SL9) have served as a motivation for exploring in depth the basic physics of atmospheric “impact erosion”—although the latter was not actually observed or predicted in the SL9 event. However, in the case of a planetary atmosphere overlying a solid surface, it still appears that these atmospheres can be eroded of substantial mass by impact events. In particular, we explore the validity of “hemispheric blowoff,” theory (Melosh and Vickery, 1989; Vickery and Melosh, 1990). This theory predicts that, for a sufficiently energetic impact, all material in a hemisphere lying above a plane tangent to the point of impact can flow radially outward and, if in excess of escape velocity, would be ejected. Many others, e.g. Zahnle et al. (1990), Zahnle (1993) and Evans, Ahrens, and Gregoire (1995), assumed that hemispheric blowoff was an accurate description of the physics making it all the more important that the dynamical behavior of impact events is accurately described.

Large scale impact events profoundly influenced the composition and dynamics of young terrestrial planetary atmospheres. Such events are often broken down into three phases (e.g. Vickery and Melosh, 1990).

1) **Entry phase.** The projectile enters the atmosphere, heating, compressing, and accelerating the atmospheric gases ahead of it. The flow resulting from meteoritic infall onto planets has been studied for many years (e.g. Bronshten, 1983). New interest in this subject was motivated
by the study of the Tunguska event (Chyba et al., 1993) and surface scars from atmospheric meteor explosions on Venus (Campbell et al., 1992; Vervack and Melosh, 1992; Zahnle, 1992). Comet Shoemaker-Levy 9 gave us some information about this phase—see Takata et al., (1994), Ahrens et al. (1994a,b), and Mac Low and Zahnle (1994). However, the entry phase of SL9 was unlike that of an impact on the surface of a solid planet with an overlying atmosphere, where the projectile diameter is comparable to the atmospheric scale height. For solid planets and massive impactors, only a few percent of the kinetic energy carried by an impactor is introduced into the atmosphere this way. However for impactors, (especially weak objects) upon oblique entry and subjected to the Kelvin-Helmholtz and Rayleigh-Taylor instabilities, the bolide disintegrates, and the shock (and, possibly, radiatively) heated air could create a regional “firestorm” effect such as that associated with the $6 \times 10^{23}$ erg Tunguska event in Siberia in June 1908, igniting fires in vegetation below. In addition, Melosh et al. (1990) has suggested that falling ejecta from larger impacts, e.g. K/T, can initiate world-wide fire storms.

(2) Creation of a Fireball and Vapor Plume. It is assumed that the surface impact event itself introduces a substantial fraction of its energy into the atmosphere at the point of collision via an explosion, the so-called “fireball.” Depending on the nature of the impactor (e.g. an iron meteorite vs. a comet), a significant part of this energy could go into vaporizing the projectile as well as “target material.” Jones and Kodis (1982) adapted existing Los Alamos codes to explore the evolution of the fireball in the Earth’s atmosphere (assuming no substantial quantity of vaporized material was present) and the implications to the Cretaceous-Tertiary (K/T) extinction event. They observed that the lateral expansion of the blast wave was self-limiting owing to the existence of an essentially exponential fall off of atmospheric density with altitude. They observed that the shocked flow would take the path of least resistance, curving upward, and that only a column of approximately 30 km radius was affected. The detailed model (including many physical effects) due to Jones and Kodis gave essentially the same results as the analytic theory due to Kompaneets (1960) for an ideal isothermal atmosphere. Significantly, the columnar shape (with a parabolic base) of the K/T impact induced atmospheric shock predicted by Jones and Kodis, and much earlier by Kompaneets for an atmospheric explosion, is similar in appearance as was predicted for Shoemaker-Levy 9 (Zahnle and Mac Low, 1994; Crawford
et al., 1995; Takata et al., 1994). The present paper will focus on this phase of evolution.

(3) Formation of a Solid and Molten Ejecta Cone. Solid and molten ejecta from a growing crater pass through the atmosphere transferring all or part of their momentum to the atmosphere by drag. Overbeck (1975) and O’Keefe and Ahrens (1977a,b) have conducted some modeling of the formation of the ejecta cone. Moreover, evidence from this type of flow is observed around 40 large impact craters on Venus where interaction with the E-W atmospheric zonal flows produces large surficial deposits with parabolic shape—see Campbell et al. (1992) and Vervack and Melosh (1992). The atmospheric effects of this phase of impact has never been fully explored.

Recently, massive impacts have come to be seen as possibly blowing off a substantial portion of the volatile content of planetary atmospheres (Cameron, 1983). Vickery and Melosh (1990) proposed that the massive release of vaporized bolide and target materials would profoundly alter the nature of the impact induced atmospheric flow. They argued that the momentum associated with the vapor cloud would completely dominate the nature of the outflow and employed a solution due to Zel’dovich and Raizer (1967—Chapter I, §29) for the sudden release of gas into a vacuum as an approximate analogue. In particular, they proposed that the vapor cloud would expand isotropically and radially above the plane tangent to the impact point, provided that the momentum of the atmosphere, vaporized bolide and surface material had sufficient velocity to escape the planet, i.e. is directed between the vertical and a specific zenith angle to escape the planet. All of the atmospheric gas above the plane if the impactor was sufficiently energetic—which they termed “hemispheric blowoff”—would escape the planet. The Zel’dovich and Raizer model was designed to describe a “nova” event. Book (1989) has shown that the emergence of a flow like that predicted by Zel’dovich and Raizer from a vapor cloud of uniform density would be a highly unstable process, characterized by substantial convective overturn and mixing. In case of expansion into a vacuum, one would expect that the isotropically expanding gas whose velocity exceeded escape velocity would depart, while the more slowly moving gas near the interior of the nova would remain gravitationally bound. Vickery and Melosh (1990) treated the overlying planetary atmosphere by considering a momentum balance equation. They considered the differential radial momentum contained in the differential solid angle $2\pi \sin \theta d\theta$ around the vertical axis of the vapor plume—hence, their momentum modeling consid-
erations are local and exclude the possibility of momentum exchange through gas dynamics. Their criterion for escape was that the differential momentum contained in that solid angle from vaporized bolide material be sufficient to accelerate both the vapor cloud and the enveloping atmosphere to escape velocity. In particular, they only included that part of the bolide mass—and its corresponding radial momentum—already traveling faster than the escape velocity, as the slower plume material ultimately will separate from the hyper-escape velocity material and fall back to the planet. Their momentum balance criterion permits one to compute, for a given planetary atmosphere, the critical angle $\theta_c$ from the vertical describing a cone containing vapor cloud and atmospheric gas that is blown off in their model.

In contrast to the hemispherical blowoff model in the case of Shoemaker-Levy 9, whose fragments encountered Jupiter, are less energetic than the K/T bolide, and yielded a pattern much like that predicted by Jones and Kodis (1982). The inertia of the atmosphere, the existence of the exponential variation of density with height, and the force of gravity could combine to moderate the initial state of hemispheric flow predicted by Vickery and Melosh. Further, there is no observational evidence for the gravitational escape of any cometary material from Jupiter. Accordingly, we need to develop a theory which would describe impacts as energetic as SL9 and the K/T bolide and, hopefully, larger events.

Simple dimensional scaling arguments (Zel’dovich and Raizer, 1967, Chapter I, §25) suggest that the atmospheric blast wave produced by the impact event would be initially spherical and have a shock radius $R_s$ that varied as

$$R_s \approx \zeta \left( \frac{E}{\rho_0} \right)^{1/5} t^{2/5}$$

where $E$ is the energy introduced into an uniform atmosphere with density $\rho_0$, $t$ is the time, and $\zeta$ is a dimensionless constant of order unity that depends on the specific heat ratio $\gamma$ of the gas involved. Newman (1977) accurately established by analytic and numerical means that $\zeta$ is 1.15167 for $\gamma = 5/3$ and 1.03278 for $\gamma = 7/5$. This scaling accurately describes the initial propagation of atmospheric nuclear explosions and the emergence of an expanding thin hemispheric shell of atmospheric gas. (The presence of a hard surface acts as a mirror, ignoring cratering, that produces a hemispherical shock.) For impact events and very large explosions, this scaling breaks down due to two physical causes: (a) the shock radius $R_s$ exceeds the atmospheric scale height $H$, or (b) the mass
of vaporized bolide and target material $M$ substantially exceeds the mass of swept up atmospheric gas. (Throughout our discussion, we will employ the pressure scale height $H = k_B T/m g$, where $k_B$ is Boltzmann’s constant, $T$ the temperature in °K, $m$ is the mean molecular mass, and $g$ is the local gravitational acceleration.) The first of these criteria is met after a time $t_H$ given by

$$t_H \approx \sqrt{\frac{H^2 \rho_0}{\zeta^2 E}};$$

our treatment will be based on identifying the nature of the flow after this time. The second of these criteria is met if any mass $M$ is introduced (e.g. a vapor cloud emerging from vaporized bolide and target material) until a time $t_M$ when the the mass accreted by the expanding shock, namely $\frac{4 \pi}{3} \rho_0 R_s^3$, becomes significantly larger than $M$, namely

$$t_M \approx \left( \frac{3M}{4 \pi \rho_0} \right)^{5/6} \left( \frac{\rho_0}{\zeta^2 E} \right)^{1/2}.$$

Theoretical models can be developed to deal with either of these two limiting cases of scale breaking. Accordingly, such models represent two logical yet “extreme” approaches to this scientific problem with the true solution lying somewhere in between. Since the atmospheric mass above a plane tangent to the Earth’s surface (Vickery and Melosh, 1990) is $3 \times 10^{18}$ gm and for a 9 km radius asteroid of $10^{19}$ gm, the second criterion is satisfied throughout much of the evolution of the flow from a K/T size event. Consequently, Vickery and Melosh assumed that the momentum carried by the vapor cloud must control the dynamics. We will refer to this as the radial “hemispheric blowoff” model.

Vickery and Melosh (1990) considered $10^{13} - 10^{23}$ gm impactors colliding with terrestrial planets and calculated how much of the impactor escaped from the planet. Accordingly, they utilized a model for an explosion and subsequent release of “gas” into a vacuum, a shock-free situation, and later adapted to astrophysical environments (Zel’dovich and Raizer, 1967, Chapter I, §29). Vickery and Melosh suggested that very large impact events can push away all atmospheric gas that was present above a plane tangent to the impact point.

We present below a revised analysis of the dynamical interaction of the vapor cloud with the enveloping medium based on the first of the scale-breaking criteria. This differs from the hemispheric blowoff model in several ways.
(1) The hemispheric blowoff model was motivated in part by recognizing that a large amount
of momentum would be transported by the vapor cloud. However, the model is based on
the assumption of local momentum conservation, while it is the total momentum which is
conserved. The horizontal component of the total momentum, moreover, is zero and can be
readily converted into thermal energy without violating any physical conservation laws due to
the violent wave motion and reverberations that would be produced—this will be discussed later.
(In particular, the Vickery and Melosh model assumes that the radially integrated momentum
density or the momentum density per solid angle is conserved. Momentum exchange can and
will occur between different radial directions due to thermal pressure effects.)

(2) The hemispheric blowoff model assumes a weak shock at the outer boundary of the vapor cloud
and neglects the effect of the low density enveloping atmosphere on the expanding vapor plume.
The interaction of the vaporized bolide with the atmosphere, however, necessarily produces a
strong shock in the atmospheric gas that has been accreted by the expanding flow. In contrast,
the front associated with the expansion of the inner weak shock corresponding to the vapor
cloud is slowed and stabilized by the mass and dynamics of the accreted atmospheric gas.

(3) The enveloping atmosphere possesses an anisotropy evident on length scales greater than an
atmospheric scale height. (Vickery and Melosh developed an approximate but different scheme
to accommodate this.) As the strong shock grows beyond a scale height in radius, the expanding
front preferentially propagates upward as a result of the usual decrease in density with altitude.
The vapor cloud and atmosphere, confined within a cylindrical region of radius \( \approx \pi H \) plus the
bolide radius, is superheated by the conversion of lateral kinetic energy into thermal energy.
Combined with the resistance offered to lateral expansion by the strong shock and relatively
dense ground-level atmosphere (compared with upper atmospheric levels), the “plume” flow
is effectively channeled up the above-defined cylinder and is thus confined to remain within a
modest (\( \approx 2\pi H \)) diameter.

Following the important initial steps taken by Vickery and Melosh in developing a theory of
atmospheric cratering, it has become clear that the hemispheric blowoff theory needed to be reviewed
in light of the other possible scale breaking argument.
Analytic Theory

The physics of gas dynamic shock waves remains a complex subject, yet one that is amenable to some simplifying—but remarkably accurate—approximations. This is an area where the Russian school including Kompaneets, Andriankin, Sakharov, Stanyukovich, Chernyi and others made important progress in the 1960’s. Suppose that an explosion (with no injected mass or radiative losses) occurs in an isotropic medium, so that Eq. (1) would apply and the flow is spherically symmetric. The pressure distribution inside the blast wave tends to become uniform—thereby eliminating noticeable accelerations inside the shock—and the pressure is simply related to the $E/V(t)$ where $V(t)$ is the volume enclosed by the expanding blast as a function of time. Meanwhile, the flow equations can be described by three “characteristics” (Courant and Hilbert, 1962, Chapter II, §1): (a) one at the material flow velocity; (b) one at the flow speed plus the sound speed—this describes the supersonic expanding shock; and (c) one at the flow speed minus the sound speed. The latter inward characteristic causes any perturbations in the flow to reverberate in towards the origin where it is geometrically focused and shock heats the center (Newman, 1977, 1980). There is an outward rebound from the origin which travels hypersonically, catching up with the blast wave, and produces another reverberating inward characteristic that returns to the origin, and so on. Each reverberation produces an entropy increase, either at the blast’s origin or at the expanding shock. The outcome of the entropy increase is that any perturbation in the flow ultimately decays and a stable self-similar flow described by Eq. (1) emerges—the mathematical description of the flow is generally credited to Von Neumann, Taylor, and Sedov (see Zel’dovich and Raizer, 1967, Chapter I, §25–27, for a summary). Ostriker and McKee (1988) and Bisnovatyi-Kogan and Silich (1995) have reviewed astrophysical blast waves in homogeneous and inhomogeneous environments, respectively. The essential feature we wish to identify here is that the shock heating of the center causes its temperature mathematically to approach infinity. Since the pressure inside the blast wave remains uniform, the density of the gas near the center is vanishingly small and virtually all of the gas resides just inside the expanding blast wave. (This scenario holds so long as the pressure inside the shock greatly exceeds that outside the shock—the Von Neumann peak overpressure is greater than 10—see Newman, 1980, for a discussion.) For all practical purposes, the expanding blast wave looks like a hemispherical “snow plow” expanding adiabatically into the surrounding medium.
Kompaneets (1960) skillfully exploited these facts in developing an analytic model for an explosion in an isothermal environment. His theory exploited differential geometry together with the Rankine-Hugoniot shock conditions to obtain the evolution of the shock wave. In particular, he employed the Rankine-Hugoniot condition that the velocity \( \vec{u}_s \) of the shock front is normal to the front and satisfies

\[
|\vec{u}_s| = \sqrt{\frac{(\gamma + 1) P(t)}{2 \rho(z)}}.
\]  

(4)

We shall now employ cylindrical coordinates, where \( z \) is the altitude and \( r \) is the radius; \( \rho(z) \) is the density of the ambient atmosphere at altitude \( z \). Thus far, this description is \textit{exact}, and we now introduce our \textit{only} approximation. We will assume that the pressure \( P(t) \) at time \( t \) behind the shock is spatially \textit{uniform}. The reverberations described earlier help bring the shock-enclosed gas to that state, although in an isothermal or adiabatic environment this assumption will ultimately break down. Andriankin et al. (1962) in the former Soviet Union and Bach et al. (1975), Oppenheim et al. (1971), and Laumbach and Probst (1969) in the United States have done much to refine this latter issue. Although these refinements will not be required for the present purpose, they can be employed to better understand the modest discrepancy that develops between analytic theory and accurate numerical simulations.

Accordingly, Kompaneets identified the spatially uniform pressure \( P(t) \) behind the shock to be

\[
P(t) = \frac{\lambda (\gamma - 1) E}{V(t)},
\]  

(5)

where \( V(t) \) is the volume contained by the shock, and \( \lambda \) is a dimensionless constant of order unity which is related to the specific heat ratio \( \gamma \) and can be accurately estimated by numerical means. (We will make the connection between \( \lambda \) and \( \xi \), introduced earlier, in the discussion below.) Now, we need to amalgamate the latter two equations into a geometric model for the shock evolution.

Suppose we define \( R(z, t) \) as the radius (in the \( x-y \) plane) of the shock as a function of the altitude—ultimately, this is the quantity that we require. Now, we define

\[
f(r, z, t) \equiv r - R(z, t) = 0
\]  

(6)

as the definition of the shock surface. Since the function \( f \) vanishes at the shock front, so must its
advective derivative, that is
\[
\frac{Df(r, z, t)}{Dt} = \frac{\partial f(r, z, t)}{\partial t} + \mathbf{v}_s \cdot \nabla f(r, z, t) = 0.
\] (7)

Korycansky (1992) employed a similar methodology to approximate the evolution of a massive impact in a deep gaseous envelope, as might have happened to the planet Uranus early in its history and could possibly account for the remarkable tilt in its spin axis. Korycansky’s model, however, was very different than that relevant to terrestrial atmospheres and his contribution will be discussed later in that context. We now exploit the fact that the shock velocity vector and a normal to the shock surface lie in the same direction and observe that
\[
\left| \nabla f(r, z, t) \right| = \sqrt{\left( \frac{\partial f}{\partial r} \right)^2 + \left( \frac{\partial f}{\partial R} \frac{\partial R}{\partial z} \right)^2}.
\] (8)

Thus, Eqs. (7) and (8) reduces by virtue of (6) to
\[
0 = \frac{\partial R}{\partial t} + v_s \sqrt{1 + \left( \frac{\partial R}{\partial z} \right)^2}.
\] (9)

We need to simplify this expression for finding an alternative method for describing the time.

Now, the shock velocity \( v_s \) depends on \( z \) and \( t \); we define a time-like variable \( y \) according to
\[
y = \int_0^t \frac{dt'}{\sqrt{\mathcal{V}(t')}} \sqrt{\frac{\lambda E (y^2 - 1)}{2\rho(0)}}.
\] (10)

Here, we evaluate \( \rho(z) \) at \( z = 0 \) since that is the altitude where the explosion is assumed to take place. We will describe later the physical meaning of the variable \( y \) and its scaling properties. It is important to note that \( y \) has the dimensions of a distance. We now replace \( t \) by \( y \), for example we let \( R(z, t) \rightarrow R(z, y) \). We then employ Eqs. (4) and (5) to eliminate \( v_s \) in Eq. (9), and observe that the resulting equation allows for the separation of terms in \( z \) and \( t \). We employ \( y \) introduced by Eq. (10) to simplify the the resulting time-like behavior and finally obtain
\[
\left( \frac{\partial R}{\partial y} \right)^2 = \frac{\rho(0)}{\rho(z)} \left[ \left( \frac{\partial R}{\partial z} \right)^2 + 1 \right]
\] (11)

defining the (cylindrical) radius vs. altitude equation (at “time” \( y \)). The remarkable feature of this partial differential equation (pde) is that we have only one equation to solve, not the normal three
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Moreover, the nonlinearity present in this equation is rather weak, and is sometimes called “quasilinear.”

To solve Eq. (11), we employ the so-called “method of separation of variables” (Courant and Hilbert, 1962, Chapter I, §3–4). (Despite a similar name, this method is different from the usual method employed in solving Helmholtz’s equation in that the solution is sought as the sum of terms instead of a product of terms.) We observe that

\[ R (z, y; \xi) = -\xi y + \int_0^z dz' \frac{\sqrt{\xi^2 \rho (z') / \rho (0) - 1}}{\xi} \]  

(12)
is a solution of the pde (11) where \( \xi \) is a dimensionless parameter selected so that \( R (z, y; \xi) \) is real, a feature that can be readily confirmed by substitution. (We assume that the atmospheric density is monotonically decreasing with altitude. Hence, it follows that \( 0 \leq \xi \leq \sqrt{\rho (0) / \rho (z)} \).) We will now derive the so-called “singular solution” to the pde from this parametric form and show that it matches the initial conditions for the flow, thereby rendering the desired solution.

For every altitude \( z \) and time \( y \), select \( \xi \) so that \( \partial R / \partial \xi = 0 \), i.e. let \( \Xi (z, y) \) be that value of \( \xi \) such that

\[ \frac{\partial R (z, y; \xi)}{\partial \xi} \bigg|_{\xi=\Xi} = -y + \int_0^z dz' \frac{\Xi \rho (z') / \rho (0)}{\sqrt{\Xi^2 \rho (z') / \rho (0) - 1}} = 0 \]  

(13)

Then, it can be shown, \( R [z, y; \Xi (z, y)] \) is also a solution to the partial differential Eq. (11). To establish the latter, note for example that

\[ \frac{d R (z, y; \Xi)}{dz} = \frac{\partial R (z, y; \Xi)}{\partial z} + \frac{\partial \Xi}{\partial z} \frac{\partial R (z, y; \xi)}{\partial \xi} = \frac{\partial R (z, y; \Xi)}{\partial z} \]  

(14)

by virtue of the condition that \( R \) be an extremum with respect to \( \xi \). A similar result holds for the \( y \) or time derivative and (11) is automatically satisfied. The physical meaning of the singular solution is that we associated it with the fastest moving solution of (12) and, hence, isolates the first arrivals emerging from the shock. As a final step, we must verify that for small \( z \) and \( y \) (i.e. where \( R < H \)) that Sedov’s isotropic solution is recovered.

Near the surface \( z = 0 \), at early times when \( R \ll H \), the density \( \rho (z) \) may be regarded as constant, namely \( \rho (0) \). We seek to identify the solution to Eq. (11) in the limiting case of a homogeneous and isotropic atmosphere. Then, Eq. (12) reduces to

\[ R = -\xi y + \sqrt{\xi^2 - 1} z \]  

(15)
and the derivative condition (13) is met when

$$\mathcal{E}(z, y) = \frac{y}{\sqrt{y^2 - z^2}}.$$  

(16)

Employing the latter in our new expression (15) for $R(z, y)$, we obtain

$$R(z, y) = \sqrt{y^2 - z^2}.$$  

(17)

Since the time-like variable $y$ has the units of length, namely $R_s$ in the homogeneous atmosphere limit, it follows that this expression defines a hemisphere for $z \geq 0$ and that (1) can now be made completely self-consistent by noting that

$$R_s = \left[ \frac{75(y^2 - 1)}{32\pi} \cdot \frac{\lambda E}{\rho(0)} \right]^{1/5} r^{2/5}.$$  

(18)

This relationship emerges from combining Eqs. (10) and (17) with the volume $V(t) = 4\pi R^3(t)/3$. This expression can now be compared directly with Eq. (1), yielding

$$\xi = \left[ \frac{75(y^2 - 1)\lambda}{32\pi} \right]^{1/5},$$

whereupon we obtain that $\lambda$ is 1.527571 for $\gamma = 5/3$ and 1.64061 for $\gamma = 7/5$. Thus, for the first time, $\lambda$ in the Kompaneets theory is evaluated from first principles. We believe that this is the first quantitative identification of $\lambda$ in the Kompaneets theory. On the other hand, $\xi$ was first estimated by Bethe et al. (1947) and was later refined by Newman (1977). Hence, we have shown that when the altitude $z$ and shock radius $R$ are small compared with the scale height $H$, the solution generated by the method of separation of variables matches the initial conditions of a hemispheric blast in a homogeneous atmosphere with density $\rho(0)$. (If the initial source is not a point, but has the form of a “pancake”, i.e. a disk with zero height and some radius $\mathcal{R}$, then the Kompaneets solution for $R(z, y)$ has $\mathcal{R}$ added to it. This piecewise continuous solution is also continuous in its derivative with respect to $R$. The discontinuity in the second derivative ultimately weakens the expanding shock at that point and causes the “flat top” of the expanding front to be smoothed out.) Thus, we have shown that the singular solution to the pde (11) matches the initial physical state of the impact problem.
There are two other models where the singular solution can be calculated in closed form. Kompaneets considered an "exponential atmosphere"—what we would call an isothermal one—where
\[
\rho (z) = \rho (0) \exp \left( -\frac{z}{H} \right) ,
\]
where \( H \) is again the scale height. By taking particular care with the branch cuts for the integrals in (12) and (13), Kompaneets showed that
\[
R (z, y) = 2H \arccos \left\{ \frac{1}{2} e^{z/H} \left[ 1 - \left( \frac{y}{2H} \right)^2 + e^{-z/H} \right] \right\} .
\]
It is noteworthy that, by taking expansions with \( y \ll H \) and \( z \ll H \), we can recover the homogeneous atmosphere result of Eq. (17)! This result has some remarkable properties, in particular this demonstrates that a limiting shock radius of \( \pi H \) always is achieved because the \( \arccos \) term can never be larger than \( \pi/2 \). This limiting feature of the K/T bolide fireball was observed by Jones and Kodis (1982) whose simulation of a K/T size event assumed no vapor cloud. Further detailed insight into the evolution of the blast wave from first principles accurate simulations may be found in Symbalisty (1990). A further result here is that \( y \leq 2H \); at \( y = 2H \) the "top" of the shock, or altitude \( z_{\text{max}} \), is no longer finite and the volume contained in the column becomes infinite. As the top of the shock pops open at very late time (in this example, after a fraction of a second), we observe that the volume \( V (t) \to \infty \) and, therefore, \( y \) will cease growing and asymptotically approaches \( 2H \), as can be obtained from Eq (20). Note, further, that this solution is independent of \( \gamma \), apart from its influence in determining \( \lambda \) in the expression for \( y \). Different adiabatic constants \( \gamma \) only change (and in a very modest way) the time for a given shock profile, and not its shape! In Fig. 1, we illustrate some of the special scaling features pertinent to the isothermal model and, especially, the role of the time-like variable \( y \). We observe in that figure that \( y \) scales like \( t^{2/5} \), at small times, essentially \( t_H \) in Eq. (3), and then approaches \( y = 2 \) at late times (as \( t \geq 4 \)). We plot \( y \) in units of the scale height \( H \). We see a similar scaling for small times for the maximum altitude of the shock, \( z_{\text{max}} \), in units of the scale height \( H \), which later rushes off exponentially fast to infinity. The time variable \( t \) is plotted in units of the time scale associated with Eq. (18) and the scale height \( H \), namely
\[
t_y = \sqrt[3]{\frac{32 \pi H^5 \rho (0)}{75 \lambda E (\gamma^2 - 1)}} ,
\]
which is approximately $1.05 \times 10^{-2}$ sec for the Earth’s atmosphere. An important feature of the Kompaneets’ model, just as in the Zel’dovich and Raizer model employed by Vickery and Melosh, is that the flow properties are qualitatively independent of the energy $E$ and other physical parameters which only affect via the scalings given above for the time and the shock radius. We will return momentarily to the isothermal model.

Before proceeding further, it is useful to review some of the features implicit to the Kompaneets solution for an isothermal atmosphere, and then consider how the underlying mathematics differs from the physical situation encountered in explosions. We recall that Kompaneets was motivated to address the first of the scale breaking issues that we identified, namely the presence of an atmosphere with an exponential density distribution with a scale height $H$. His solution embraced four basic steps.

(a) The blast wave is approximated by a surface moving according to the Rankine-Hugoniot relations, i.e. Eq. (4). This was the fundamental intellectual contribution made here which we will refer to as the “Kompaneets approximation” and the behavior of such a surface is a problem in differential geometry.

(b) The pressure behind the shock front is assumed to be proportional to the contained energy divided by the volume, see our Eq. (5). This was a standard approximation for blast waves in the Russian school of hydrodynamics, and is often referred to as the “Chernyi approximation” (Chernyi, 1957) in later works by Sedov and Stanyukovich. A good summary of the latter in English may be found in Zel’dovich and Raizer (1967). This approximation was refined later by Andriankin et al. (1962) to accommodate variability in the constant of proportionality.

(c) The method of solution for the quasilinear partial differential equation, namely Eq. (11), was developed during the 19th century; one identifies a parametric family of additively separable solutions, and observe that the extremum to this family of solutions is also a solution (sometimes called the “singular solution”) and corresponds to our physical constraints.

(d) Kompaneets solved this quasilinear partial differential equation in the case of an exponential or isothermal atmosphere in closed form. We refer to his solution for the exponential atmosphere as the “Kompaneets solution.”

Korycansky (1992) employed the Kompaneets approximation in radial geometry to explore an
atmosphere with power-law density stratification. Although the physical significance of such an approximation to the atmosphere is unclear, that approximation is amenable to the added complexity introduced by the radial geometry. However, unlike the isothermal approximation which is applicable to terrestrial atmospheres and the Kompaneets solution for that case, Korycansky’s solution for the evolving surface becomes ill-defined as the shock wraps around itself. Nevertheless, Korycansky’s work provides some useful insight into another model atmosphere.

While the mathematical solutions discussed are useful approximations to the underlying physics, it is important to explore their limitations. First, these models belong to a class of solutions called “intermediate asymptotics” (Barenblatt and Zel’dovich, 1972; Barenblatt, 1996). These models become applicable after an interval time needed for the underlying scaling to manifest itself, namely Eq. (2), and cease to be applicable once other physical limitations become significant. In the case of blast waves, the latter limitation emerges as the shock continue to expand with its associated pressure becoming close to the pressure of the enveloping atmosphere. Empirically, it is well-known that the ratio of the pressure inside the shock relative to that outside—sometimes called the “peak overpressure”—must exceed 10 for such approximations to be valid. Consider now an explosion at a substantial altitude in the atmosphere. As the shock envelope expands and weakens, the bottom of the shock front is first to recognize the diminished overpressure and evolves from a strong shock, with discontinuities in the physical flow quantities, to a weak shock, with discontinuities in derivative in these quantities. Finally, as it essentially “runs out of energy” from shock heating the increasingly dense atmosphere below, it devolves into an acoustic wave. Thus, the shock front will cease to propagate downward at a finite height. Similarly, the nature of the shock at the top of the shock front also encounters different physics, e.g. the long collision lengths in the exosphere which obfuscates the nature of the shock. Nevertheless, the “intermediate asymptotic” approximation remains a highly useful description of the flow for a large fraction of its lifetime. A second noteworthy issue relating to the physical description relates to the issue of inertia and the description of the flow. The Kompaneets approximation renders the shock front infinitesimally thin, unlike the physical shock front which is \( \approx 1/10 \) the thickness of the shock radius. As a consequence, the Kompaneets approximation generates solutions that can change direction instantaneously, unlike the extended fronts that occur in nature. Intuitively, some have argued that the Kompaneets approximation ne-
neglects inertia (Zahnle, private communication). Formally, the Kompaneets approximation contains
the inertial flow terms \((\vec{v} \cdot \nabla) \vec{v}\) in the transport equations; the fundamental issue is the ability to
describe a shock front, which has finite thickness, by a surface. The answer to that question, as
well as the earlier one posed above, has been obtained many times in first principles hydrodynamic
simulations which embrace all inertial terms in terrestrial atmospheres: the Kompaneets solution
provides an unexpectedly accurate approximation to the evolution of the shock front.

The adiabatic model occupies an extremely important role in describing the tropospheres of
planetary atmospheres (Chamberlain and Hunten, 1987). The density profile for an adiabatic atmo-
sphere has the form

\[
\rho(z) = \rho(0) \left[ 1 - \frac{z}{H'} \right]^{1/(\gamma - 1)},
\]

where the modified scale height \(H'\) satisfies

\[
H' = H_0 \frac{\gamma}{\gamma - 1}
\]

where \(H_0\) is the isentropic, density scale height at the surface. (In the adiabatic model, the emergence
of a strictly linear lapse rate forces the temperature to vanish at \(z = H\) and, hence, so does \(P\) and \(\rho\).) The integrals in (12) and (13) can be performed for this model in terms of incomplete beta
functions (Abramowitz and Stegun, 1972; Bateman et al., 1955) and can be evaluated numerically.
Remarkably, this insight can be employed to produce a numerical method based on the singular
solution obtained from the method of separation of variables.

To develop such a numerical method, we observe that \(R(z, \gamma, \xi)\) exists, if \(\xi\) is greater than some
minimum value, so that the quantity inside the radical in (12) never becomes negative. Hence, we say
that \(R\) as a function of \(\xi\) exists and is “convex” over some continuous range of \(\xi\). Second, we observe
that \(R\) is a “concave” function of \(\xi\), i.e. \(\partial^2 R / \partial \xi^2 \leq 0\) as can be readily verified. General results from
real analysis (Rudin, 1987) show that a concave functional over a convex space has a maximum,
and it is unique. Eq. (11) can be numerically integrated as a function of \(\xi\), although substantial
care must be taken to assure convergence. The concavity result above guarantees that a numerical
optimization routine, such as the “golden search algorithm” (Press et al., 1992), will be absolutely
convergent in finding the numerical value of \(\xi\) that maximizes \(R\) and, hence, determines the radius.
Applying this semi-analytic procedure (since it is the numerical realization of a completely analytic
method) to fitted atmospheric density vs. altitude profiles, e.g. those in Chamberlain and Hunten (1992), yields fireball expansion profiles which are almost indistinguishable from the adiabatic and, especially, the isothermal ones. Accordingly, we will continue to focus on isothermal atmospheres in the rest of this discussion.

One aspect of the present solution never previously explored for terrestrial atmospheres are the particle trajectories that are associated with the Kompaneets’ solution. Korycansky (1992) used the methodology we describe below to solve the problem of calculating trajectories in a radially stratified atmosphere with power-law density distribution; he succeeded in finding closed form solutions by exploiting a transformation that he discovered which converted the problem into one with uniform density! The essential feature which identifies particle trajectories is that they are orthogonal to the contours defining the location at different times of the blast wave. First, we observe that the “time” $y$ when a shock passes through coordinates $(r, z)$ can be obtained by inverting Eq. (20), namely

$$y (r, z) = 2H \sqrt{1 - 2 \exp \left( - \frac{z}{2H} \right) \cos \left( \frac{r}{2H} \right) + \exp \left( - \frac{z}{H} \right)}.$$  

(24)

Suppose, then, that $Z (r, y)$ describes the altitude of a particle trajectory at radius $r$ and time $y$. We now employ an elementary result in differential geometry, namely two curves are orthogonal if the product of their derivatives is $-1$. This yields a differential equation for the particle trajectories, namely

$$\frac{dZ}{dr} = - \frac{dR [Z, y (r, Z)]}{dz}.$$  

(25)

We use, in turn, Eqs. (20) and (23), first to find $dR [Z, y (r, Z)]/dz$ and then $dR [Z, y (r, Z)]/dz$. The algebra involved is formidable, but ultimately reduces to the differential equation

$$\frac{dZ}{dr} = \frac{\cos \left( \frac{r}{2H} \right) - \exp \left( - \frac{z}{2H} \right)}{\sin \left( \frac{r}{2H} \right)}.$$  

(26)

After some further manipulation, this equation can be converted into a linear ordinary differential equation whose complete solution can be explicitly obtained and then expressed as a function of the particle’s initial trajectory. Assuming that the particle’s trajectory is initially at an angle $\theta$ from the zenith, we then obtain

$$Z (r, \theta) = 2H \ln \left[ \frac{\sin \left( \frac{r}{2H} + \theta \right)}{\sin (\theta)} \right].$$  

(27)
Note that, for a given $r$ and $\theta$, we can now compute the altitude $z$ which is intersected by the particle’s trajectory and, by (24), calculate the corresponding “time” $y$. (Here, $\theta$ is an integration constant, but has the physical meaning given above.) Thus, a particle which passes through $(r, z)$ does so at time $y$ given by Eq. (23) and has a trajectory with an initial zenith angle of

$$\theta (r, z) = \arctan \left[ \frac{\sin \left( \frac{r}{2H} \right)}{\exp \left( \frac{r}{2H} \right) - \cos \left( \frac{r}{2H} \right)} \right] .$$

(Put another way, the coordinates $(y, \theta)$ define an orthogonal system, just as the cylindrical coordinates $(r, z)$ are orthogonal. However, there are no conformal properties shared between the two sets of orthogonal curves, since Cauchy-Riemann like conditions are never involved, in contrast with the methodology often employed in obtaining stream functions in two-dimensional incompressible flows.)

The physical explanation underlying the mathematics is that, as the flow expands isotropically to one scale height in radius, it begins to see the diminished resistance in the vertical direction. The reverberations now tend to focus the inward moving shocks onto the $z$-axis which is then superheated. The conversion of the lateral momentum (whose total is zero) into heat is very efficient. Meanwhile, the gas begins to move in the direction of least resistance, curving upward in its path, and limiting lateral expansion to $\pi H$. The trajectories taken by particles is observed from (26) to be quickly deflected from the vertical due to the high temperature which emerges along the $z$-axis and no material is observed to escape, i.e. possess an upward component in its velocity in excess of the escape velocity. Note that these particle trajectories are universal, i.e. are independent of the energy $E$ involved. In order to convert these trajectories into physical velocities, the scalings described earlier must be invoked. Nevertheless, a common feature of these trajectories is that virtually no material escapes.

In Fig. 2, we show the shock contours as well as particle streamlines at uniformly separated values of $y = 0.0H, 0.1H, 0.2H, \ldots$. (The distance units used are dimensionless and in terms of the scale height $H$.) The solid, ellipsoidal lines describe the evolution of the shock, with the heavy paraboloidal line describing the $t \to \infty$ (or $y \to 2H$) limit for the flow. The dashed lines describe particle trajectories whose initial zenith angles were $0^\circ, 10^\circ, 20^\circ, \ldots$. The heavy dashed line denotes the “surface” and the solution is also shown below this level, relevant to situations where a bolide explodes in the atmosphere. As mentioned earlier, the shapes of the shock profiles are independent
of $\gamma$, apart from its influence on the time-like variable $y$. There are some interesting qualitative features shared by our results for an isothermal atmosphere, germane to terrestrial planet modeling, and the results obtained by Korycansky (1992) for the mathematical model that he was exploring. It is especially interesting to contrast these results for the particle trajectories with, for example, the analytic model by Laumbach and Probstein (1969) which assumes that the flow field is locally radial. We note from this figure that an air burst does not propagate downward more than $\approx 1.4H$. First principle simulations of airbursts also cease to propagate below some altitude as a shock, becoming an acoustic wave, unless they are sufficiently energetic for reasons detailed earlier. We will later make explicit comparisons between the Kompaneets solution-associated particle trajectories, Laumbach and Probstein’s model assumptions, and accurately computed trajectories germane to the impact problem.

Although the present solution has some relevance to SL9, its applicability is limited as there is substantial evidence that the SL9 fragments effectively disintegrated or even exploded in Jupiter’s atmosphere not at a point but along a line, fundamentally complicating our assumed scenario (Takata et al., 1994; Crawford et al., 1994; Zahnle and Mac Low, 1994). Nevertheless, the theoretical prediction that essentially no particles would escape from the atmosphere was a fundamental surprise. [Interestingly, no particle escape from SL9 was detected by Takata et al. (1994).] In order to better understand if the results in Fig. 2 were genuinely appropriate and under what conditions atmospheric escape would occur, we turned to an accurate hydrodynamic code designed to simulate energetic gas dynamic explosions in atmospheric environments.
Computational Method and Numerical Results

We employed Los Alamos’ CAVEAT, a high accuracy hybrid Eulerian-Lagrangian finite difference code for flow problems with large distortion and internal slip (Addessio et al., 1992). The CAVEAT computational fluid dynamics code, combined with a discrete ordinate radiation transport algorithm, has been used to model the effects of atmospheric nuclear explosions. The radiation transport is only important for the early time history of these fireball simulations, but is not important to the early time evolution of impact events. The numerical results, early and late time, provide quite accurate agreement with fireball yield estimates and shock front evolution. The application of the code to energy sources in the $10^{30}$ erg range, comparable to those of the K/T extinction bolide, produces a shock that is much more energetic than previous applications of this code. Consequently, considerable numerical experimentation was required to obtain an appropriate parameterization of the internal slip, diffusion, and other computational quantities associated with the algorithm to assure the correctness of the simulations. Significant modifications were made to the code and to its mode of utilization in order for it to accurately compute the hydrodynamic flow.

The behavior of the algorithm was examined by studying the shock evolution, at a given time, as a function of time step size and zoning parameters until convergence was achieved. It was necessary to use the multi-material option of CAVEAT in order to treat the interface between the bolide and the atmosphere as a Lagrangian surface. As a result there are two grids in the simulation—one fitted to the bolide and the other fitted to the surrounding air. The mass per unit area of the bolide is much greater than that of the atmosphere in the computational grid and as a result the bolide grid needed fine zoning for an accurate solution. The Arbitrary Lagrangian Eulerian parameter known as ALEC0EF was set to allow for a near Lagrangian simulation and thereby capture the shock with minimal diffusion. The parameter ALEC0EF (Adessio et al., 1992; pp. 80, 97, 109, 148) varied between 0.96 and 0.99 in our simulations, with the particular value employed selected to minimize computational error. ALEC0EF can range between 0 and 1 with the end points corresponding to pure Eulerian and pure Lagrangian computations, respectively. Thus, we found that the most accurate computations emerged when the code was functionally close to being a pure Lagrangian finite difference method. We provide other details underlying our numerical examples, both for the Earth’s atmosphere and that of Venus.
The shock that first develops at the bolide/atmosphere interface and soon progresses through the atmosphere is extremely strong (i.e. satisfies the Von Neumann overpressure criterion of internal to external pressure is greater than 10) and required that the Arbitrary Lagrangian Eulerian parameter be employed with selectively weighted rezoning in order to provide a reasonable grid at all times during the simulation. The weighting function for the rezoning routine was chosen to be the absolute temperature, as a practical device for accommodating the Courant-Friedrichs-Lewy condition (Richtmyer and Morton, 1967). The parameter WFGR (Adessio et al., 1992; pp. 4, 162) was selected to make temperature the weighting function in contrast with the defaults of density or pressure. The choice of temperature as the weighting function insures that the grid will move to and, thereby, resolve in a fine scale both the air shock and the bolide-air interface. The parameter ITREZN (Adessio et al., 1992; pp. 84, 158) is the maximum number of iterations for the Jacobi solver of the rezone generator equation. Since we used an ALECOEF less than one, we were employing a continuous rezone, i.e. rezoning on every time step! The default value for ITREZN was increased from 3 to 10 to obtain the desired result of a nearly orthogonal computational grid at all times.

Convergence of the numerical solution was established in an *ad hoc* but exacting manner. We are most interested in the shock evolution. The shock is moving fastest in the vertical direction. We chose an altitude of 100 km and studied how fast the shock reached this altitude as a function of bolide zoning and total zoning (i.e. bolide and atmosphere). We found that convergence was achieved when the bolide air interface was chosen as Lagrangian, and when the bolide was modeled with about 1000 zones. The large number of atmospheric zones was chosen in order to prevent large aspect ratio zones everywhere in the grid and to compare, with some precision, the shock locations according to the theory and simulation. We use the term “accuracy” to represent shock front resolution. The chosen ALECOEF, ITREZN, and WFGR parameters maintained shock resolution to the order of 1 km or less.

We performed dozens of simulations with differing initial vapor cloud configurations, and a vapor cloud with $10^{19}$ gm and $5 \times 10^{30}$ erg, commensurate with that associated with a K/T size event. The long-term behavior of the flow did not depend sensitively upon the initial placement.
of the vapor cloud. Following O’Keefe and Ahrens (1977a, b), we assumed that the initial vapor cloud had a pancake or pillbox shape, with a density approximating that of lithospheric material. A representative case, initialized with a 8 km high × 10 km radius pillbox, is described in Fig. 3 for Venus and Fig. 4 for Earth. The total computational grid for the Venus pillbox simulation had a 10 km radius and 8 km height with 95 radial by 220 vertical zones. The initial pillbox was resolved with 25 × 50 rectangular zones—initially 400 m horizontally and 160 m vertically. The remaining 70 radial zones increased by a constant ratio out to the maximum height of 160 km. Therefore, there were 1250 quadrilaterals resolving the bolide, and 19650 quadrilaterals resolving the enveloping air. The initial Venus atmosphere was taken from Fegley (1995). We used a constant $\gamma = 1.4$ equation of state. The total computational grid for the Earth pancake simulation of 20 km radius and 2 km height was 120 × 220 zones. The initial pancake was resolved with 50 × 50 rectangular zones—initially 400 m horizontally and 40 m vertically. The remaining 70 radial zones increased by a constant factor to a maximum height of 160 km. Therefore, in this case the bolide always contained 2500 zones and 23900 zones resolved the surrounding air. The initial Earth atmosphere was taken from a 1965 CIRA profile. The air equation of state was developed by many groups mainly in the 1950’s. The tabular equation of state data, used in these simulations, was derived from the results of Gilmore (1955) and Hilsenrath et al (1957,1959). It was compared against the more recent SESAME (Lyon and Johnson, 1992) equation of state tables, developed at Los Alamos, and found to be nearly identical.

For computational reasons, we employed an approximation to the “rock equation of state” that made our computed solutions an upper bound to the amount of material that would be lost by the expansion of the vapor cloud. The condensation of vaporized rock materials results in a density change of a factor of order $10^3$, and essentially causes the computation to grind to a halt—particularly as our code was largely Lagrangian in character. (Ideally, a multi-phase, multi-fluid code would be required; we are developing this capability for future work.) Accordingly, we treated the vapor cloud as though it were “heavy air,” and that it could not lose its reservoir of thermal energy to condensation. Since this would artificially preserve an excess amount of thermal pressure, we can realistically expect that substantially less atmospheric erosion would be observed than we predict here.
Fig. 3 shows density contours 5.9 seconds after impact (with \( y \approx 2 \)) of a K/T size bolide (\( 5 \times 10^{19} \) gm and \( 10^{30} \) erg initially in thermal energy) in the Venus atmosphere. The most striking feature in this figure is the anisotropy and the limited expansion in the horizontal. (Qualitatively similar behavior was obtained for a K/T size impactor in the Earth’s atmosphere.) Since the Venus surface scale height is \( \approx 11.2 \) km, we expect a column to form that is \( \pi \times 10 \) times that (or about 35 km) plus the initial pancake radius of 10 km. The extension to Kompaneets’ theory predicts a column to form that is about 45 km in radius—the computations reveal a column with 55 km radius. (We performed checks on late time evolution of the vapor plume, extending the simulation until the plume height exceeded 400 km. We observed no significant further expansion of the column formed, but do not report those results here since ionization and radiative processes become important in those late stages, albeit in a self-limiting way from the standpoint of the expansion of the plume, and those physical processes were not fully incorporated into our code.) The Kompaneets approximate solution at late time is overlaid on this figure showing the remarkable agreement between the computed and the theoretical flow patterns. The departure between the two observed near the base is a relic of the energy still resident in the vaporized bolide. Importantly, this material does not have sufficient energy to escape from the planet. The qualitative lack of dependence of our results on the initial disposition of the vapor cloud provides further evidence for the remarkable role that the strong shock has in confining the vapor cloud’s horizontal expansion.

In Fig. 4, we explicitly compare the velocities in the precise numerical solution with those obtained from the hemispherical blowoff model applied to Earth (evaluated at very large time, so that all of the thermal energy would have been converted into kinetic energy and assuming minimal residual atmospheric inertia). The hydrocode results for a K/T size event with an initial pillbox distribution of vaporized bolide of 8 km in height and 10 km in radius are described much more closely by our analytic theory than the Vickery and Melosh model and shows the scale height’s influence on regulating the evolution of the gas dynamic shock, notably the asymmetry in the vertical direction, producing a “fountain-like” flow which may play a role in the global distribution of volatilized bolide and target material, which ultimately may effect the K/T event’s climatic influence, i.e. \( \text{H}_2\text{O}, \text{SO}_2, \text{CO}_2, \text{SiO}, \text{FeO}, \text{CaO}, \text{MgO} \), and the well-known platinum-group elements, Ru to Ir. Our simulation does not include the role of the atmospheric shock induced by the entry of
the bolide on the atmosphere (e.g. Takata et al. 1996) nor the partial atmospheric evacuation of a column cleared by the incoming bolide. Both effects influenced the flow from the SL9 impact. However, it should be noted, since the Kompaneets’ theory shows how the flow proceeds in the direction of the density gradient, that any atmospheric preparation will further confine the emerging plume to the column which forms. It should be noted, as discussed in association with Fig. 10, that the Vickery and Melosh model applied to the K/T impact would have only about 25% of the vapor plume escape from the atmosphere, a factor five more than our first principles calculation. (Many published papers erroneously suggest that the Vickery and Melosh model would produce complete radial hemispheric blowoff for a K/T size event—this is inconsistent with the detailed calculations that Vickery and Melosh presented.) Importantly, the fast-moving gas in the simulation exists only at very rarefied altitudes, and very little atmosphere escapes, showing that moderate events remain an important source of volatiles, unlike the predicted effect of atmospheric erosion from Mars-size Earth impactors of Vickery and Melosh.

Figure 5 provides a convincing illustration of how insensitive the late-time evolution of the vapor cloud is to its initial conditions as well as the degree to which it conforms with the Kompaneets solution. We considered for Earth two K/T size impactors with $10^{30}$ erg and a $10^{19}$ gm vapor cloud. In the first instance, we employed an initial pillbox distribution (as in Fig. 3 and 4) that was 8 km in height and 10 km in radius—this is shown in panel (a). The evolution of the density distribution in this panel closely parallels the velocity plot shown for the same simulation in the Fig. 4. In the second instance, we employed an initial pancake distribution that was 2 km in height and 20 km in radius—this is shown in panel (b). The initial expansion in panel (b) was essentially one dimensional (in the vertical) until expansion over several scale heights emerged, whereupon the flow assumed its signature Kompaneets appearance. In order to solidify the observation that the flow becomes Kompaneets-like, we have superposed on both panels the Kompaneets approximation solution at late time.

The preceding figures and, especially, the last two in this paper are presented to resolve issues attending to the erosion of terrestrial planetary atmospheres and K/T sized bolides. However, as we have provided a “snapshot” in the previous figure of the distribution of velocities at a given time for both the vapor cloud and the enveloping atmosphere, it would be especially insightful to understand
the dynamics of individual particles over the lifetime of the simulation. This is especially significant in trying to develop an appreciation of how the dynamics differs among particles deep inside the vapor cloud, particles at the interface between the vapor cloud and the enveloping atmosphere, and particles that are initially far from the vapor cloud. In Fig. 6, we provide a schematic describing the tracer particles and the original pillbox geometry for the vapor cloud employed in our computational experiments. The 8 km high by 10 km radius pillbox appears as a shaded rectangle. To simulate the behavior of particles deep within the cloud as well as particles well outside the cloud, we created two sets of 91 particles—at 1° intervals from 0° to 90°—arranged so as to describe “hemispheres” at 5 km and 15 km initial radius. These tracers are labeled (a) and (d) in Fig. 6. In order to describe the dynamics near the vapor cloud-atmosphere interface, we employed two sets of tracers. The first set of interface-related tracers, analogous to the hemispherically distributed ones described above, were situated as above in a 10 km arc and are labeled (b) in the schematic. The second set of interface-related tracers are situated precisely on the interface; the 76 tracers employed are labeled (c) in Fig. 6.

The results obtained can now be compared directly with those Kompaneets solution-based predictions developed earlier. In Fig. 7, we present the evolution of the 5 km inner hemisphere particles denoted (a) above. To parallel the results presented for the Kompaneets-solution model in Fig. 2, we use solid lines at representative times to show the “front” associated with the initial hemispherical distribution of particles. We select representative tracer particles, at 5° intervals, whose evolution we follow in detail and describe using dashed lines. The behavior of the inner hemisphere tracers, depicting particles deep inside the vapor cloud, has two basic features. First, they exhibit a tremendous resistance to moving. (That explains why the arc near 5 km appears so dark; the tracer particles are initially stagnant.) Second, once they start moving, the tracer motion is essentially radial.

In Fig. 8, we present the two sets of interface-related tracer particles in panels denoted (b) and (c). In panel (b), we show the evolution of the “interface hemisphere” tracers. We observe a remarkable degree of complexity in the flow including the appearance of mixing of tracer particles. Similarly, in panel (c), we see the interface pillbox tracers undergo a complex evolution. In that case, particles far from the corner of the pillbox, travel perpendicular to the respective pillbox faces.
A void begins to emerge at the (artificial) corner, and nearby tracers rush in to fill the gap and also undergo complex behavior. (In future work, we hope to present a movie describing the behavior observed.)

In Fig. 9, we present the final group of tracer particles, the set associated with the outer hemisphere (d). This is the region where we expect the Kompaneets solution to be most accurate. The characteristic Kompaneets solution shape is seen, although the lateral momentum of the enclosed vapor cloud is having a significant influence on the lateral expansion of the tracer particles. (However, it must be emphasized that the apparent bulge does not provide the atmospheric mass with the momentum it would need to escape, in contrast with the hemispheric blowoff model.) There are two other features shown in this figure that merit discussion.

(a) The upward moving tracers, i.e. those near 90°, are accelerating. This feature was explored by Laumbach and Probstein (1969), using analytic approximations, as well as by Mac Low et al. (1989) by computational means. In the latter work, Mac Low et al. explored an astrophysical problem involving supernova explosions occurring outside the galactic disk, which has an essentially exponential interstellar medium density distribution and did make qualitative comparisons with the Kompaneets solution. However, unlike the situation relevant to the early phase of evolution of explosions in planetary atmosphere environments, radiation effects were a particularly important component of their simulations. Mac Low et al. obtained a density jump across the shock more than two orders of magnitude, which is a tell-tale sign of a radiation-induced thermal and/or Vishniac instabilities. They claimed that other features near the top of their shock front were due to the Rayleigh-Taylor instability, although they acknowledge that the thermal and Vishniac instabilities could be its source. In our simulation which contains no radiation, we see no sign of Rayleigh-Taylor instability and believe that the effects observed by Mac Low et al. are radiation induced and occurred at substantially later times.

(b) The particle trajectories are complex and conform crudely to the Kompaneets solution based calculation only near the top of the atmosphere. Indeed, one could argue that the radial flow assumption in Laumbach and Probstein (1969) is an equally-valid approximation for particle motion. The key to this dilemma is resolved by examining the grid of solid and dashed lines depicted in Fig. 9. Unlike the situation present in fluid dynamics, we are not looking at a
system of streamlines intersecting uniform-velocity potential lines. The principal limitation of the Kompaneets solution, addressed by Andriankin et al. (1962), Bach et al. (1975), Laumbach and Probstein (1969), and Oppenheim et al. (1971) is the assumption that the pressure behind the shock front is uniform. In their refined theoretical models, the coefficient $\lambda$ employed in our Eq. (5) is allowed to vary with time and angle. This is done to accommodate the time it takes for pressure information to be communicated upward in the rapidly evolving front. In practical terms, the effect this has on the shape of the shock front is modest, because the Rankine-Hugoniot conditions directly drive the flow. However, the effect of variable pressure is more important in driving the motion of individual particles behind the shock. The non-orthogonal nature of the grid that we have obtained in Fig. 9 also implies that an assumption we employed in deriving the Kompaneets solution trajectories is only partly correct: as mass is accreted onto the shock, it is not permanently “glued” in place but will respond by moving along the shock in response to the gradually emerging interior pressure distribution. Thus, while the Kompaneets solution remains a useful descriptor of the evolving shock front, particle trajectories follow paths that lie between the predictions made by the Kompaneets model and the Laumbach and Probstein model.

Having explored the behavior of individual particles in different regimes within the evolving shock, we return now to the issue of atmospheric erosion.

In Fig. 10, we show what fraction of vapor cloud (and atmosphere) escapes as a function of time for two different initial partitionings of energy, a situation where all of the initial energy is in thermal form and where $1/2$ of the initial energy is in upward directed motion. The total amount of gas which escapes, expressed as a fraction of the initial bolide mass, converges exponentially to a value ranging between 4% and 7%, which brackets a reasonable range of possible initial distributions. It is difficult to make a direct comparison with the hemispheric blowoff model of the amount of atmospheric gas that escapes—the mass fraction expressed above reflects the total amount of vapor cloud and atmospheric gas relative to the original vapor cloud that escapes—it is our expectation that virtually none of the escaping material represents terrestrial volatiles. This follows owing to the manner in which the atmospheric gas was pushed aside by the rising vapor plume, leaving very little that could be propelled outward. We have, nevertheless, estimated what fraction of bolide mass
and atmospheric mass would escape in the hemispheric blowff model. For a $10^{19}$ gm bolide with $5 \times 10^{30}$ erg, the innermost 61% (by radius) of the vapor cloud travels at speeds below the escape velocity of 11.2 km/s; only 31.8% of its mass is traveling above the escape velocity. The critical angle $\theta_c$ which defines the cone above which all atmospheric gas escapes is 77.4° for a bolide of this size in the Earth’s atmosphere. The atmospheric mass contained within this cone is $6.93 \times 10^{16}$ gm, compared with the $3.05 \times 10^{18}$ gm that lies above the tangent plane—the Earth’s atmosphere has a total mass of $5.30 \times 10^{21}$ gm. In contrast, of the $3.18 \times 10^{18}$ gm in the vapor cloud traveling faster than escape velocity, 78.2% has sufficient momentum in the Vickery and Melosh model to expel the radially aligned atmospheric gas—hence, $2.49 \times 10^{18}$ gm of the original vapor cloud (i.e. 24.9%) will escape. From this perspective, the atmospheric mass lost in the Vickery and Melosh model is inconsequential; however, their work is often misquoted as guaranteeing that virtually all of the mass above the tangent plane will be eroded. In contrast, the fraction of the vaporized impactor and target material returned to space is a factor five larger than that shown by our more accurate hydrodynamic calculation. We summarize our comparison of models in the cartoon shown in Fig. 11. As a consequence, we conclude that K/T-size and smaller events remain a major source of volatiles to young planetary environments such as that of the Earth.

Discussion

Cameron (1983) was the first to suggest that impact events could lead to planetary atmospheric erosion, a possibility with monumental importance (Ahrens, 1993). Vickery and Melosh (1990) were the first to propose quantitative physical models, based on the significance they attributed to the lateral momentum carried by the vapor cloud, and suggested that hemispheric blowoff would ensue. We pursued another set of model assumptions, based on the controlling influence that a strong shock produced by the interaction of the vapor cloud with the atmosphere might have. In so doing, we found vapor particle trajectories that were inconsistent with any atmospheric erosion.

We were sufficiently surprised by the new analytic results that we extended state-of-the-art hydrocodes to be able to treat a K/T size event and found that precise numerical results confirmed our approximate analytical ones for the evolving shock front. However, we were further surprised to discover that particle trajectories in the vapor cloud and enveloping atmosphere environment could be dynamically complex, depending upon the physical region explored, and we suggest that this
may have observable consequences particularly as a result of the condensation of the vapor cloud mixed with atmospheric gases. After many numerical tests (only a sampling of which are presented here), we concluded that K/T and smaller size events remain an important source of volatiles for terrestrial planets. The challenge ahead is to explore the influence of much larger events on planetary atmospheres, particularly those of Earth, Venus, and Mars.

Acknowledgments

We thank the Institute of Geophysics and Planetary Physics at Los Alamos National Laboratory for support (IGPP Grant #605 to WIN and EMDS) and a NASA Grant #NAGW-1941 to TJA. Contribution #5668, Division of Geological and Planetary Sciences, California Institute of Technology. We are grateful to Mordecai Mac Low for an insightful discussion of an earlier version of this manuscript. We are particularly grateful for the comments and discussion by two extremely conscientious referees, Kevin Zahnle and Gary McCartor. This manuscript benefitted substantially from all of their suggestions.

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Figures

1. Time-like variable $y$ (in units of scale height $H \approx 8.6$ km on Earth) versus time $t$ (in units of $t_y = \sqrt{\frac{32 \pi H^2 \rho(0)}{75 \lambda E (y^2 - 1)}} \approx 1.05 \times 10^{-2}$ sec for Earth’s atmosphere). Inset uses a log-log plot to show both $y$ and corresponding shock height $z_{\text{max}}$ (both in units of $H$) versus $t$ (in units of $t_y$). Also shown in inset is theoretical $2/5$ power law curve expected from similarity argument underlying (1). Note that at late time, i.e. $t \gg 1$, $z_{\text{max}} \to \infty$ while $y \to 2$.

2. Solid ellipsoidal lines describe expansion of blast into enveloping isothermal atmosphere (Kompaneets, 1960). Heavy paraboloidal line describes shock at late time. Light dashed lines, orthogonal to ellipsoidal contours, describe analytic particle trajectories, showing that little material is eroded from atmosphere. Heavy horizontal dashed line describes ground, for impacts on a terrestrial planet surface, with light solid and dashed lines below describing expansion of shock and trajectory of particles below point of explosion that is relevant to comet impacts onto a major planet. The model ignores atmospheric flow induced by entry of impactor.

3. Contour of logarithm (base 10) of density in gm/cm$^3$ approximately 5.9 sec after a K/T size event on Venus. The Kompaneets approximate shock solution for the above is overlaid.

4. Vector plot showing velocity vectors $\approx 4$ s after K/T size impact on Earth using hydrocode (left panel) and hemispherical blowoff model (right panel). Escape velocity is 11.2 km/sec; maximal velocity obtained in simulation was 76 km/sec, and arrows in both panels are scaled according to this maximum (shown in vertical axis label). The maximal velocity in Vickery and Melosh model is 18.3 km/sec, and corresponds to denser altitude regime. Solid lines employed in both panels identify hyper-escape velocity flow zones in the simulation and in Vickery and Melosh model.

5. Contours of logarithm base 10 of density in gm/cm$^3$ of two K/T size impacts on Earth, with differing initial vaporized bolide geometries. Panel (a) corresponds to pillbox distribution with initial height 8 and radius 10 km which has evolved over 4.08 s, while panel (b) corresponds to a 2 km high by 20 km radius “pancake” which has the same volume and has evolved over 3.75 s. The Kompaneets approximate solution is overlaid on each panel.

6. Schematic illustrating initial placement of pillbox-shaped vapor cloud and tracer particles chosen to describe (a) inner hemisphere at 5 km, (b) interface hemisphere at 10 km, (c) interface
pillbox at vapor cloud boundary, and (d) outer hemisphere at 10 km.

7. Solid ellipsoidal lines describe expansion of tracer particles (a) initially distributed in a hemisphere at 5 km distance while radial dashed lines describe tracer particle trajectories.

8. Solid ellipsoidal lines describe expansion of tracer particles associated with the interface distance while radial dashed lines describe tracer particle trajectories. Tracer particles correspond to cases described in Fig. 5, namely (b) a hemisphere with radius 10 km near vapor cloud-atmosphere interface and (c) precise interface between the vapor cloud and the atmosphere.

9. Solid ellipsoidal lines describe expansion of tracer particles (d) initially distributed in a hemisphere at 15 km distance while radial dashed lines describe tracer particle trajectories.

10. Mass eroded during cloud evolution relative to bolide mass for K/T size event where initial vapor cloud is a $10^{19}$ gm pillbox 10 km in radius and 8 km high for different sets of initial conditions.

11. Cartoon highlighting differences between the Vickery and Melosh model (left panel) and present analytic and computational model (right panel). In the former case, only vapor cloud material traveling faster than the escape velocity plus the radially outlying atmosphere which it is able to expel against the gravitational barrier escapes. The escaping material is contained within a cone, designated by heavy solid line at an angle $\theta_c$ measured from the vertical (around 77.4° for a K/T size event), is able to escape. In the present model, the vapor cloud expands rapidly upward, essentially pushing aside atmospheric gas. The vapor plume whose vertical velocity exceeds the escape velocity (i.e. above the horizontal solid black line) escapes pushing only a small part of the overlying atmosphere away. The net effect is that present analytic and computational model predicts very little atmospheric gas is eroded, in contrast with the hemispheric blowoff model.
Figure 1
Figure 2
Figure 3


Impact Erosion
Figure 4
Figure 5
Figure 6
Figure 7
Figure 8
Figure 9

(d) Outer Hemisphere
Model with 1/2 initial energy in upward directed motion.

Model with all initial energy in thermal form.

Figure 10
Figure 11