Giant impact-induced blow-off of primordial atmosphere

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ABSTRACT
The surface motion from a large impact upon an attenuation-free fluid sphere was studied and numerically simulated. An analytic solution for the free-surface velocity shows that close to the source, the acoustic wave due to the free-surface interaction (a "quasi-surface wave") is not separable from the direct wave, Ai > 90°, the quasi-surface wave separates and has a larger amplitude than the direct body wave. Near the antipode the quasi-surface wave amplitude is much larger than the direct body wave and is comparable to the direct wave amplitude immediately near the source at 0°. The resulting solution covers both he wave interference range that defined in the asymptotic theory of near-source surface wave propagation developed by Russian physicist V.S. Buldyrev reported in 1968, as well as in the geometric ray range. The geometric range theory was developed in several papers in terms of multi-geometry reflection by R. Bull ridge, H. Jeffrey, and E.R. Lapwood in England in 1957 through 1963. For a large surface excitation (e.g., O.1MPac impact) a portion of the atmosphere above a plane tangent to Earth at the impact point is launched to velocities greater than the escape velocity. The resulting antipodal free-surface velocity achieved is ~10 km/s, which is sufficient to launch a comparable fraction of the atmosphere to escape.

Keywords: impact, blow-off, atmosphere.

INTRODUCTION
It has long been recognized that the terrestrial planets are depleted relative to carbonaceous meteorites in volatile molecules (H2, CO, CO2, SO2, NH3, and CH4) as well as noble gases (Ringwood, 1979).

Experiments conducted on serpentine and other water-bearing minerals (Ahrens et al., 1985) suggest that for planetesimals impacting Earth at >1 km/s, partial devolatilization takes place. As a result, during accretion of Earth and other terrestrial planets, primitive atmospheres whose major component is water (plus CO2 and other gases) form. Because of the infrared opacity of a water-bearing atmosphere, the impact energy of accretion is unable to be radiated efficiently to space and a molten rock (magma) ocean forms after Earth accretes >0.1 of its final mass, as described by Abe and Matsui (1985). During the period of planet formation, both planets and planetesimals impactors grow via accretion. Larger impactors colliding with the newly grown Earth are hypothesized to have been Mars-sized objects. Such large-scale accretionary collisions result in large amplitude body wave and even larger amplitude surface waves. These large amplitude waves resulted in free-surface motion of the magma ocean-covered Earth. As a result, the motion of Earth's free surface drives a wave into the proto-atmosphere. Because of the exponential density distribution of the atmosphere, the particle velocity in the atmosphere is accelerated to higher values with increasing altitude (Zel'dovich and Raizer, 1967). Upon sufficient energetic impact, particle velocities at high altitude that exceed the Earth's escape velocity are achieved. Therefore, a portion of the upper atmosphere can be ejected upon accretion of the primordial Earth and other objects. Numerical studies of compressional body wave driven motion (Chen and
Ahrens, 1997; Genda and Abe, 2003) show that a lunar-sized impactor (with an energy of \( E \)) would have blown out a significant portion of the entire (paleo) Earth atmosphere to greater-than-escape velocity. Therefore, Earth and its sister planets could have lost a large portion of their initial, primordial atmospheres. Further, Ahrens (1993) proposed that the impact-induced mechanism that erodes an atmosphere most efficiently is related to the planet-free-surface-atmosphere interactions. There appears to be at least two regimes for atmospheric blow-off relating to planetary impact.

Regime A. This regime corresponds to the flow in the vicinity of the impact, where the bolide radius \( a \) is \( a < h \). Here, \( h \) is the atmospheric scale height. Initially, Meoleh and Vickery (1989) discussed this regime in connection with the loss of Mars’s primordial atmosphere. Later analysis and simulations suggested that atmosphere is instead eroded via a Regime A impact. However, the assumption of self-similar gas flow used in Meoleh and Vickery’s model may overestimate the mass of atmosphere blown-off in this regime. Newman et al. (1999) showed that impact-induced gas flows in the atmosphere are strongly affected by gravity.

Regime B. In Regime B, \( a > h \). To relate the motion of the atmosphere to the energy of the impactor, we calculate the acoustical wave field from a subaqueous explosion in a cavity of radius \( a \) (Fig. 1). This approximates the initial energy distribution from a giant impact. In this case, impact-induced flows of the atmosphere induce atmospheric blow-off in the vicinity of the impact, as in Regime A. Also, in Regime B, a wave is transmitted radi ally through Earth. Upon interaction of this wave with the solid phase, a new surface-atmosphere interface, a shock wave is launched into the atmosphere which may induce further global atmospheric blow-off.

Jeffreys and Lapwood (1957), and Burridge (1966) used asymptotic properties of Legendre functions to study reflections of waves (which are approximated by geometrical rays) against the free-surface of a sphere. They derived asymptotic results for seismic phases that evolve several reflections, e.g., PP, PP and PPP. For a large number of reflections (e.g., \( P \) where \( n \) is very large), only a limiting number of reflections \( M \) defined by Buldyrev (1968). If \( n > M \), ray theory is no longer applicable near the free-surface because of the existence of a boundary layer where wave interference predominates. Therefore, it is desirable to obtain a solution which reduces to the above solution for single-pass waves and reverberations, so that the free-surface motion can be studied in detail for a realistic excitation.

**METHOD**

The transfer of energy into a magma-covered planet from an impactor is complex. We approximate the impact by the detonation of a shallow buried charge in the acoustic approximation (Fig. 1). Thus, we effectively only address impact on the magma ocean-covered Earth. The relation of near-surface explosive coupling and stress wave attenuation in the simulation of impact was previously studied in porous media (Obertacke, 1971). Hughes et al. (1977) verified that the shock from a buried explosive source is a good approximation to that of a large impact for the case of

\[
\varepsilon = p \left( \frac{\partial p}{\partial \rho} \right)
\]

has a value equal to \( \rho \). The pressure at \( t = 0 \) is \( \rho \) in the spherical source zone. The pressure is zero outside the source zone. The radius of the source sphere is \( a \) and the center of the source sphere is located at \( (x = r = 0) \) (spherical coordinate) (Fig. 1). From fluid dynamics, the linearized equations for conservation of mass and momentum are:

![Figure 1](image-url)
\[
\begin{align*}
\frac{\partial \psi}{\partial t} + \rho v \cdot \nabla \psi &= 0 \\
\rho \frac{\partial v}{\partial t} + \nabla P &= 0
\end{align*}
\]

and the isentropic equation of state is \( e^{\text{is}} = (\partial P/\partial \rho)_{s} \), where \((\partial P/\partial \rho)_{s}\) is the isentropic partial derivative of pressure \((P)\) with respect to density \((\rho)\). \(v\) is the unperturbed density, \(\psi\) is the particle velocity vector, and \(\nabla\) is the isentropic acoustic sound speed. Equations (1) and (2) yield:

\[
\frac{\partial^2 P}{\partial t^2} + \frac{\partial^2 \psi}{\partial t^2} = c_s^2 \psi
\]

Initial and boundary conditions are:

- \(\psi(r,\theta,t)\big|_{t=0} = 0\); all particles are at rest;
- \(\psi(r,\theta,t)\big|_{t=\infty} = 0\); free surface;
- \(\psi(r,\theta,t)\big|_{r=a} = \psi_{in}\) if \((r,\theta)\) in the source zone defined by spherical region of radius \(a\), c.f. Fig. 1;
- \(= 0\), otherwise.

The solution of wave Equation (3) is:

\[
P(r,\theta,t) = 2\pi \sum_j A_j \frac{\psi_{in}}{R} J_0(\psi_{in}) \cos\left(\frac{k_n a}{R}\right)
\]

The radial particle velocity is:

\[
u_r(r,\theta,t) = 2\pi \sum_j A_j \frac{\psi_{in}}{R} \cos(\psi_{in}) \cos\left(\frac{k_n a}{R}\right)
\]

where \(\phi_j\) is the \(j\) th spherical Bessel function \((Watson, 1922)\) which is defined as

\[
j_0(x) = \frac{\sin x}x
\]

and \(j_1(x)\) is the Bessel function. \(j_0\) is the derivative of \(j_0\). \(P_j(\psi_{in})\) is the ordinary Legendre polynomial.

\[
k_n = \text{zero of } j_0(x), \text{ and is usually called the wave number.}
\]

\[
A_n = \frac{2\pi}{R^2} \int_0^\infty (k_0 P_{in} R) P_{10} R = 0 \sin(k_0 R) R^2 dR
\]

where \(A_n\) is the excitation coefficient.

For the special case of a spherical source with uniform pressure, \(A_n\) has the form \((\text{Ni and Ahrens, } 2005)\):

\[
A_n = \frac{P_{10} R}{R} \left( \frac{\sin k_0 a}{k_0 a} \right) \left( k_0 a_0 \right) \left( k_0 a \right)
\]

Utilizing the recurrence relation for spherical Bessel functions, we obtained a solution that is simplified for radial velocity at the free surface:

\[
u_r(r = R, \theta, t) = 2\pi \frac{P_{10}}{R} \left( \frac{\sin k_0 a}{k_0 a} \right) \left( k_0 a_0 \right) \left( k_0 a \right) P_{10} \cos(\psi_{in}) \cos\left(\frac{k_n a}{R}\right)
\]

Equation (7) and (7) are the main analytical results of this paper and \(\psi_{in}\) is plotted in Figure 2. Here, \(P_{10}\) is the pressure in the source region (in Pa), \(a\) the sound velocity (in km/s), and \(R\) is the radius of the planet (in meters). The given solution of \(\psi_{in}\) is km/s. A more detailed mathematical treatment of this problem is given by Ni and Ahrens \((2005)\) who have examined the excitation of body and surface waves for different values of \(r/R\) and \(a/R\).

The method of expressing \(u_r(r,\theta)\) and \(P(r,\theta)\) as sums of basis functions is called the normal mode summation method \((Kamotani, 1997)\). Sato \((1961)\) and Sato and Usami \((1967)\) define the methodology for studying normal modes of homogeneous and radially heterogeneous spheres. These authors do not give explicit values of \(A_n\). Bateman \((1930)\) obtained the solutions of waves excited by sources distributed on the free-surface of a sphere. Numerical evaluation of \(\psi_{in}\) is possible if the appropriate algorithm for calculating \(w_n\) and \(j_n\) is used. In Equation 5, \(\psi_{in}\) is given as an infinite summation.

\(\sin c\) is the error function. This convergence can be demonstrated because the asymptotic behavior of \(j_n\) is known. Numerical tests show that, for given \(i\), when \(\psi_{in} \geq M\psi\), \(M \geq 5\), truncation at \(i\) terms results in an error of less than \(e\). When \(i < M\psi\), the error due to truncation of \(i\) is less than \(e\).

**RESULTS AND DISCUSSION**

We calculated a nominal case where \(r_s = 0.96R\) and \(a = 0.02R\). \(\text{Ni and Ahrens, } 2005\) present results for various plowed sources. Scaling to Earth, the source region has a radius of 127.4 km and contains a mass of rock of \(4.78 \times 10^{10} \text{ kg}\) (assuming an average Earth density of \(p = 5.51 \times 10^3 \text{ kg/m}^3\)), bulk sound velocity of \(c = 10 \text{ km/s}\), the pressure in the source region is \(5.51 \times 10^3 \text{ Pa}\), which is derived from \(P_{10} = k_0 c_0 = c, \text{ where } k_0\) is the mean isentropic bulk modulus). Here \(u_r < 0\), \(P(r,\theta)\) is plotted in Figure 2. It is shown for \(t\) from \(0\) to \(180^\circ\), at \(10^\circ\) intervals. The peak amplitude (\(\psi_{in}\)) at each angle is given along the right-hand side of the figure. The curve plotted in the figure shows arrivals predicted by geometrical rays. The first arrivals are direct waves. For large angles (\(<90^\circ\)), there are later arriving waves corresponding to propagation along the free surface. We note that the particle velocity at the antipode \((180^\circ)\) reaches its maximum as time \(t\) increases.

Thus, a quasi-surface wave in a compressible fluid sphere is excited by a surface impact. The free-surface velocity amplitude associated with this wave is larger than
the direct waves for angles greater than 90° on an Earth-sized planet. The energy density within the spherical source of Figure 1 may be estimated from the Grüneisen formula:

$$\Delta E = K_v (\gamma \rho)$$

(8)

where $\gamma$ is the Grüneisen parameter within the spherical source region. Assuming that $K_v/\rho = 100$ (km/s)^2 and $\gamma = 2$, the impact yields an energy density $5.0 \times 10^9$ J/kg. The $127.42$ km radius source region that contains $2.39 \times 10^{21}$ J. For this energy, the antipodal surface point achieves a peak velocity of $1.9$ km/s. Notably, we expect that the energy radiated seismically is $10^{-2}$ times the energy of the impact. Hence we expect that an equivalent seismic source of $2.39 \times 10^{21}$ J might be equivalent to an impact with some $10^2$ times more energy than expected for planetary impactors in the $10$–$40$ km/s velocity range (for the values of Fig. 2). According to the calculation (Figure 4) of Chen and Ahrens (1997), this antipodal velocity will cause escape of $\sim 10^2$ of the...
ambient Earth atmosphere. Recently, Genda and Abe (2003) have examined atmospheric loss as a function of impact energy.

In the present study, quasi-surface waves are excited by multiple reflections, with a propagation velocity of \( c \).

Budyko (1968) called these "surface waves of interference nature." Watson (1922) found that the \( j(x) \)'s first zero is at \( x = 1.85 \times 10^6 \) sec, where \( v = 1 + \frac{1}{2} \), and the first maximum of the ray-trace is at \( x = 0.808 \times 10^6 \). Upon asymptotic expansion as \( x \to x \), it appears that \( J(x) \) has a width of \( 10^6 \) when \( x = 1 \). From Equation (5) and the recurrence relations for \( J(x) \), we find that the quasi-surface wave has a skin depth of \( k \). This coincides with Budyko's result.

### Elasticity Scattering of Present Results

In an expansion of the present paper, Ni and Ahrens (2005) examined the ground motion that is induced by varying the radius of the source center from \( r_s \) to \( r_s = 0.90 \) to \( 1.10 \), also the effect of varying the size of the source from \( 0.01 \) to \( 0.05 \). We briefly discuss the scaling of the elastic results and do not consider the scaling of large velocity impacts. The present result given in mathematically form by Equations (6) and (7) and shown for the case of \( r_s = 0.96 \) and \( x = 0.02 \) and \( x_s = 10^6 \) (in km/s) in Figure 2 are scalable by varying two parameters, \( r_s, \mu, \kappa, p, \) and \( y \). We, in this paper, scaling of distance and time stems from Earth's radius \( R \) and mean sound velocity \( c \), and mass scaling stems from mean density \( \rho \). We note that \( r_s \) describes the placement of the equivalent explosive source. The effects of varying \( r_s \) and \( x \) are not studied in this work. For a huge impact, we assumed \( r_s = 0.96 \). The ratio from the center of Earth to the source region, in part, controls the volume of the source region, and might be considered approximately equivalent to a characteristic dimension of the terrestrial crater from a giant high-velocity impact. Since \( P = \rho c^2 \), the mean inelastic bulk modulus and the density control both, the wave propagation velocity and, via Equation 8, the energy density of the source. Finally, the value of Grötzinger parameter \( y \) specifies the internal energy which is associated with the gas pressure \( P \).

### Comparison of Shock and Elastic Wave Propagation. Limits to Present Approach

As the present wave propagation model is elastic, it is useful to make some limited comparisons as to the expected effect of an impact-induced shock propagation versus the present elastic assumption. As discussed above, the source region assumed is a spherical 127 km region with an energy density of \( \Delta E = 5 \times 10^{10} \) J/kg. If we assume that the peak particle velocity achieved at the source \( u_0 \) corresponds to \( \Delta E/2 \), we have

\[
\frac{u_0}{c} = \frac{\Delta E}{2}
\]

or \( u_0 = 2.2 \) km/sec. We note the free-surface velocity in Figure 2, immediately above the source, is 2.6 km/sec, corresponding to a particle velocity of 1.5 km/sec. Thus, the elastic solution at \( 0^\circ \) is 1.3 km/sec, compared to a shock particle of 2.2 km/sec, a deficit of \( 0.45 \). We note that above peak amplitude attenuation over distances greater than a factor of \( 10^2 \) occur when the radius of the source is small compared to propagation distance, as for example, at 45-50°.

\[
\frac{u}{u_0} = \left( \frac{r}{R} \right)^n
\]

where \( r \) is the chord distance to the source and \( u_0 \) and \( u \) are the free field particle velocity and particle velocity at the source. Here \( n = 11 \) to 12 between the elastic wave of \( n = 1 \), but less than the \( n = 2 \) value expected of strong shock propagation (Melosh, 1989).

Although a fluid material may withstand strong compression, a fluid can withstand only very weak tensional loading. Hence, upon application to the present results (Genda and Abe, 2003) to pumping of the atmosphere, we restrict tensional unloading of the free surface velocity, \( u_0 \), to be:

\[
\frac{3u_0}{R} = \frac{2 \times GM}{R^2}
\]

where \( G \) is the gravitational constant and \( M \) planetary mass. Therefore, we use the results summarized in Figure 2 to describe only the peak particle velocity upon compression. Decompression is assumed to occur ballistically as specified by Equation 11.

### CONCLUDING REMARKS

Previously, Vickery and Melosh (1990) calculated that energetic impactor will remove a substantial portion (10%) of the atmosphere (note the impact point, \( x = 0 \)) of a smaller planet. This energy level (at \( 1^\circ \)) is similar to that considered in the present paper, and corresponds to an asteroidal impactor some 100 km in diameter (This is \( \approx 10^6 \) more energetic than that resulting in such-terrestrial impact structures as Chicxulub, Vestafltd, and Sudbury, and Imbrium on the Moon). Chen and Ahrens (1997) demonstrated that a giant impact-induced solid Earth free surface velocity \( \approx 8 \) km/sec produces an upward particle velocity in the atmosphere in excess of the 112 km/sec escape velocity of Earth, and hence, virtually the entire atmosphere escapes. We finally note that for an impact with an energy of \( < 10^{12} \) J (a 10 km radius projectile), although some atmosphere is ejected, the impact brings more mass to the Earth than is ejected, and hence, the impact remains an accretional event (O'Keefe and Ahrens, 1977).

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