THE DYNAMIC TENSILE STRENGTH OF ICE AND ICE-SILICATE MIXTURES

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Abstract. We determined the dynamic tensile strength of ice and ice-silicate mixtures at strain rates of \(10^{-2}\) s\(^{-1}\) to \(10^{3}\) s\(^{-1}\). At these strain rates, ice has a tensile strength of \(\sim 0.1\) MPa, and ice-silicate mixtures with 3 and 20 wt. % sand content have strengths of \(\sim 0.25\) and \(2.3\) MPa, respectively. These values lie significantly above tensile strengths of \(\sim 0.5\) MPa for ice and \(\sim 1\) MPa for frozen soil, measured at strain rates of \(10^{-3}\) to \(10^{3}\) s\(^{-1}\), but are significantly below values found for a variety of rocks at comparable strain rates. Results of the present experiments are used to derive parameters for continuum fracturing models in icy media, which are used to determine relations between tensile strength and strain rate, and to predict stress and damage histories as well as size frequency distributions for ice and ice-silicate fragments. It is found that tensile strength \(\sigma_{t}\) is related to strain rate by \(\sigma_{t} \propto \epsilon^{1.25}\) to \(1.75\), similar to results obtained for other geological materials. The increase of small fragments relative to larger fragments with increasing strain rate, as predicted by the continuum model, is a result which parallels findings in laboratory impact experiments.

Introduction

The strength of crystalline solids is dependent on the mode and rate of loading. Compressive and tensile strengths of many geological materials vary by as much as 1 order of magnitude when subjected to other stress (at strain rates of \(10^{-5}\) to \(10^{-1}\) s\(^{-1}\)) or dynamic (at strain rates of \(10^{2}\) to \(10^{3}\) s\(^{-1}\)) tests (Grace and Hennell, 1974). Knowledge of the dynamic tensile strengths of rocks is essential for an understanding of fracturing and fragmentation processes. Arctic and Antarctic industrial processes involving rock breakage, such as quarrying and drilling (Watkins, 1978), impinge on the problem of explosive crater formation (O'Keefe and Ahrens, 1976), and processes related to the excavation of planetary bodies (Nakamura and Mitsui, 1977).

Although water ice is a common geological material, very little has been done to determine its mechanical properties over a wide range of stress and strain rates. Voyager discoveries of impact cratered surfaces on the satellites of Jupiter and Saturn (e.g., Wash et al., 1979, 1981) have focused attention on the study of physical properties of ice and ice-silicate mixtures. The processes related to the origin and evolution of the icy moons of Jupiter and Saturn require the knowledge of fracturing and fragmentation properties of icy substances. Our goal in previous studies has been to establish some data and scaling laws related to impact crater formation and fragmentation of ice and ice-silicate mixtures (Lange and Ahrens, 1981, 1982a, b). A central observation, pertinent to most of these experiments, was the occurrence of tensile failure as the principal mechanism in the fragmentation of impacted icy targets.

The goal of the present study is the determination of the dynamic tensile strength of icy media at strain rates of \(10^{-2}\) to \(10^{3}\) s\(^{-1}\). These data will help to understand better the macroscopic phenomena observed in our impact experiments and will be used in the derivation of scaling laws for fragmentation and crater formation in icy substrates.

The dynamic tensile strength of ice and ice-silicate samples with 3 and 20 wt. % sand content at temperatures between -230 and 250 K was obtained by carrying out experiments in which plexiglass plates impacted target pellets consisting of these materials. Upon wave reflection, tensile stress pulses of \(0.75 \pm 0.25\) MPa per meter of plate were measured, using a laser interferometer. Impact velocities were 30 cm s\(^{-1}\) with a 30-g projectile.

Experimental Techniques

Sample Preparation and Analysis

All samples were prepared by compressing finely crushed ice (mean grain size \(\sim 0.1\) to 0.5 mm) or ice-silicate mixture containing specified amounts of silica sand (mean grain sizes of \(0.1\) to 0.9 mm) into sample pellets of 20 mm diameter and 6 mm thickness. The grain size of the sand was reduced using a chilled food blender. The ice was placed under a homogenizer in a 1:1 mixture of ice and silica sand was then filled in a mold and compacted by use of a hand-operated hydraulic press. During compaction, the sample was evacuated, thus avoiding extensive trapping of air bubbles. This procedure gave mostly transparent or semitransparent samples with no observable voids. Extensive recrystallization, along grain boundaries of single ice grains took place during sample compaction. The sample pellets were pressed into stainless-steel target plates which were cooled via the cooling coil circuit surrounding the sample chamber (Figure 1). Temperatures were monitored with a thermocouple attached to the target plate, close to the sample. Since the target plate was thinner than the sample (3 mm versus 6 mm, respectively) the plexiglass flyer

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place could impact the sample and be stopped by the target plate.

The complete sample assembly (Figure 1) consists of the target plate attached to a steel catcher tank. An opening in the catcher tank is aligned with the target plate such that the impacted sample is driven into a cloth sleeve covering the opening in the tank. The sleeve closes itself off and is in turn driven into a mass of ice powder in the catcher tank. This avoids further damage of the sample and provides cooling of the impacted sample until recovery. The target plate and catcher tank are cooled via chilled methyl alcohol which is pumped through copper piping from a solid CO₂ and acetone bath (Figure 1). This provides sample temperatures of approximately -130° to 250° K.

The catcher tank and the attached target plate are placed in a larger aluminum target container which is aligned with the barrel of a 40 mm compressed-air gun. The target container can be closed with a plexiglass window and is evacuated until immediately prior to firing the gun. This prevents the build-up of frost layers on the sample and target plate and improves the temperature control of the sample. The window also serves as a reference for aligning the target with respect to the gun barrel. Since the target plate and sample are always at a fixed position with respect to the target container (see Figure 1), alignment of the container assures plane impact of the projectile onto the sample, a condition important in these experiments.

Experimental Procedure

Many of the experimental procedures we used were similar to those of Cohn and Ahrens (1961). As in their experiments, a tungsten projectile carrying a plexiglass flyer plate at its front surface is accelerated by the expansion of compressed air. Projectile velocities varied between 4.7 and 23.3 m/s depending on the pressure of the compressed air. Projectile velocities are measured in two ways. Interception of three laser beams, positioned at known distances along the path of the projectile, allow determination of two velocities of the accelerating projectile. The
interception of the final laser beam and a signal of a stress-gage, positioned at the target plane and actuated by impact of the projectile, allow measurement of the final impact velocity.

Uniaxial tensile loading of the sample is achieved by the interaction of two relief waves traveling into the sample from the surfaces of sample and flyer [Cohn and Abrams, 1981]. They originate as reflections of initial compression waves generated by impact of the flyer onto the sample. Knowledge of the wave velocities for compression and relief waves in both materials determines the thickness of the flyer plate to be used in order to provide a state of maximum tension approximately in the mid-plane of the sample. For tensile stresses below the strength of a sample no observable damage or only incident spallation will result. For stresses exceeding the tensile strength, either the sample will split in two or a few larger fragments, or complete fragmentation will occur. Analysis of the recovered sample allows an assessment of the damage done by tensile stresses and knowledge of the projectile velocity yields the stress experienced by the sample (see below).

Determination of Experimental Parameters

A number of assumptions are made in the present study. We assume that shock and relief wave velocities \( V_R \) and \( V_L \) in sample and flyer plate can be approximated by the longitudinal elastic wave velocities of these materials. We also assumed that the tensile stress level is equal to that of the initial compression waves [see Cohn and Abrams, 1981].

We calculate the thickness of the plastically deformed region necessary to generate maximum tensile stress in the mid-plane of the samples. From this knowledge, computational wave velocities in flyer and sample materials were measured. The wave velocity of plasmas (\( V \), 2.8 km/s, density = 1.8 g/cm\(^3\)) was obtained from the study of Barker and Hollenbach [1970] (see Table 1).

For the tensile waves to meet in the mid-plane of the sample the reflected wave in the flyer plate must arrive at the sample-flyer interface when the primary compressive wave in the sample reaches the free surface of the sample. This requires the condition

\[
\frac{d_2}{d_1} = \frac{V_L}{V_R}
\]

where \( d_2 \) and \( d_1 \) are the thicknesses of the flyer plate and sample, respectively. With \( d_2 = 8 \) mm and wave velocities as given in Table 1, \( d_1 \) varies from 2.2 to 2.4 mm. For most experiments, a flyer plate thickness of 2.21 mm was used, which falls in this thickness range.

Following Cohn and Abrams [1981] derivation, the dynamic tensile stress \( \sigma_t \) in the sample is given by

\[
\sigma_t = \frac{\rho_s V_s^2 c_f}{\rho_f V_f^2 c_f} \sigma_f = \frac{R c_f}{\rho_f V_f^2}
\]

where \( \rho_s \) and \( \rho_f \) are sample and flyer densities, respectively. The material constant \( K \) for impacts of the present sample types with plasmas flyer plates are given in Table 1. Multiplication of \( K \) by \( \rho_s \) (g/s) yields the tensile stress in megapascals (MPa) for each experiment (Table 1). Cohn and Abrams [1981] showed that the major uncertainty in \( K \) is caused by neglecting the difference between ultrasonic \( V \) wave velocities and shock velocities in the sample material. They demonstrated that, for their experiments, an increase in \( V \) by 10% would change the value of \( K \) by less than 2%, a fact caused mainly by the low shock impedance of plasmas. Following their derivation, we have

\[
\rho_f V_f^2 \int_0^L \frac{d\sigma_f}{\sigma_f} = C_f
\]

Using the appropriate parameters for the sample material in the present study, we find that

\[
\frac{\rho_s V_s^2}{\rho_f V_f^2} C_f = 0.5
\]

which means that variation of \( V \) by 10% results in changes of \( K \) by 5%. Although these favor blower than Cohn and Abrams’ [1981] result, mainly caused by a lower shock impedance of our sample material as compared to rock samples, the assumption of shock wave velocities equal to the \( V \) velocity of the sample materials does not affect our results significantly.

The strain rate in our experiments was estimated as follows. Strain \( \epsilon \) in shock-loaded solids is given by

\[
\frac{\epsilon}{\rho_s V_s^2} = \frac{\Delta t}{\rho_f V_f^2}
\]

where \( \rho_s \) and \( \rho_f \) are the particle and shock velocity, respectively. By using first-order principles for the continuity of stress and displacement across the flyer plate-sample interface one arrives at an equation which gives \( c_f \) as a function of known material parameters and \( V_s \);

\[
c_f = \frac{\rho_f V_f^2 \sigma_f}{\rho_s V_s^2}
\]

The strain rate \( \epsilon \) is approximately given by the ratio between strain \( c_f \) and the time interval \( \Delta t \) needed to impose tensile strain in the entire sample:

\[
\Delta t = \frac{\epsilon}{\rho_f V_f^2} = \frac{\epsilon}{\rho_s V_s^2}
\]

which can be written as

<table>
<thead>
<tr>
<th>TABLE 1. Sample Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample</td>
</tr>
<tr>
<td>5 g/cm³</td>
</tr>
<tr>
<td>10 g/cm³</td>
</tr>
<tr>
<td>20 g/cm³</td>
</tr>
</tbody>
</table>

\( \rho_s \) and \( \rho_f \) are sample and flyer densities, respectively.
### Table 2: Experimental Data

<table>
<thead>
<tr>
<th>Shot No.</th>
<th>Sand Content, wt %</th>
<th>Target Temperature, °C</th>
<th>Projectile Velocity, m/s</th>
<th>Tensile Stress, MPa</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-2</td>
<td>240</td>
<td>23.3</td>
<td>39.7</td>
<td>completely fragmented</td>
<td></td>
</tr>
<tr>
<td>I-3</td>
<td>229</td>
<td>21.9</td>
<td>37.7</td>
<td>completely fragmented</td>
<td></td>
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<tr>
<td>I-4</td>
<td>230</td>
<td>9.2</td>
<td>13.7</td>
<td>fragmented</td>
<td></td>
</tr>
<tr>
<td>I-5</td>
<td>265</td>
<td>6.7</td>
<td>8.0</td>
<td>intact, no visible cracks</td>
<td></td>
</tr>
<tr>
<td>I-6</td>
<td>242</td>
<td>9.5</td>
<td>16.2</td>
<td>fragmented</td>
<td></td>
</tr>
<tr>
<td>I-7</td>
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<td>7.3</td>
<td>12.4</td>
<td>intact, no visible cracks</td>
<td></td>
</tr>
<tr>
<td>I-8</td>
<td>261</td>
<td>10.1</td>
<td>17.2</td>
<td>fragmented</td>
<td></td>
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<tr>
<td>I-9</td>
<td>267</td>
<td>8.6</td>
<td>14.6</td>
<td>intact, no visible cracks</td>
<td></td>
</tr>
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<td>16.9</td>
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<td></td>
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<tr>
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<td>24.1</td>
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<td></td>
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<tr>
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<td>8.8</td>
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<td></td>
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<tr>
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<td>36</td>
<td>8.6</td>
<td>12.5</td>
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<td></td>
</tr>
<tr>
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<td>266</td>
<td>6.6</td>
<td>19.2</td>
<td>intact, no visible cracks</td>
<td></td>
</tr>
<tr>
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</tr>
<tr>
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<td>21.7</td>
<td>intact, spall visible</td>
<td></td>
</tr>
<tr>
<td>15-9</td>
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<td></td>
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<td>15-11</td>
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<td>22.7</td>
<td>intact, no visible cracks</td>
<td></td>
</tr>
<tr>
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<td></td>
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<td>8.8</td>
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<td></td>
</tr>
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<td>9.5</td>
<td>15.7</td>
<td>intact, no visible cracks</td>
</tr>
<tr>
<td>15-14</td>
<td>5</td>
<td>267</td>
<td>12.3</td>
<td>20.4</td>
<td>fragmented</td>
</tr>
</tbody>
</table>

\[ x = 50 \left( \frac{\Delta T}{P} \right)^2 \]  

(Eq. 2)

Here, \( x \) is the sample thickness. The terms \( (V_0 + \Delta V) \) and \( (V_0 - \Delta V) \) in (2) give the velocities of longitudinal sound waves in the sand and the superposition of these velocities caused by the difference in sample and sand, velocity caused by compaction of the sample, i.e., by applying a stress \( \sigma \), to the sample (see equation (1)). We evaluated \( \Delta T \), based on data for the B-wave dependence of sound velocities in ice, when \( \sigma = 7 \times 10^5 \) MPa (Kentler, 1975). This gave a range of stresses \( x \), corresponding to the range in projectile velocities, with a mean value of 2 x 10^5 m/s.

### Results

The main results of the present experiments in terms of macroscopically observable damage imposed on each sample as a result of tensile loading are given in Table 2. Four different damage categories have been defined, which are illustrated in Figure 2. The results of Table 2 are illustrated in Figure 3, where the dynamic tensile strength for each of the sample types can be determined directly. The tensile stress corresponding to the transition from intact to spalled or fragmented samples is taken as the dynamic tensile strength for each of the samples. We thus infer a tensile strength of 17 MPa for ice and 20 and 22 MPa for ice and sand samples with 3 and 30 wt % sand, respectively. Allowing for an 5% uncertainty in \( x \) and accounting for an experimental uncertainty in the projectile velocity by ±0.5 m/s results in a maximum uncertainty for the tensile strength of ±1 MPa (see equation (2)). The accuracy in the given tensile strength depends on the accuracy in the tensile stress values of the experiments and on the precision with which the transition between intact and fragmented shocked samples can be determined. As can be seen in Figure 3, with the exception of the 5% sand samples, this transition can be well defined for our samples, i.e., within 1 MPa. Thus, together with the uncertainty for the tensile stresses (±0.5 MPa), we derive a mean uncertainty for all of the materials under consideration of ±1.1 MPa for the tensile strength.

These results, together with data for ice (Hankes and Muller, 1972), frozen silt (Haynes et al., 1975), which we believe is comparable to our ice-ice-silicate samples, and ice-ice-silicate mixtures (Goughnour and Anderland, 1983), are given in Figure 4. As can be seen, the dynamic tensile strengths of ice and ice-ice-silicate mixtures lie well above the values found in quasi-static tests. Dynamic tensile strength even exceeds values of the compressive strength for ice and ice-silicate mixtures significantly. These results demonstrate that ice and ice-ice-silicate mixtures, like rocks, show significantly different strength properties when stress loading is applied dynamically versus quasi-static loading. The increase in strength with increasing sand content in ice-ice-silicate mixtures is in good agreement with results found in compression tests by Goughnour and Anderland (1983). It also agrees with previous observations in impact crating experiments which yielded decreasing crater dimensions with increasing sand content for a given impact (Scott et al., 1979; Lange and Ahrens, 1982). These samples not fragmented in our experiments were cut directionally, and core sections were
Fig. 2. Classification of sample damage due to tensile loading. Major divisions of the scales are millimeters; numbers on labels represent experiment numbers, and white numbers in lower right corners are tensile stresses in megapascals as experienced by each sample. The four classifications are (a) intact, no visible cracks, (b) spalled, (c) fragmented, and (d) completely fragmented.
Peak tensile stress, MPa

Fig. 3. Degree of fragmentation as a function of peak tensile stress. The tensile strength of each substance is determined from the stress where transition from intact to spalled or fragmented samples occurs.

Investigated under a microscope. Typical examples of cross sections of each of our target types are shown in Figure 5. Of these, only one cross section, 15-4, shows clear indications of incipient fracturing; the outer surface markings are artifacts of the cutting process. The fact that not more of the intact samples show signs of incipient fracturing might be explained by the rapid smearing of small cracks and fissures prior to examination (Daclos, 1962).

Continuous Modeling of Fracturing in Icy Media

In order to put our results in a somewhat broader perspective and to develop limited prediction capabilities, an attempt is made to generalize our findings in terms of a continuum fracturing model (Grady and Fipp, 1980). The two major elements in the continuum modeling of fracturing which will be explained to the present results are (1) the distribution of flaws and fissures as sources of weaknesses which lead to activation and growth of cracks under tensile loading and (2) a kinematic parameter, the rate of tensile loading of a specimen. At quasi-static loading conditions a single large crack is responsible for failure. At high rates of loading, however, a larger number of cracks is necessary to relieve the increasing tensile stress, and additional flaws must participate in the fracturing process, leading to more numerous and smaller fragments.

In the following, we will adopt most of the definitions of Grady and Fipp [1980], and only those relations essential for an understanding of the continuum model, will be given here.

First, a measure of the fracture damage experienced by a stress-loaded specimen has to be defined. This parameter $D$, where

\[ D = \frac{1}{2} \]

defines the reduction of the elastic bulk modulus $K$:

\[ K_D = K (1 - D) \]

where $K$ is the bulk modulus of the intacted material and defines a state of an intact ($D = 0$), partly fractured ($0 < D < 1$), and completely fragmented ($D = 1$) specimen. $D$ can be described by

\[ D = \frac{N}{V} \]

where $N$ is the number of cracks per unit volume and $V = 4/3 \pi r^3$ is the spherical region around a flaw of radius $r$ which approximates the stress-relieved volume due to the traction-free boundary associated with each crack.

The activation of cracks is described by a two-parameter Weibull distribution, an approach which provides a satisfactory description of brittle fracture in crystalline solids (Hager and

Fig. 4. Tensile and compressive strengths as a function of strain rate for icy substances. The data of Goupilow and Anderlak [1966] are for ice-sand mixtures with 3 (open symbols) and 10 wt % (solid symbols) sand content.
Fig. 5. Cross sections of intact, shocked samples. The main divisions on the scales are millimeters, and the numbers designate sample numbers. Only the sample IS-4 shows indications of incipient spall. Figures 5a, 5b, and 5c show ice and ice-silicate samples with 30 and 5 wt % sand, respectively.
The number of flaws, \( n \), which are activated at or below a tensile stress level \( \sigma \) is given by

\[
n = k \sigma ^n
\]

where \( k \) and \( n \) are material parameters.

The growth of an activated flaw is assumed to proceed at a constant velocity \( v \). Thus, the increase in the stress-relieved\(^\text{15}\) volume \( V \) surrounding the flaw as a function of time \( t \) is given by

\[
V = (t - 4) \left( \frac{\sigma}{C} \right) ^t
\]

for the easy case of constant strain rate loading, i.e., \( e(t) = \text{const} \). Grady and Kipp (1980) derive an equation which relates the damage growth \( D(t) \) to time \( t \) and constant strain rate \( \dot{e} \).

\[
D(t) = \frac{1}{2} \dot{e} t
\]

where \( n \leq \frac{3}{2} \) and

\[
\gamma = (\text{m} + 1)(\text{m} - 2)/2
\]

On the basis of this relation, the stress history \( \sigma(t) \), the maximum or fracture strength \( \sigma_f \), the time at which maximum stress accumulation is reached, \( t_f \), and the time at which failure occurs, \( t_i \), can be obtained from

\[
\sigma = K \gamma \dot{e} t
\]

\[
\sigma_f = \dot{e} (m+1)/(m+2) = 3/(2 m)
\]

\[
\gamma = (m+1)/(m+2) = 1/(m+2)
\]

Finally, Grady and Kipp (1980) derive relations which allow determination of a fragment size distribution \( F(L) \) and the dominant fragment size \( L_f \) for constant strain rate loadings:

\[
F(L) = \frac{1}{4} \frac{L^2}{L_f^2} (L - L_f)^2
\]

\[
C = \frac{L_f^2}{\dot{e}} \frac{\gamma}{L_f^2}
\]

The principal variables in the equations (4)-(21) are the strain rate \( \dot{e} \) and the major parameters of the velocity decay growth \( C \), the material parameters \( m \) and \( k \) in the Weibull distribution (equation (12)), and the bulk modulus \( K \). The bulk modulus can be obtained from the measured \( P \) wave velocities \( V_p \) (M. A. Lange, unpublished data, 1962) in each of our sample types by

\[
K = \frac{4}{3} \rho V_p^2
\]

where the assumption is made that the shear wave velocity \( V_s = 0.5 V_p \) (Duljan, 1970, p. 75).

Grady and Kipp (1980) find values for the shear strength \( V_s \) for all soils by fitting equations (17) and (10) to experimental data which relates principal fragment size \( L_f \) and fracture stress \( \sigma_f \) to strain rate. They find that \( V_s \) is about 0.5 times the \( P \) wave velocity of soil shaft, a result in good agreement with data of Shockey et al. (1971), who find \( C = \frac{1}{15} V_p \) for Arkansas cobbles. We assume that this relation holds for most crystalline rocks and define

\[
\gamma = \frac{C}{\rho} \frac{V_p}{L_f^2}
\]

for our sample materials.

The remaining parameters needed to define a continuous shocking model for the ice-oxide mixtures are the constants \( k \) and \( m \) (equation (12)). We used the following conditions to evaluate values for these parameters: (1) the value of the tensile strength \( \sigma_t \) for each of our largest materials at a strain rate of \( 2 \times 10^3 \text{ s}^{-1} \), (2) the principal fragment size \( L_f \) in completely fragmented samples (3) the total duration of the tensile stress pulse in the samples, and (4) the tensile strength of ice at a strain rate of \( 10^3 \text{ s}^{-1} \) (Hawkes and Melton, 1972).

The principal fragment size \( L_f \) was found by measuring the long axis of some 200 fragments of plate I-2 (Figure 6a) and II-2 (Figure 6b) on photographs taken of the fragments directly after the experiment. The lack of data for smaller fragments is due to the limit in the resolution of the photographs. On the basis of these measurements we define as the principal fragment size, i.e., the size range which covers the majority of the measured particles, \( 0.1 \leq L_f \leq 0.35 \) mm for the present experiments. For ice at a strain rate of \( 2 \times 10^3 \text{ s}^{-1} \). Although this size range encompasses the main phase of the ice and sand grains from which our samples were made, we do not believe that \( L_f \) simply reflects the initial grain sizes. As mentioned above, our method of sample preparation provided ample recrystallization of ice in the sample pellets.

The duration of tensile stress pulses at \( \dot{e} \) in the experiments varied between \( 0.3 \) and \( 0.75 \) s, depending on the peak tensile stress reached in the sample. This can be used to constrain the continuous shocking model at which maximum stress is reached in our sample \( C \), equation (13) and the time at which failure occurs \( t_f \), equation (19) by requiring that \( t_f \leq t_i \).

The tensile strength of ice at a strain rate of \( 10^3 \text{ s}^{-1} \) lies between \( 1.5 \) and \( 1.6 \) MPa (Hawkes and Melton, 1972). Since there are no tensile strengths for ice-oxide mixtures with between 50 and 75% nonmetallic contents, as distinct from iron-saturated sand data available, we assume that the ratio between tensile strengths of ice and three of ice-oxide mixtures are approximately homogeneous of that rate. This allows, based on our measurements at \( 2 \times 10^3 \text{ s}^{-1} \) and the tensile strength of ice at \( 10^3 \text{ s}^{-1} \), to estimate strengths values for ice-oxide mixtures at a strain rate of \( 10^3 \text{ s}^{-1} \), which yields values of 1.08 MPa and 2.07 MPa for ice-oxide with 30 and 50 wt % sand, respectively. The tensile strength for iron-oxide at \( 10^{-4} \text{ s}^{-1} \) is 6.0 MPa (Haynes et al., 1975)) is used as an upper bound for the strength of our ice-oxide samples at a strain rate of \( 2 \times 10^3 \text{ s}^{-1} \).

We varied the parameters \( m \) and \( k \) for each sample type systematically and used a computer program to compute \( C \), \( \rho \), \( V_p \), \( V_s \), and \( L_f \) for strain rates of \( 2 \times 10^3 \text{ s}^{-1} \) and \( 10^3 \text{ s}^{-1} \) until the combination of \( m \) and \( k \) was found which gave satisfactory agreement of the numerically derived values with those obtained in our experiments or from sources in the literature. Figure 7 illustrates the range of values of \( V_s \) for ice computed for \( 6 \leq k \leq 10 \) and \( 0.32 \times 10^{-4} \leq k \leq 0.32 \times 10^{-3} \) at a strain rate of \( 2 \times 10^3 \text{ s}^{-1} \). The values of \( m = 6.7 \) and \( k = 0.32 \times 10^{-3} \) gave the
Fig. 6. Photographs of completely fragmented (a) ice (1-2) and (b) ice-silicate samples (13-2) and size distribution of some 200 fragments as determined by measuring fragment sizes on photographs. The scale in both photographs is in centimeters.
most satisfactory agreement of $\sigma_m (= 17.5 \text{ MPa})$ with the value obtained in our experiment ($= 17 \text{ MPa}$) as well as with respect to the other criteria mentioned above. Table 3 gives the values of $m$ and $k$ for all of the present sample materials and the values of $\sigma_m$, $\sigma_n$, $\sigma_{0.25}$, and $\tau_0$, as computed by use of equations (17)-(19) and (22). As can be seen, the fits between computed and measured material parameters in Table 3 for each combination of $m$ and $k$ can be made quite close. This suggests that the continuous fracturing model provides a useful tool in evaluating and generalizing experimental fracturing data in icy media.

Discussion and Conclusions

The parameters derived in the previous section (Table 3) can now be used to predict fracturing behavior of icy media and related properties. Figure 8 gives fracture stresses (tensile strengths) for each of the materials under consideration as a function of stress rate. Following the results of Grady and Whillans [1979] and Grady and Kipp [1980], we propose a relation between tensile strength $\sigma_m$ and stress rate $\dot{\varepsilon}$ in icy media:

$$\sigma_m = \sigma_0 \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^m$$

which is close to relations found for other geological materials (Grady and Kipp, 1980). Also given are tensile strengths for a number of different rocks, as obtained in similar experiments (Cohen and Ahrens, 1981; Cohen et al., 1982), which lie above the strength values for icy media.

The continuous fracturing model also allows computation of stress and damage histories during tensile loading (equations (14)-(16)). Damage growth is negligible until a critical time $t^* = \frac{\sigma_m}{\sigma_{0.25}}$ at which damage growth becomes catastrophic. This
Fig. 8. Tensile strength versus strain rate for ice and ice-illinites. The tensile strength for four different rock types represent results of shock wave experiments by Cohn et al. [1982] (San Marcos Gabbro) and Cahn and Ahrens [1981] (chert) at strain rates between 10^3 and 10^5 s^-1.

corresponds to a gradual increase in tensile stress to a maximum (fracture stress) at which the drop-off in stress proceeds rapidly. Increase in strain rate results in shifts of c(x) and c(x) to shorter periods of time; i.e., c(x) and c(x) decrease (see Figure 4 of Grady and Kipp [1980]).

The distribution of fragments resulting from the complete fragmentation of ice substrates can be computed as a function of strain rate from equation (9). We observe trends qualitatively similar to those found by Grady and Kipp [1980] (see their Figure 3). With increasing strain rate, the number of small fragments relative to the number of larger fragments increases, and the prismatic fragment rate decreases (see Table 3). This agrees with impact fragmentation experiments on ice and ice-illinites which demonstrate an increase in the number of small fragments relative to large fragments with increasing impact velocity (strain rate) Lange and Ahrens, 1981, also unpublished manuscript, 1982).

Dynamic strength values for ice and ice-illinites mixture lie significantly above values found in quasi-static tests. This is surprising in the case of ice, since previous [Hooke and Mullin, 1972] measurements over a relatively wide range of strain rates (10^-3 to 10^-2 s^-1) showed an independence of tensile strength on strain rate.

Acknowledgments. We appreciate the skillful help of R. Gollis, M. Olson, and M. Long in the experiments. Special thanks to T. Yang, who spent many hours of sample preparation in the cold lab. The use of the cold lab facility and helpful advice proffered by P. Kamb are appreciated. Grateful reviews by S. K. Creat and R. Cimelis are gratefully acknowledged. M. Lange is supported by a stipend from the Deutsche Forschungsgemeinschaft. Work was supported under NASA grant NGL-05-002-103. Contribution 3806, Division of Geological and Planetary Sciences, California Institute of Technology.

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(Received August 12, 1982; revised November 11, 1982; accepted November 19, 1982.)