SHOCK TEMPERATURE MEASUREMENTS FOR METALS

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The effects of heat conduction, melting, and radiation, at metal-window interfaces on shock temperature measurements of metals such as iron are quantitatively described using an example, previous research employing Fe/AlO₃ and Fe/U₃O₈ targets shocked to the pressure range of 110 to 210 Gbars. The mathematical description of the effects of thermal conduction induced weld-line initiation or cavity formation in metal samples and the melting of transparent windows are described in detail. These effects cannot be considered in earlier analyses.

We also analyze the properties of release welding and superheating which occur when shocked solid metal bars are partially provided as result of laser reflections at the metal-window interface. It is concluded that the thermal properties of the metal and window (specific heat and thermal conductivity at high pressure) for ideal metal films, and bricks, shock interface temperature, and yield metal-halogen temperatures, along with melting do not take place under the target interface. In contrast, the metal-window interface temperature is in the partial melt regime. In other words, shock-induced partial melting in metal and window will also occur due to thermal conduction across the metal-window interface. When this occurs, the experimental interfacial temperature can be used to estimate pressure-temperature state along the shock wave front. At very high pressure, however, the metal is completely melted, and the laser temperature can only be used to infer Hugoniot temperatures provided the window material remains transparent and cooler than the metal.

KEY WORDS: Shock temperatures, Hugoniot, melting, iron, pyrometry.

INTRODUCTION

The determination of temperatures of shock-compressed metals, although of great interest to constraining high pressure scaling behavior and the pattemeters in thermal equation of state such as specific heat, Gruntensius parameter and entropy of melting, as well as thermal and electrical conducting processes, has long been a challenge. Kroeyer first reported shock-temperature measurements in transparent materials such as alkali halides and water

The techniques used by Korker and co-workers were to view the explosively driven shocked material through the transparent unshocked sample and measure the radiation versus time at a series of wavelengths to obtain black or grey body temperatures.

Later, experimental techniques developed by Lyzenga and Alven's employed four to six channel photodiode systems which operated with light gas guns and

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were used to study a series of transparent materials, including forsterite (Mg$_2$SiO$_4$)$_2$, crysol and fused quartz (SiO$_2$)$_2$, Calcium (CaO)$_2$, water (H$_2$O)$_2$, anorthite glass (CaAl$_2$Si$_2$O$_8$)$_2$, periclase (MgO)$_2$, diopside and diopside glass (CaMg$_2$Si$_2$O$_6$)$_2$. More recently, Williams et al.\cite{1} and Bass et al.\cite{2}, have modified the geometry of the earlier experiments using a thin film of the metal of interest (iron) plated onto a transparent window material as shown in Figure 1. A major problem with these experiments is the difficulty in both making ideal interfaces between the metal and the transparent window and, in the case of metal films, determining that the metal films do not contain undetected initial porosity which would result in actual temperatures to be greater than for non-porous metals. The question of the effect of film porosity and a reanalysis of the Williams et al.\cite{1} and Bass et al.\cite{2} shock temperature data for iron are addressed in a companion paper\cite{3}.

![Figure 1](image-url) Flyer-driver-metal film-window system used in shock temperature measurement. Observed thermal radiation is from the metal-window interface assuming window material remains transparent after shock wave is transmitted into window.

In the ideal experiment, the observed temperature is related to the spectral radiance from the metal–window interface, provided the shock transmitted into the window does not produce opacity at the shock front and the temperature behind the shock front in the window remains substantially below the temperature of the shocked metal film. The film is usually the same metal as the driving plate depicted in Figure 1. Provided the film is thick enough, the film effectively thermally isolates the film-window interface from thermal perturbation at the driver plate–film interface. A virtually ideal interface can be achieved.

The relation of the radiative interface temperature to the initial Hugoniot temperature in the metal is dependent upon the shock temperatures in the window, the state in the film, which, because of the mechanical impedance mismatch between the metal film and the window, is at a different pressure than the initial Hugoniot state, and the heat conduction process and possible melting process at the interface. Previously Grover and Urtiew\cite{4,5}, analyzed data from an earlier attempt to carry out such experiments for magnesium. These experiments did not
employ films sputtered or metal vapor condensed onto the window but relied on optical contact between a metal foil or plate and window. In these experiments ~ 1 μm interface gaps caused double shocking to occur at the metal–window interface. Recently, Álvarez and Ahrns analyzed in detail the thermal radiation from shock-compressed material interfaces. Their studies, however, did not take into account possible re-solidification of the melted metal film or melting in the window due to heat conduction at the interface. These effects take place when the temperatures at the metal–window interface achieve the melting temperature of metal sample, or when the observed interface temperature is above the estimated high-pressure melting temperature of the window material.

In the present paper, we examine the effects of melting the sample, upon pressure release at the window-sample interface the possible resolidification of a film of melted material also at the window-sample interface, and possibly melting window material via conduction from the interface.

MODELS OF METAL SHOCK TEMPERATURE EXPERIMENTS

The major temperature-pressure regions of interest in metals such as iron, for which the shock temperatures are studied in detail in this paper, are in the region of shock-induced melting. Two window materials which have been employed are single-crystal sapphire (Al₂O₃) and lithium fluoride (LiF). Urtiew showed that crystal sapphire remained transparent under shock up to 80 GPa and transmitted light to at least ~ 100 to 130 GPa above which some loss of transparency were observed. Earlier, Hawke (see Urtiew, 1974) reported that Al₂O₃ remained transparent under dynamic isentropic leading to 500 GPa. The temperatures achieved by Bass et al. similarly imply transparency of Al₂O₃ to ~ 230 GPa. LiF is reported by Chhabhia to remain transparent up to 600 GPa.

The strong shock wave driven into the metal sample will, in general, be reflected upon reaching the metal–window interface. In the case of an iron sample with a sapphire (Al₂O₃) or lithium fluoride (LiF) window, this reflected wave is a rarefaction wave, because of the highest shock impedence of iron with respect to the window. Although the pressure and particle velocity on both sides of the interface are equal, because of the usual situations that the iron or other metal sample is more compressible, the temperatures on both sides of the interface may differ enormously. Figure 2 shows schematically the path of the interface upon interaction of the shock at the metal–window interface and the subsequent path upon thermal relaxation in the temperature-pressure plane. Let us consider the release process at the material interface first.

A. Release Process Approximation in Mixed Phase Region

The pressure, \( P_\text{r} \), at the iron-window interface, which is equal to the Hugoniot pressure of the shock wave transmitted into the window material, \( P_\text{w} \), is obtained via the particle velocity, \( u_\text{r} \), at the interface using the usual shock impedence match
Figure 2: Preserve release and thermal inhobiting path of metals sample at the heat-window interface. Six different progressions labeled as "A" to "F". "A"—Initial state in solid, release, and thermal conduction process remains in solid phase region. "B" to "E"—Processes having phase change along release and conduction path. "E"—Initial state in solid, release heating occurs but conduction sets in solid. "G"—Initial state in mixed phase region, release remains in mixed phase region, Conduction induced solidification occurs. "H"—Initial state in mixed phase region, release heating is completed and entire sample in liquid phase region, conduction induced solidification occurs. "I"—Initial state is liquid, upon release, sample remains in liquid but conduction induced solidification occurs. "J"—Initial state in liquid, release and conduction process remain sample in liquid phase region.

solution. The release isentrope of iron, HRS, of Figure 3a can be approximated by assuming that in the β-α plane (Figure 3b) the release isentrope is the mirror image reflection of the principal Hugoniot of iron. The material velocities before and after the simple release wave, Δu = u_e - u_i, and the pressure change are then related via the Riemann integral.
\[ \Delta u = \gamma \int_{V_0}^{V} \left[ 1 - \left( \frac{dP}{dV} \right) \right]^{-\gamma} dV \]  

or approximately

\[ V_r = V_0 + \frac{(\Delta u)^2}{p_r - p_a}, \]  

where subscripts \( H \) and \( R \) refer to shock state and the release process. The release temperature, \( T_r \), can be related to the initial shock-temperature, \( T_0 \), in terms of

\[ T_r = T_0 e^{-\frac{\gamma}{\gamma-1} \frac{\Delta u}{V_0}}. \]

Here \( \gamma \) is the Grüninich parameter. Shock temperature, \( T_0 \), is estimated by means of the method described in Alboes \( ^4 \), assuming the Grüninich equation of state is valid.

Figure 3: (a) Release path in \( P-V \) plane and (b) Shock impedance match solution in \( P-u \) plane. The particle velocity at the material interface, \( u_0 \), can be determined from the intersection \( S \) of the hyperbola of the principal Hugoniot of metal (line) with the Hugoniot of window material. \( H \)—initial Hugoniot state in metal; \( ES \)—principal Hugoniot of initial WKB; \( ES \)—principal Hugoniot of window material; \( RS \)—release path of metal material upon interaction of the initial shock wave with the interface.
The above approximation, however, is poor if the initial state is the metal (e.g., iron) is solid or partially molten and further release melting occurs. The mirror-image release path in the $p-v$ plane will shift from $R$ to $R'$ because of the release melting transition (Figure 4a). If the initial shock state is in the solid phase region, release path HNR corresponds a "super-heating" release process (Figure 4b). This "super-heating" release temperature, $T_r$, can be estimated from Eqs. (1) to (3) via the mirror-image path HNR. If we assume a Lindemann type of melting curve for iron, the melting temperature $T_m$ and volume $V_m$ takes a form

$$d \ln V_m = \frac{\gamma}{3} - T_m. \tag{4}$$

An energy balance consideration for release melting state $R$ and super-heating release state $R'$ gives the melted mass fraction $x_m$

$$x_m = \frac{C_p(T_m - T_0) \Delta H_m}{\Delta U_0}. \tag{5}$$

where $C_p$ is the specific heat at constant pressure (here $P = P_f$) and $T_m$ is the melting temperature at the melting pressure $P_f = P_0$, $\Delta H_m$ is the enthalpy change upon fusion. In the actual determination of $x_m$, because of lack of knowledge of $C_p$ at
high pressures and temperatures, it is approximated by

$$\chi_{cr} = C_r (T_o - T_l) / \Delta E_{cr},$$

(6)

where $C_r$ is the Debye specific heat at constant volume, $\Delta E_{cr}$ is the internal specific energy change during fusion at constant pressure $P_c = P_l$, and is related to $\Delta H_{cr}$ via the Clausius-Clapeyron equation

$$\Delta H_{cr} = \frac{\Delta V_c}{\Delta T_c} \Delta E_{cr},$$

(7)

with $\Delta V_c$ denoting the volume change during the fusion. Subscript $M$ denotes the derivative taken along the assumed Lindemann melting line.

For the process described via paths "3" and "4" in Figure 2, the initial shock state is in the partially shock-melted region. The totally melted mass fraction is the sum of that initially melted mass fraction, $\chi_{m}$, and the melted mass fraction due to release.

$$\chi_{m} = \chi_o + C_r (T_o - T_l) / \Delta H_{cr}$$

(8)

$\chi_o = 1$ gives the condition at which the release path intersects with the liquids and $\chi_o \geq 1$ implies that the release path enters the liquid region. The volume $V_o$ achieved via release to pressure $P_o$ is

$$V_o = V_c + \chi_o \Delta V_c.$$  

(9)

If $\chi_o < 1$, for the case $\chi_o > 1$, which is calculated from Eqs. (5) to (8), the final release temperature and volume can readily be deduced via the corresponding values given by $\chi_o = 1$ and via the simple release calculation in the liquid, single-phase region.

Table 1 gives some calculated typical shock temperatures, release temperature, melted mass fraction for Fe/AlO, and Fe/UFe. In Figure 5 and Figure 6, we plot the relations between shock temperature, release temperature, melting temperature, and relaxation temperature of Fe/AlO, and corresponding shock temperature and melting temperature of window materials. The relaxation temperature, $T_r$, at the interface is the directly observable temperature in the experiments. As is discussed below, $T_r$ is closely related to the thermal conduction process of the materials at the interface. All the parameters used in the model calculations are listed in Table 2.

B. Thermal Relaxation at the Material Interface

The large temperature difference existing between the release temperature, $T_o$, in the metal sample and the shock temperature, $T_s$, in the window material, will cause thermal relaxation across the interface. Libert and Guest ($^3$) first examined this process by solving the heat conduction equation for an initial step-function
Table 1: Relative temperature-appropriation model calculations

<table>
<thead>
<tr>
<th>Sample</th>
<th>Shock pressure (GPa)</th>
<th>Release pressure (GPa)</th>
<th>Shock temp. (K)</th>
<th>Release temp. (K)</th>
<th>Mel temp. fraction</th>
<th>Relative volume change (m^3/m^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SiO2</td>
<td>0.84</td>
<td>0.82</td>
<td>0.84</td>
<td>0.82</td>
<td>0.84</td>
<td>0.84</td>
</tr>
<tr>
<td>CaO</td>
<td>0.65</td>
<td>0.64</td>
<td>0.65</td>
<td>0.64</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>FeO</td>
<td>0.50</td>
<td>0.49</td>
<td>0.50</td>
<td>0.49</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>MnO</td>
<td>0.35</td>
<td>0.34</td>
<td>0.35</td>
<td>0.34</td>
<td>0.35</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Temperature distribution. Their results predicted an interface temperature, \( T_i \), in a form of

\[ T_i = \frac{T_e - T_r}{1 + \alpha} \]

(10)

where \( \alpha \) is a parameter connected with the density \( \rho \), specific heat \( c \) and thermal conductivity \( K \), of metal and window material, respectively.

\[ \alpha = \frac{K_K \rho^2 c}{K_{w} \rho^2 c_{w}} \]

(11)

High-pressure thermal conductivities for most metals have not been obtained. The Wiedemann-Franz relation between thermal and electric conductivity,

\[ K = L_0 T \alpha \]

(12)

is used here to evaluate the thermal conductivity of metal, from its corresponding high-pressure electric conductivity \( \alpha \). The Lorentz number, \( L_0 \), for most metals, has a relatively constant value of \( 2.45 \times 10^{-8} \) watt-ohm/K^2. Kedem22 and Mackinney22 reported the electric conductivity of shock compressed iron in a plot of the form \( (\sigma / \sigma_0)T = \alpha P \). We approximate their data in a form

\[ \frac{\sigma}{\sigma_0} = a + b \frac{T}{P} \]

(13)

where \( a = 0.2742 \times 10^{6} \) ohm-m and \( b = 1.1698 \times 10^{6} \) ohm-m-GPa. Since, at
Figure 5. Calculated shock temperature, incident temperature, melting temperature for Fe/AlO, and the conduction temperature of material interface using Model 1 (see Figure option 4).
Figure 6  Typical shock-temperature, release-temperature melting temperatures for Fe-50% and 10% coordination temperatures. Led markers illustrate melting behavior. (See figure caption for details.)
existing pressures, the locus of release temperature are higher than the locus of shock temperature (Figures 5 and 6), the electric conductivity \( \sigma \) is related to conductivity \( \sigma_0 \) at release pressure \( P_r = P_0 \) via a model given by Mott and Jones\(^{22} \):

\[
\frac{\sigma_0}{\sigma} = \frac{(\theta_0/\theta)^2}{1 + \theta_0/\theta}
\]

where \( \theta \) is Debye temperature. We assume that \( \theta \) is only a function of volume and takes a form

\[
\theta(v) = \theta_0 v^{-3/2} \exp \left(-\frac{\gamma}{\theta_0 v^{1/2}}\right).
\]

Equations (12) through (15) give the thermal conductivity for iron at release pressure \( P_r \):

\[
K_r = L_r \theta_0^{1/2} (a + b P_r) v^{-1/2} e^{-\gamma/\theta_0 v^{1/2}}.
\]

Thermal conductivity for sapphire is estimated using the method described by Bass et al.\(^{23} \). The typical values of \( a \) of interest in the pressure-temperature region of Figure 5 are between 4 and 5. This method, however, may be inadequate in evaluating the thermal conductivity of LiF because the shock temperatures achieved by LiF are more than a factor of 2 greater than the zero-pressure melting temperature, 133 K. We infer that the thermal expansions obtained at thermal pressure should not be used to estimate the high pressure thermal conductivity. Since the zero-pressure thermal conductivity of LiF is about a factor of 100 less

<table>
<thead>
<tr>
<th>Material</th>
<th>( \alpha ) (% vol)</th>
<th>( C )</th>
<th>( S )</th>
<th>( \gamma )</th>
<th>( T_0 )</th>
<th>( T_r )</th>
<th>( \Delta T )</th>
<th>( \theta_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ni</td>
<td>7.55</td>
<td>3.95</td>
<td>1.36</td>
<td>1.37</td>
<td>1500</td>
<td>2800</td>
<td>0.134</td>
<td>315(^o)</td>
</tr>
<tr>
<td>Al(_2)O(_3)</td>
<td>7.77</td>
<td>8.67</td>
<td>0.95</td>
<td>1.30</td>
<td>2100</td>
<td>2690</td>
<td>0.267</td>
<td>320(^o)</td>
</tr>
<tr>
<td>LiF</td>
<td>2.64</td>
<td>5.48</td>
<td>1.52</td>
<td>1.63</td>
<td>1130</td>
<td>2350</td>
<td>0.426</td>
<td>640(^o)</td>
</tr>
</tbody>
</table>

\(^{1}\)From refs. 14 and 15.\(^{2}\)From ref. 16.\(^{3}\)From ref. 17.\(^{4}\)From ref. 18.\(^{5}\)From ref. 19.\(^{6}\)From ref. 20.\(^{7}\)From ref. 21.\(^{8}\)From ref. 22.\(^{9}\)From ref. 23.
than that of iron. We have assumed that LiF has a thermal conductivity an order of magnitude lower than iron, at high pressure, on the basis of the high pressure on thermal conductivity of halides [14,15] and metals [16]. Typical values for are approximately 1 in the pressure-temperature region of interest for Fe/LiF [Fig. 6].

We note that, in general, Eq. (10) can only be strictly applied in the situation where melting of the metal upon release of pressure does not occur, or melting is not induced, in the window material upon thermal conduction from the metal into the window. We note that the interface temperature will not drop down below the melting temperature of metal until the latent heat of fusion is conducted away. The direct result of this effect is that the apparent release-dominated region observed in the present shock temperature measurements on metals will be wider than that expected from the previous treatment (see section A, Figures 2, 5, and 6). Moreover, even if, upon pressure release, the metal enters the liquid phase region, or in some cases, the initial shock wave has encroached the liquid phase region (e.g., process labelled as 5 in Figure 2), the interface temperature will still remain in the melting region due to the latent heat of fusion. We first consider the simple situation where only solidification in metal sample occurs.

Model I. Conduction induced solidification in metals; no melting in window

This situation may occur when the initial release temperature, $T_0$, behind the reflected wave from metal-window interface (Figures 5 and 6) is equal to, or not much higher than, the melting temperature $T_m$ at the release pressure, $P_r$. Adopting the mathematics of Carslaw and Jaeger [16] and solving the heat conduction equations with the boundary conditions illustrated in Figure 7, the interface temperature $T_I$ at $x = 0$ is

$$ T_I = T_0 + \frac{T_m - T_0}{1 - \alpha_0 \alpha_f} $$

(17)

Here

$$ \alpha_0 = \frac{[K_f C_f + C]}{[K_0 \rho_0 C]} $$

with $K$, $\rho$, and $C$ denoting thermal conductivity, density and specific heat, respectively. The parameter $\alpha_0$ specifies the rate of solidification or the position of the solidification surface $X_d$ (Figure 7),

$$ X_d = \frac{2 \mu (\alpha_0 t)^{1/2}}{K_0 \rho_0 C} $$

(18)

as defined by the coordinate of Figure 7, $T_m < 0$. The thermal diffusivity is given as $\kappa = K/\rho C$. Thus $\mu$ must be determined from the solution of
\[
\frac{\frac{\alpha_s(T_0 - T_1)}{1 + \text{erf}\left(\frac{x}{\sqrt{2}}\right)}}{\frac{1 + \text{erf}\left(\frac{x}{\sqrt{2}}\right)}{0.5}} \exp\left(-\frac{\mu}{\mu_0}\right) \cdot \frac{1}{1 - \alpha_0 \text{erf}\left(\frac{x}{\sqrt{2}}\right)} \cdot \exp\left(-\frac{\mu}{\mu_0}\right)
\]

Here, subscripts 1, 3 and 4 denote solid, solid, solid and liquid metal material respectively. \(\Delta H_0\) is the latent heat of fusion for metal and \(x_1\), the realted mass fraction immediately after the releasing from the material interface. For the convenience of mathematical analysis, we rewrite Eq. (19) as:

\[
\frac{\frac{\alpha_s(T_0 - T_1)}{1 + \text{erf}\left(\frac{x}{\sqrt{2}}\right)}}{\frac{1 + \text{erf}\left(\frac{x}{\sqrt{2}}\right)}{0.5}} \cdot \frac{\exp\left(-\frac{\mu}{\mu_0}\right)}{\exp\left(-\frac{\mu}{\mu_0}\right)} \cdot \exp\left(-\frac{\mu}{\mu_0}\right)
\]

having
\( \omega \equiv \frac{1}{T_S - T_A} \) \( b_\omega \equiv \frac{\Delta \omega}{\Delta \mu} \frac{T_S - T_A}{T_S - T_U} \)

Examination of Eq. (20) indicates:

i) when \( T_S = T_M \), i.e., when the release process ends in the mixed phase region, Eq. (20) has a solution \( \omega = 0 \), such that:

\[
\mu = \frac{1}{b_\omega} (1 - \mu_0) \text{ or } \mu_0 = \frac{1}{b_\omega} \mu
\]

(21)

ii) when \( T_S > T_M \), Eq. (20) has a solution \( \mu_0 < \mu < 0 \) if and only if:

\[
\omega < 1
\]

(22)

Consider a simple case for Fe/Al4O3, when \( T_S = T_M \). If \( \omega_0 \) is very small \( \omega_0 \to 0 \), then \( b_\omega \to 0 \) and \( \omega \to -1 \). The interface temperature \( T_H \) will, according to Eq. (17), reduce to:

\[
T_H = T_M + \frac{T_S - T_U}{1 + s_0} \quad \omega_0 \to 0, \quad T_H = T_M
\]

(23)

which is equivalent to Eq. (10). In contrast, when \( \omega_0 \to 1 \), i.e., when the initial release-melting is just completed, as is the case when \( T_S = 100^\circ \text{C} \) in Figure 3, we estimate that \( b_\omega \to 4, \mu_0 \to -0.25 \).

\[
T_H = \frac{T_S - T_U}{1 + s_0} \left( 1 - \frac{T_S}{T_M} \right) - 0.95 \quad (\omega_0 \to 1, \quad T_H = T_M)
\]

(24)

We are, in fact, observing a melting temperature in this case. This \( \mu_0 \) also predicts a thickness of solidified iron of \( \sim 2.5 \text{ mm} \), from Eq. (18), for an experimental time scale of \( t \sim 0.5 \text{ s} \).

The situation changes when \( T_S > T_M \). Since in this case \( \omega_0 \to 1 \), we always have \( \mu_0 < \mu < 0 \), the observed temperature will be higher than predicted by Eq. (24):

\[
0.95 < \frac{t}{T_H} < 1 \quad (\omega_0 \to 1, \quad T_S > T_M)
\]

(25)

In the actual data reduction, we infer the melting temperature \( T_M \) from the observed interface temperature \( T_H \):

\[
T_M = T_H + \left( T_S - T_H \right) \left( 1 - \mu_1 \text{ or } \mu_0 \right)
\]

(26)
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if release temperature is equal to the melting temperature, we find \( \mu \) from

\[
\mu = \left(1 - \alpha \right) \frac{T_r - T_m}{T_r - T_m} + \frac{\alpha \frac{T_r - T_m}{T_r - T_m}}{\Delta T_m} \frac{1}{T_m} \quad (T_r = T_m).
\]

Equation (27) can be solved iteratively by the Newton-Raphson method\(^\text{31}\) using \( \mu_0 \) as a starting value. On the other hand if \( T_r > T_m \), we can simply take

\[
T \approx T_r, \quad \left( T_r > T_m, \mu > 0 \right)
\]

and obtain \( T \) from Eq. (23).

\[
T_r = T_r - \frac{T_r - T_m}{\alpha_0},
\]

Equation (28) implies \( \mu \to 0 \) while Eq. (29) gives the upper limit for which this model holds. In our model calculation for iron, release shock shock pressure \( P_r \approx 205 \text{ GPa} \) for Fe/Al\(_2\)O\(_3\) and \( P_r \approx 195 \text{ GPa} \) for Fe/LiF configurations; no solidification, due to thermal relaxation at the interface, was assumed if the initial shock pressure is higher than \( \approx 200 \text{ GPa} \) for Fe/Al\(_2\)O\(_3\), and higher than \( \approx 290 \text{ GPa} \) for Fe/LiF, respectively, according to Eq. (28).

Model II. Conduction melting is induced in window; no phase transition due to heat conduction in metal

Figure 8 shows the basic configuration of this model. The interface temperature in this case will be again time-dependent,

\[
T = T_r - T_m - \frac{T_r - T_m}{1 + \alpha_0 \alpha} \frac{T_r - T_m}{\Delta T_m}.
\]

Here \( \alpha \) is a parameter related to the melting rate in window material or the position of melting surface \( X_m \).

\[
X_m = 2 \lambda X_r^2.
\]

By definition (Figure 8), \( X_r > 0 \); subscript "2" denotes "window" material. \( \lambda \) must be determined from

\[
\frac{\alpha \frac{T_r - T_m}{T_r - T_m} e^{-2 \frac{T_r - T_m}{T_r - T_m}}}{1 + \alpha_0 \alpha} \frac{T_r - T_m}{T_r - T_m} e^{\frac{T_r - T_m}{T_r - T_m}} \frac{1}{\lambda} \Delta T_m \frac{\Delta T_m}{T_m} \lambda = \frac{\alpha_0 \alpha \frac{T_r - T_m}{T_r - T_m}}{1 + \alpha_0 \alpha} e^{\frac{T_r - T_m}{T_r - T_m}}.
\]
Figure 8: Model II: Shock-induced melting in iron and ablation-induced melting of window equation—calculated temperature profile at t = 31 μs after shock arrival at Fe-AlO₃ interface (λ = 3). Initial fragmentary pressure in Fe is 327 GPa and transmitted shock pressure in Al₂O₃ is 247 GPa. Heavy line shows the initial temperature distribution (t = 0) on both sides of the material interface. The vertical temperature, T₀, deviates above the melting temperature of iron due to heat neutralization across the optical path. A melting surface (Tₘ) separates the solid region (1) and the melted region (2) window material. It moves to right, as shown by the arrow. The calculated thickness of melt window material is about 0.7 μm at t = 31 μs.

\[
\frac{df}{dt} = \frac{\rho v_0}{1 - \alpha_f(\rho_0/\rho_f)^{1/3}} \left[ \frac{1}{k_f} \right] \left( 1 + \alpha_f \text{ref 2} \right) b_f \cdot \lambda,
\]

\[
\sigma_f = \frac{T_f - T_0}{\alpha_f(T_f - T_0)}
\]

\[
b_f = \frac{\sigma_f \Delta S_f}{C_f} \frac{T_f}{T_f - T_0}
\]

Here, \( T_0 \) is the melting temperature, \( \Delta H_f \) the latent heat of fusion and \( \Delta S_f \), the entropy increase upon fusion at atmospheric pressure, for the window material. We assume \( \Delta H_f = T_f \Delta S_f \).
Equation (33) predicts that if \( a_{06} < 1 \), we have a solution for \( \lambda \) which satisfies the physical condition \( \lambda > 0 \) such that

\[
0 < \lambda < \lambda_0
\]

(34)

where \( \lambda_0 \) is the root of Eq. (35)

\[
\lambda_0 \exp(1 + a_{12}) \exp(1/2) \lambda = 0
\]

(35)

Consider the Fe/Al\(_2\)O\(_3\) configuration at \( P_0 = 190 \text{ GPa}\) and assuming \( a_{12} = a_{15} = -4 \) (Figure 5), then \( \lambda_0 = 0.45 \) can be calculated from Eq. (35). Since \( \lambda < \lambda_0 \), we estimate the melted layer thickness for Al\(_2\)O\(_3\) will be less than 1.5 \( \mu \text{m} \) at \( t = 0.1 \mu \text{s} \), from Eq. (31), and \( \lambda \lambda_0 \) at \( t = 0.5 \mu \text{s} \), it is expected that such a thin layer of melted window material will have little contribution to the total observed spallation radiation.

In the actual data reduction, \( T_0 \) can be deduced in this case from Eq. (30)

\[
T_0 = \frac{T_1 - T_0}{a_{12} \exp(1/2) \mu}
\]

(36)

and \( \lambda \) is evaluated from

\[
\frac{a_{12} (T_1 - T_0)}{\exp(1/2) \mu} = \frac{1}{\exp(1/2) \mu} \left[ \frac{T_0}{a_{12} \exp(1/2) \mu} - \frac{T_0}{a_{12} \exp(1/2) \mu} \right]
\]

(37)

Equation (37) can be simplified if we assume that \( a_{12} = 1 \) and \( \mu = \kappa \).

Model III. Conduction-induced solidification in metal and melting in window occur simultaneously

Three boundaries separating the four phase regions are shown in Figure 9: the melting surface, \( X_M \), in window material (\( X > 0 \)), the solidification surface, \( X_M \), in metal region (\( X < 0 \)) and the material interface at \( X = 0 \). The temperature distributions in the four regions are obtained for the boundary conditions shown in Figure 9:

\[
T = T_1 + \frac{T_0 - T_1 \left[ 1 - \exp\left( \frac{X}{2(k_1 \mu)^{1/2}} \right) \right]}{1 - \exp\left( \frac{k_2}{k_1} \right)^{1/2} \lambda} \quad (z = X_M)
\]

(38)
Figure 9 Model 10: Solidification in contact with material moving in axial direction simultaneously. The shaded area shows the initial temperature distribution (t=0) on both sides of the material interface. Calculated isotherm profiles at t=1.0 s after solidification to the limit (X=0). The initial ingot temperature is \( T_0 = 2373 \, ^\circ\text{C} \) and the phase transition temperature for Al-Ga is \( 211 \, ^\circ\text{C} \). Although the release temperature \( T_0 \) is lower than the melting temperature \( T_m \), the thermal contraction induced shrinkage will occur at the material interface.

\[
\begin{align*}
T_1 = T_0 + \frac{\alpha_0 \left( T_0 - T_w \right)}{\alpha_3 \, \text{erf} \, \lambda - \text{erf} \, \mu} \left[ \text{erf} \, \lambda - \text{erf} \, \frac{x}{2 \left( \lambda \, \mu \right)^{1/2}} \right] \quad (0 < x < X_w) , \quad (39) \\
T_0 = T_0 + \frac{\left( T_0 - T_w \right)}{\alpha_0 \, \text{erf} \, \lambda} \left[ \text{erf} \, \frac{x}{2 \left( \lambda \, \mu \right)^{1/2}} \right] \quad (X_w < x < 0) , \quad (40) \\
T_0 = T_0 - \frac{\left( T_0 - T_w \right)}{1 + \text{erf} \, \frac{x}{\left( \lambda \, \mu \right)^{1/2}}} \left[ 1 + \text{erf} \, \frac{x}{2 \left( \lambda \, \mu \right)^{1/2}} \right] \quad (x < X_w) . \quad (41)
\end{align*}
\]

The material interface temperature \( T_0 \) at \( X=0 \), by Eq. (39) or (40),

\[
T_0 = T_0 + \frac{\alpha_0 \, \text{erf} \, \lambda}{\alpha_3 \, \text{erf} \, \lambda - \text{erf} \, \mu} \left( T_0 - T_w \right) \quad (42)
\]
or
\[ T_a = T_b + \frac{\text{erf} \mu}{\alpha_2 \text{erf} \lambda - \text{erf} \mu} (T_b - T_w) \]  \hspace{1cm} (43)

is time-independent. \( \lambda \) and \( \mu \) are parameters related to the rate of melting in window material and solidification in metal,

\[ X_a = 2 \cdot \lambda \cdot (\alpha_2)^{1/2} \]  \hspace{1cm} (44)
\[ X_b = 2 \cdot \mu \cdot (\alpha_2)^{1/2} \]  \hspace{1cm} (45)

By definition, \( X_a \geq 0 \) and \( X_b \leq 0 \). Unlike the simple case in Models I and II, \( \lambda \) and \( \mu \) are related to each other and must be determined by solving simultaneously

\[ \frac{a_0(T_b - T_a)}{a_2 \text{erf} \lambda - \text{erf} \mu} \frac{a_2(T_b - T_a)}{a_2 \text{erf} \lambda - \text{erf} \mu} e^{\frac{\lambda}{\alpha_2 \text{erf} \lambda - \text{erf} \mu}} = \frac{\Delta H_a}{C_1} \frac{\Delta T}{T_a} \]  \hspace{1cm} (46)
\[ \frac{T_b - T_a}{a_2 \text{erf} \lambda - \text{erf} \mu} e^{\frac{\mu}{\alpha_2 \text{erf} \lambda - \text{erf} \mu}} = \frac{\Delta H_b}{C_1} \frac{\Delta T}{T_a} \]  \hspace{1cm} (47)

To obtain \( \lambda \) and \( \mu \), we rewrite Eqs. (46) and (47) as:

\[ \frac{A_w}{1 + \text{erf}(\frac{k_1}{\alpha_2 \text{erf} \lambda - \text{erf} \mu})} = \frac{1}{e^{\frac{\lambda}{\alpha_2 \text{erf} \lambda - \text{erf} \mu}}} - B_w \lambda \]  \hspace{1cm} (48)
\[ \frac{A_w}{1 + \text{erf}(\frac{k_1}{\alpha_2 \text{erf} \lambda - \text{erf} \mu})} = \frac{1}{e^{\frac{\mu}{\alpha_2 \text{erf} \lambda - \text{erf} \mu}}} - B_w \mu \]  \hspace{1cm} (49)

where

\[ A_w = \frac{T_b - T_a}{\alpha_0 (T_m - T_w)} \]  \hspace{1cm} \( B_w = \frac{\Delta T}{\alpha_2 C_1} \) \frac{T_w}{T_w - T_a} \]

and

\[ A_w = \frac{T_b - T_a}{T_m - T_w} \]  \hspace{1cm} \( B_w = \frac{\Delta T}{\alpha_2 C_1} \frac{T_w}{T_w - T_a} \)
We point out that solutions of Eqs. (48) and (49) for \( \lambda \) and \( \mu \) will satisfy the conditions

\[
0 \leq \lambda \leq \lambda_i, \quad (50)
\]

\[
\mu_i \leq \mu \leq 0, \quad (51)
\]

where \( \lambda_i \) and \( \mu_i \) should be found from

\[
\lambda_i \text{erf} \lambda = \frac{1}{f_i} \sum_{\Delta S_{\text{int}}} \frac{T_w - T_s}{T_w}, \quad (52)
\]

\[
\mu_i \text{erf} \mu = \frac{1}{f_i} \sum_{\Delta S_{\text{int}}} \frac{T_n - T_i}{T_n}, \quad (53)
\]

In the model calculation, we solve Eqs. (48) and (49) iteratively by the Newton-Raphson method and using \( \lambda_i \) and \( \mu_i \) as the starting value for \( \lambda \) and \( \mu \).

Table 3 lists some calculated values for \( \lambda \), \( \mu \), and \( T_r \) for typical situations for which this model is applicable.

<table>
<thead>
<tr>
<th>Stock pressure (GPa)</th>
<th>Stock temp. ( T_r ) (K)</th>
<th>Melt pressure ( P_r ) (K)</th>
<th>Melt temp. ( T_i ) (K)</th>
<th>Melt front ( T_f ) (K)</th>
<th>Solidification ( \Delta T ) (K)</th>
<th>Window parameter ( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>5493</td>
<td>109</td>
<td>453</td>
<td>3877</td>
<td>0.1073</td>
<td>-1.229</td>
</tr>
<tr>
<td>257</td>
<td>6244</td>
<td>186</td>
<td>579</td>
<td>537</td>
<td>0.1999</td>
<td>-0.8719</td>
</tr>
<tr>
<td>263</td>
<td>6474</td>
<td>254</td>
<td>5920</td>
<td>5780</td>
<td>0.2677</td>
<td>-0.2414</td>
</tr>
<tr>
<td>270</td>
<td>6520</td>
<td>269</td>
<td>5989</td>
<td>5858</td>
<td>0.2672</td>
<td>-0.2166</td>
</tr>
</tbody>
</table>

In the case of \( P_r = 220 \) GPa for Fe/Al₂O₃, as in Figure 5, we estimate that \( \lambda = 0.55 \) and \( \mu = 0.52 \) from Eqs. (51) and (53). \( \lambda = 0.28 \) and \( \mu = -0.11 \) from Eqs. (48) and (49). These yield

\[
\frac{T_i}{T_i} = 1 + \frac{\text{erf} \lambda}{\Delta S_{\text{int}} \text{erf} \lambda - \text{erf} \mu} - \frac{T_i}{T_i} = -0.98.
\]

As is pointed out in Model II, this value of \( \lambda \) predicts a very thin layer of melted window material which we assume has little effect on the total observed radiation. On the other hand, if we use \( \lambda \) and \( \mu \) to approximately estimate \( T_r/T_n \).
\[ \frac{T_r}{T_0} = 1 + \frac{\varepsilon}{\alpha_2} \left( \frac{T_r - T_0}{\frac{T_r - T_0}{\alpha_2}} \right)^{0.95} \]

we obtain a result lower than the value of 0.98 for \( T_r/T_0 \) given above.

For the experimental data, we deduce \( T_r \) from observed interface temperature \( T_i \) and the calculated high pressure melting temperature, \( T_m \), of the window material,

\[ T_r = T_i + \left( T_i - T_m \right) \frac{\varepsilon}{\alpha_2} \frac{\alpha_2}{\alpha_2} \]  

and determine \( \lambda \) from

\[ \frac{T_r - T_0}{T_r - T_0} = \frac{\alpha_2 (T_r - T_0)}{\alpha_2 (T_r - T_0)} \left( \frac{T_r}{T_0} \right)^{0.95} \left( \frac{T_r}{T_0} \right)^{0.95} \]

If \( T_r - T_0 \) can be calculated by solving

\[ \frac{T_r - T_0}{\alpha_2} \frac{\alpha_2}{\alpha_2} \left( \frac{T_r - T_0}{\alpha_2} \right)^{0.95} \left( \frac{T_r - T_0}{\alpha_2} \right)^{0.95} \]

once \( \lambda \) has been determined from Eq. (55).

The observed interface temperature \( T_i \) in this model is not directly related to the release temperature \( T_r \). This implies that, in general, we cannot infer \( T_r \) from observed \( T_i \) in the case where \( T_i > T_r \). If we assume \( T_i \) is equal to \( T_0 \), however, the release temperature \( T_r \) as is then given by Eqs. (43) and (47),

\[ T_r = T_0 + \frac{T_r - T_0}{\alpha_2} \left( \frac{T_r - T_0}{\alpha_2} \right)^{0.95} \]  

Equation (57) then defines an upper limit for \( T_r \). For high release temperature \( T_r \), or equivalently for high initial Hugoniot pressure \( P_i \), solidification in metal will not occur. We point out that this Hugoniot pressure limit for Fe/\( \alpha_2 \)-O, is about 305 GPa, if parameters listed in Table 2 are used. Finally, we point out that Eq. (57) is equivalent to Eq. (30) for model II as it implies \( \mu = 0 \).

CONCLUSIONS

Shock-temperature measurements of metals in which interfaces of metals with lower-impedance transparent windows are observed via high speed radiometry yield interface temperatures which are related to the metal fusion curve, rather than Hugoniot temperature in the regime of shock-induced melting. At lower shock pressures (e.g., 140 GPa) in any of the regime where melting can occur either along the Hugoniot or upon pressure release against a transparent Al/\( \alpha_2 \)-O or LIF
window, it is completely feasible to directly relate the observed temperature to the release temperature, and hence, the shock temperature of metals. Moreover, at high shock pressures (310–320 GPa in iron) where shock melting has been completed and the release temperature is so high that no solidification due to thermal conduction at the interface will occur, the interface temperature is again completely insensitive to the shock temperature.

Thus for ideal interfaces of a higher impedance but lower melting point samples (foil or film) with a transparent window material, there are six possible outcomes of the shock interaction at the window. The measured interface temperatures can be used in the following ways:

1) Upon shocking and partial release both the metal and the window remain in the solid state. A Hugoniot temperature of the metal can be obtained.

2) Upon initial shocking, the metal remains solid, partial melt occurs upon partial release but the metal rapidly refreezes upon conduction of heat into the window. A point on the melting curve of the metal can be obtained.

3) Upon initial shocking, the metal is partially melted and remains partially melted upon partial release. Upon conduction of heat into the window, freezing occurs. Again a point on the melting curve of the metal can be obtained.

4) Upon initial shocking, partial melting of the metal occurs and complete melting occurs upon partial release. Again refreezing occurs upon transfer of heat into the transparent window. A point on the melting curve of the metal can be obtained.

5) Upon initial shocking complete melting occurs and the complete melt remains upon partial release. Transfer of heat into the window occurs and a metal film freezes. Again a point on the melting curve of the metal can be obtained.

6) Upon shocking the metal, complete melting occurs and the metal remains molten upon partial release. Transfer of heat into the window does not result in refreezing of the metal. In this case the interface temperature can be used to obtain a Hugoniot temperature of the molten metal regime.

**LIST OF SYMBOLS**

- \(P_a\) Hugoniot pressure in metal material (iron)
- \(P_r\) Release pressure in metal material which equals to the shock pressures transmitted into window material
- \(P_m\) Melting pressure of metal material
- \(P_i\) Isentropic pressure
- \(T_h\) Hugoniot temperature in metal material
- \(T_m\) Hugoniot temperature in window material
- \(T_i\) Melting temperature in metal material
- \(T_w\) Melting temperature in window material
- \(T_f\) Temperature at the material interface
SHOCK TEMPERATURE MEASUREMENTS FOR METALS

\( T_R \) Release temperature of metal material

\( T_e \) Isentropic temperature

\( V_M \) Volume of metal material under shock pressure \( P_S \)

\( V_M \) Volume of metal material under release pressure \( P_R \)

\( V_M \) Melting volume of metal

\( K \) Thermal conductivity

\( \rho \) Density

\( \gamma \) Grüneisen parameter

\( \rho_0 \) Initial melt mass fraction upon shock

\( \rho_0 \) Total melt mass fraction upon release

\( \rho_0 \) Parameter related to the solidification rate, due to thermal conduction, in metal material

\( \lambda \) Parameter related to the melting rate, due to thermal conduction, in window material

\( \alpha_0 \) Parameter defined through the thermal conductivity, density and specific heat of both metal sample and window material

Subscripts

1 Solid state of window material

2 Liquid state of window material

3 Solid state of metal sample

4 Melting state of metal sample

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References


