Three-dimensional FEM derived elastic Green’s functions for the coseismic deformation of the 2005 $M_w$ 8.7 Nias-Simeulue, Sumatra earthquake

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[1] Using finite element models (FEMs), we examine the sensitivity of surface displacements to the location of fault slip, topography, and three-dimensional variations in elastic moduli in the context of a 2-D infinite thrust fault. We then evaluate the impact of these factors and fault geometry on surface displacements and estimates of the distribution of coseismic slip associated with the 2005 $M_w$ 8.7 Nias-Simeulue, Sumatra earthquake. Topographic effects can be significant near the trench, where bathymetric gradients are highest and the fault is closest to the free surface. Variations in Young’s modulus can significantly alter predicted deformation. Surface displacements are relatively insensitive to perturbations in Poisson’s ratio for shear sources, but may play a stronger role when the source has a dilatational component. If we generate synthetic displacements using a heterogeneous elastic model and then use an elastic half-space or layered earth model to estimate the slip distribution and fault geometry, we find systematic residuals of surface displacements and different slip patterns compared to the input fault slip model. The coseismic slip distributions of the 2005 earthquake derived from the same fault geometry and different material models show that the rupture areas are narrower in all tested heterogeneous elastic models compared to that obtained using half-space models. This difference can be understood by the tendency to infer additional sources in elastic half-space models to account for effects that are intrinsically due to the presence of rheological gradients. Although the fit to surface observations in our preferred 3-D FEM model is similar to that from a simple half-space model, the resulting slip distribution may be a more accurate reflection the true fault slip behavior.

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1. Introduction

Space-based geodetic techniques including Global Positioning System (GPS) and Interferometric Synthetic Aperture Radar (InSAR) now provide observations of crustal deformation with unprecedented spatial coverage. This increase in observation quantity and quality motivates a reappraisal of standard modeling assumptions. A common approach to modeling different phases of the earthquake cycle relies on the use of simple elastic dislocation modeling, where one commonly assumes that the earth can be modeled locally as either an elastic half-space [Okada, 1985, 1992] or a horizontally layered elastic half-space [e.g., Singh, 1970; Rundle, 1980; Savage, 1998]. This approach frequently results in models that match observations reasonably well, while also being computationally simple due to the semianalytic nature of elastic Green’s functions describing the response at any point on the surface to a unit of slip at a given location for these one-dimensional elastic structures. Numerous studies using elastic or layered half-space dislocation models have successfully explained how accumulated elastic strain released in major fault zones and produced reasonable fits to the surface displacements [e.g., Savage, 1983; Reilinger et al., 2000; Yu et al., 2001; Simons et al., 2002; Hsu et al., 2003].

However, in many geologic environments, we may have sufficient constraints on the elastic structure to warrant consideration of models that are more complex than the Poissonian layered half-space commonly adopted. Even if the resulting models fit observations no better than those using 1-D elastic structures, the resulting slip models may reflect the true fault behavior with more fidelity. For instance, finite fault source inversions based on seismic and geodetic data show that inaccurate Green’s functions (elastic half-space or 1-D layered approximations) not only introduce errors into the source rupture complexity, but also limit the recovery of the slip pattern due to the blurring from increased smoothing [Graves and Wald, 2001; Wald and Graves, 2001]. In the 1989 Kalapana, Hawaii earthquake, the focal depth determined by seismic studies is 5 km deeper than that inferred from geodetic data [Du et al., 1994]. The authors conclude that the discrepancy is possibly associated with variations of elastic modulus. The study of the coseismic deformation of the 1992 Hector Mine, California earthquake shows that the main effect of the rigidity stratification is to increase the magnitude of inferred slip at depth by about 20% to 30% [Simons et al., 2002]. Similarly, comparisons of the modeling results of the 1999 Izmır, Turkey earthquake using elastic half-space and layered earth models indicate that the Coulomb stress changes at depth, the recovered centroid depth and the seismic potency are larger in a layered earth model relative to those in a uniform half-space model [Hearn and Burgmann, 2005].

Previous works demonstrated the extent to which predicted surface displacements are sensitive to assumptions of material homogeneity, elastic isotropy, and Poisson solid for subduction earthquakes [e.g., Masterlark et al., 2001; Masterlark, 2003; Masterlark and Hughes, 2008]. Their results are generally consistent with those that consider the impact of material heterogeneity on geodetic analysis of the Japan subduction zone [Sato et al., 2007]. Sato et al. [2007] also find that Young’s modulus affects surface displacements more than Poisson’s ratio. In California, interseismic velocities are asymmetric across the San Andreas Fault, which can also be attributed to the effects of lateral variations of crustal rigidity [Fialko, 2006]. Generally, analysis of the effect of material heterogeneity in surface deformation is probably best explored in the context of deformation associated with large earthquakes, given the high signal-to-noise ratio.

Previous studies have also shown that irregular surface topography has significant effects on surface displacements and source location when the rupture source and the topography are in phase or the source is shallow [McTigue and Segall, 1988; Huang and Yeh, 1997; Williams and Wadge, 1998]. For subduction zone earthquakes, topography generally has a small influence on the surface displacement and fault slip distribution [Masterlark,
2003] because the rupture sources are usually deep compared to the surface topography. However, the topographic effect could be significant in some cases and the accuracy of modern geodetic techniques is sufficient to detect this discrepancy. We should consider this effect in the coseismic deformation modeling.

[5] As opposed to previous studies of modeling deformation of a specified earthquake with a finite element approach and different material parameters, we first examine the sensitivity of the location of fault slip, topography, and three-dimensional variations in elastic moduli on surface displacements using a 2-D infinite fault. We use this simple model to demonstrate how inversions for earthquake slip parameters may be significantly biased when the material gradients and strain are large and the material gradients are not sufficiently well represented in the model. We also show that the influence of Young’s modulus on surface deformation may be larger than that of Poisson’s ratio for mode II or mode III behavior associated with faulting, while Poisson’s ratio may play a more important role compared to Young’s modulus if the rupture source is dilatational. We then consider the impacts of topography, Poisson’s ratio, and elastic heterogeneity on inferences of coseismic slip distributions and surface displacements for the specific case of the 2005 $M_w$ 8.7 Nias-Simeulue, Sumatra earthquake. This earthquake occurred offshore Sumatra and ruptured the megathrust in the Sunda subduction zone (Figure 1a) where the plate convergence between the Indian-Australian plates and the Sunda plate is about 5.7 cm/yr. This subduction zone has generated numerous large earthquakes in the past three centuries [Chlieh et al., 2007], including the 2004 $M_w$ 9.1 Sumatra-Andaman earthquake [Subarya et al., 2006]. The spatial coverage of coseismic observations from coral measurements and GPS displacements for the 2005 earthquake allow us to quantify the influence of heterogeneity on inferences of coseismic deformation (Figure 1). Beyond the exploration of the role of 3-D elastic heterogeneity, we also investigate how uncertainties in the assumed fault geometry can complicate evaluating the effect of elastic heterogeneity on the inferred distribution of fault slip.

2. Method

[6] We use the finite element method (FEM) and the 3-D finite element code, PyLith v. 0.8.3 [Williams et al., 2007], to compute coseismic surface displacements and Green’s functions for the 2-D and 3-D elastic models. PyLith is designed for simulating lithospheric deformation over a wide range of spatial and temporal scales (http://www.geodynamics.org/cig/software/pylith). In PyLith, the governing equations for the static problem are the equations of motion, the strain-displacement equations and the constitutive law for linear elasticity. We use boundary conditions with fixed displacements perpendicular to the boundary on the sidewalls and bottom of the model. Kinematic coseismic slip is implemented using a split-node technique [Melosh and Raefsky, 1981]. We have validated our FEM models by comparing predicted FEM surface displacements for uniform coseismic slip in a homogeneous material model for the 2005 earthquake against those predicted by the analytical solution of Okada [1985]. Maximum differences in predicted surface displacement from these two approaches are typically less than 2%. Note that the Green’s functions generated by PyLith (tapered slip distribution on a linear element) cannot exactly match the Okada slip (uniform slip on a square fault patch). One has to find a slip patch with the same potency as the Okada slip patch, but the slip patch in FEM will have a tapered slip distribution and lead to differences relative to the Okada solutions for shallow displacements near the slip patch [Lohman, 2007].

2.1. Two-Dimensional Model Setup

[7] We require an assumed geologic structure, from which we generate a finite element mesh. In our case, we develop a simplified 2-D plane strain conceptual model for testing purposes. We construct the necessary meshes using the CUBIT Geometry and Mesh Generation Toolkit (http://cubit.sandia.gov). The topography of the 2-D FEM model is a step function with elevations equal to 0 and 2 km at distances less than and greater than 200 km, respectively. The mesh resolution decreases with increasing distance from the origin. The model extends 1200 km eastward and westward from the center and 1000 km downward (Figure 2a).

2.2. Three-Dimensional Model Setup

[8] The 3-D model approximates topographic relief and bathymetry using a global digital elevation model with a horizontal grid of 30 arc seconds [Sandwell and Smith, 1997]. We interpolate the elevation with grid spacing of 5 km in the near field of the 2005 coseismic rupture and gradually increase the grid size to 100 km in the far field. The
Figure 1
dimensions of the model are 2700 km × 1800 km × 1200 km in length, width, and height, respectively (Figure 2b). The top surface corresponds to real topography. Assuming the 2005 earthquake ruptured the plate interface between the Sunda plate and the Indian-Australian plates, we use a cubic spline representation to describe the fault geometry (fault B in Figure 3) as opposed to the kinked fault adopted in the work by Hsu et al. [2006] (Figure 3). The dip of the modeled fault increases from 5° at the trench top to 30° at 100 km depth (fault B, Figure 3). Constraints on the dip angle of the shallower portion come from the joint analysis of coseismic geodetic and seismic data [Hsu et al., 2006]. The bottom of the model fault is defined by relocated seismicity in this region over the past few decades [Engdahl et al., 2007]. The fault plane is discretized using a total of 504 split nodes (28 and 18 nodes in the along-strike and downdip directions, respectively) on the plate interface. We partition our model into six regions with potentially different density and Young’s modulus (Table 1 and Figure 4a), with values chosen in accordance to data from seismic refractions in the Sunda trench and nearby fore-arc.
Figure 3. A 50 km wide, trench-perpendicular transect across the Nias Island and the Sumatra region. The circles denote the seismicity between 1964 and 2005 [Engdahl et al., 2007]. Grey lines indicate modeling curved faults used in finite element models. The black line denotes the fault geometry used in the work by Hsu et al. [2006]. The star is the hypocenter of the 2005 earthquake.

Figure 4. (a) A simplified heterogeneous elastic model in the Sumatra region. Six material properties (Table 1) are represented by different color blocks. The 3-D topographic relief is included in the model. (b) Magnified view of the blue box in Figure 4a. Numbers indicate ratios of Young’s modulus ($E/E_{\text{min}}$). Four elastic models are tested in this study.
We examine the sensitivity of source locations \( G \) and \( n \) We also test the sensitivity between source \((\text{km/s})\) Density \((\text{cm}^3/\text{g})\) The Preferable Material Model Constrained by Data From Seismic Refractions in the Sunda Trench and Nearby Fore-Arc Basins [Kieckhefer et al., 1980, 1981]. These blocks are meshed using hexahedral elements. The rupture area of the 2005 earthquake is divided into small hexahedra with a size of about 3.5 km × 3.5 km × 1 km, grading to a larger element size of 100 km × 100 km × 2 km in the far field. The final FEM model consists of 68412 nodes and 64662 elements (Figure 2b).

### 3. Sensitivity of Source Locations to Topography, Poisson’s Ratio, and Material Properties on Surface Displacements of Infinite 2-D Dipping Thrust Fault

\[ V_p \text{ (km/s)} \quad \rho \text{ (g/cm}^3) \quad \nu \quad G \times 10^{10} \text{ Pa} \quad E \times 10^{10} \text{ Pa} \quad \text{Ratio (E/E}_{\text{min}}) \]

<table>
<thead>
<tr>
<th>ID</th>
<th>Material</th>
<th>( V_p )</th>
<th>( \rho )</th>
<th>( \nu )</th>
<th>( G \times 10^{10} )</th>
<th>( E \times 10^{10} )</th>
<th>( \text{Ratio (E/E}_{\text{min}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Upper wedge</td>
<td>4.8</td>
<td>2.5</td>
<td>0.25</td>
<td>1.92</td>
<td>4.80</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>Lower wedge</td>
<td>6.6</td>
<td>2.7</td>
<td>0.25</td>
<td>3.92</td>
<td>9.80</td>
<td>2.0</td>
</tr>
<tr>
<td>3</td>
<td>Oceanic plate</td>
<td>8.0(^b)</td>
<td>3.0</td>
<td>0.25</td>
<td>6.40</td>
<td>16.00</td>
<td>3.3</td>
</tr>
<tr>
<td>4</td>
<td>Mantle</td>
<td>8.2</td>
<td>3.4</td>
<td>0.25</td>
<td>7.62</td>
<td>19.05</td>
<td>4.0</td>
</tr>
<tr>
<td>5</td>
<td>Upper crust</td>
<td>6.1</td>
<td>2.7</td>
<td>0.25</td>
<td>3.35</td>
<td>8.37</td>
<td>1.7</td>
</tr>
<tr>
<td>6</td>
<td>Lower crust</td>
<td>7.5</td>
<td>3.1</td>
<td>0.25</td>
<td>5.81</td>
<td>14.53</td>
<td>3.0</td>
</tr>
</tbody>
</table>

*aKieckhefer et al. [1980, 1981]. The index for each block is shown in Figure 4a with a magnified view in Figure 4b (model B). Abbreviations: \( V_p \), \( P \) wave velocity; \( \nu \), Poisson’s ratio; \( G \), shear modulus; \( E \), Young’s modulus; \( (E/E}_{\text{min}} \), the ratio of Young’s modulus to the minimum value.\n
\(^b\)Note that the \( V_p \) of 8.0 km/s may be too high for oceanic plate. If we use a value of \( V_p \) of 6.5 km/s for oceanic plate the resulting \( E \) (8.45 × 10\(^{10}\) Pa) is about half of its current value (16 × 10\(^{10}\) Pa), while our FEM tests with \( E = 8.45 \times 10^{10} \) Pa for oceanic plate show insignificant influences on the conclusions related to this change.

We examine the sensitivity of source locations to variations of topography, Poisson’s ratio, and material properties using a simplified 2-D plane strain model (Figure 2a). We use three different rupture models with unit dip slip applied to segments with depth ranges between 10 and 20 km, 20 and 30 km, and 30 and 40 km, respectively ((1)–(3) in Figure 2a). Two topographic models, including a flat top surface and a step function change of topography on the surface, are used to calculate surface displacements resulting from the three rupture sources (Figures 5a and 5b). The effect of topography on surface displacements is significant above the rupture zone and when the fault is closest to the free surface (Figures 5a and 5b). Because the primary change of surface topography is far from the rupture source, the differences of vertical displacements between two topographic models are small (Figure 5b). The influence of topography on vertical displacements is less than that in horizontal displacements. Additionally, we calculate surface coseismic displacements with Poisson’s ratio of 0.25 and 0.3, respectively. The results show that the difference between these two models is very small (Figures 5c and 5d). Surface displacements are not very sensitive to perturbations in Poisson’s ratio in substrata.

We also test the sensitivity between source locations and Young’s modulus. We estimate surface displacements of different rupture sources using two material models including a uniform and a heterogeneous elastic model (model B, Figure 4b). The surface displacements computed from a shallow rupture source and a heterogeneous elastic model are very different from those in a homogeneous model (Figures 5e and 5f) because of distinct changes of Young’s modulus at the depth range less than 30 km (Figure 2a). However, vertical
Figure 5
displacements are less affected by the complexity of the material heterogeneity at shallow depths. Note that if the source is steeply dipping, the vertical deformation would be more sensitive. These experiments suggest that the influences of topographic relief and material complexity on surface displacements depend on the position of rupture sources as well. To demonstrate this idea, we consider the constitutive equation for a linear elastic material in terms of Young’s modulus \(E\) and Poisson’s ratio \(\nu\), which may be written as

\[
\sigma_{ij} = \frac{E \varepsilon_{ij}}{(1 + \nu)(1 - 2\nu)} + \frac{E \varepsilon_{ij}}{(1 + \nu)}
\]

(1)

For plane strain conditions this yields

\[
\sigma_{xx} = \frac{\nu E (\varepsilon_{xx} + \varepsilon_{yy})}{(1 + \nu)(1 - 2\nu)} + \frac{E \varepsilon_{xx}}{1 + \nu}
\]

\[
\sigma_{yy} = \frac{\nu E (\varepsilon_{xx} + \varepsilon_{yy})}{(1 + \nu)(1 - 2\nu)} + \frac{E \varepsilon_{yy}}{1 + \nu}
\]

\[
\sigma_{xy} = \frac{E \varepsilon_{xy}}{1 + \nu}
\]

(2)

In the case of homogeneous material properties, the equilibrium equations in the absence of body forces are

\[
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0 = \frac{\nu E}{(1 + \nu)(1 - 2\nu)} + \frac{E \varepsilon_{xx}}{1 + \nu} \frac{\partial \varepsilon_{xx}}{\partial x}
\]

\[
+ \frac{\nu E}{(1 + \nu)(1 - 2\nu)} \frac{\partial \varepsilon_{xy}}{\partial x} + \frac{E \varepsilon_{xy}}{1 + \nu} \frac{\partial \varepsilon_{xy}}{\partial y}
\]

\[
\frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{yy}}{\partial x} = 0 = \frac{\nu E}{(1 + \nu)(1 - 2\nu)} + \frac{E \varepsilon_{yy}}{1 + \nu} \frac{\partial \varepsilon_{yy}}{\partial y}
\]

\[
+ \frac{\nu E}{(1 + \nu)(1 - 2\nu)} \frac{\partial \varepsilon_{xy}}{\partial y} + \frac{E \varepsilon_{xy}}{1 + \nu} \frac{\partial \varepsilon_{xy}}{\partial x}
\]

(3)

If we consider the case where Young’s modulus is no longer constant, but varies spatially \(E = E(x, y)\), these equations can be written as

\[
\frac{\nu E}{(1 + \nu)(1 - 2\nu)} + \frac{E \varepsilon_{xx}}{1 + \nu} \frac{\partial \varepsilon_{xx}}{\partial x}
\]

\[
+ \frac{\nu E}{(1 + \nu)(1 - 2\nu)} \frac{\partial \varepsilon_{xy}}{\partial x} = \frac{E \varepsilon_{xx}}{1 + \nu} \frac{\partial \varepsilon_{xx}}{\partial y}
\]

\[
\frac{\nu E}{(1 + \nu)(1 - 2\nu)} + \frac{E \varepsilon_{yy}}{1 + \nu} \frac{\partial \varepsilon_{yy}}{\partial y}
\]

\[
+ \frac{\nu E}{(1 + \nu)(1 - 2\nu)} \frac{\partial \varepsilon_{xy}}{\partial y} = \frac{E \varepsilon_{yy}}{1 + \nu} \frac{\partial \varepsilon_{xy}}{\partial x}
\]

(4)

In the case where Poisson’s ratio is no longer constant, but varies spatially \(\nu = \nu(x, y)\), these equations can be written as

\[
\frac{\nu E}{(1 + \nu)(1 - 2\nu)} + \frac{E \varepsilon_{xx}}{1 + \nu} \frac{\partial \varepsilon_{xx}}{\partial x}
\]

\[
+ \frac{\nu E}{(1 + \nu)(1 - 2\nu)} \frac{\partial \varepsilon_{xy}}{\partial x} = \frac{E \varepsilon_{xx}}{1 + \nu} \frac{\partial \varepsilon_{xx}}{\partial y}
\]

\[
\frac{\nu E}{(1 + \nu)(1 - 2\nu)} + \frac{E \varepsilon_{yy}}{1 + \nu} \frac{\partial \varepsilon_{yy}}{\partial y}
\]

\[
+ \frac{\nu E}{(1 + \nu)(1 - 2\nu)} \frac{\partial \varepsilon_{xy}}{\partial y} = \frac{E \varepsilon_{yy}}{1 + \nu} \frac{\partial \varepsilon_{xy}}{\partial x}
\]

(5)

Comparing the expressions where \(E\) and \(\nu\) are constant (equation (3)) with those where they are spatially variable (equations (4) and (5)), we see that terms associated with spatial variations of material properties on the right hand sides of equations (4) and (5) will act as if they were distributed sources. Thus, equations (4) and (5) show that inversions for earthquake slip parameters may be significantly biased when the material gradients and strain are large and the material gradients are not sufficiently well represented in the model.

Alternatively, we can examine the deformation in terms of shear modulus \(G\) and bulk modulus \(K\), which are often used to describe the material resistance to shearing strain (e.g., faulting) and to a change in volume (e.g., volcano deformation), respectively. The shear modulus \(G\) can be written as

\[
G = \frac{E}{2(1 + \nu)}
\]

(6)

According to the quotient rule in calculus

\[
\frac{\partial G}{\partial E} = \frac{1}{2(1 + \nu)} - \frac{E \varepsilon_{xx}}{2(1 + \nu)^2} \frac{\partial \varepsilon_{xx}}{\partial E}
\]

\[
\frac{\partial G}{\partial \varepsilon_{xx}} = \frac{2(1 + \nu)}{E} \left[ \frac{\partial E}{\partial \varepsilon_{xx}} - \frac{E \varepsilon_{xx}}{2(1 + \nu)^2} \frac{\partial \varepsilon_{xx}}{\partial E} \right]
\]

(7)

Assuming \(\nu = 0.25\), \(\frac{\partial G}{\partial E} = \frac{\partial E}{\partial \varepsilon_{xx}} - 0.2(\frac{\partial \varepsilon_{xx}}{\partial E})\), the same amount of perturbation in \(E\) can give rise to five times larger perturbation in \(G\) compared to that in \(\varepsilon_{xx}\). Since most earth materials have a value of \(\nu\) between 0.22 and 0.35, the same amount of perturbation in \(E\) has a larger contribution to \(G\) compared to that in \(\varepsilon_{xx}\).
On the other hand, the bulk modulus $K$ can be written as

$$ K = \frac{E}{3(1-2\nu)} $$  \hspace{1cm} (8)$$

According to the quotient rule

$$\frac{\partial K}{K} = \frac{\partial E}{E} + \frac{\partial \nu}{\nu} - \frac{\partial \nu}{1-2\nu} $$ \hspace{1cm} (9)$$

Assuming $\nu = 0.25$, $\frac{\partial K}{K} = \frac{\partial E}{E} + \frac{\partial \nu}{\nu}$, the perturbations in $\nu$ and $E$ have the same contribution in $K$. If $\nu = 0.35$, the same amount of perturbation in $\nu$ can cause 2.3 times larger perturbation in $K$ compared to that in $E$. These equations show that the variation of Poisson’s ratio may play an important role when $\nu$ is larger than 0.25 and the rupture source is dilatational such as volcano deformation.

### 4. Results and Discussion: 3-D FEM Models of the 2005 Nias–Simeulue Earthquake

In this section, we discuss different parameters in the finite element modeling of the 2005 earthquake and summarize all tests, their respective goals, and numbers of related figures in Table 2.

#### 4.1. Impact of Topography and Poisson’s Ratio on Surface Displacements

Inversions on fault slip distributions using elastic half-space or layered earth models often assume that the top surface is flat. However, topographic relief near the Sunda subduction zone can be as large as 6 km (Figure 3). To examine the effect of topographic relief on coseismic surface deformation of the 2005 earthquake, we compare surface coseismic displacements from a model with a flat surface to one with more realistic topographic relief, both using the same input slip model [Hsu et al., 2006]. The predicted horizontal surface displacements from the model with realistic topography are generally larger than those in a flat earth model (Figure 6b). Predicted vertical displacements from the two models show systematic differences in predicted surface displacements as a function of distance from the Sunda trench (Figure 6e). The model with realistic topography shows that the actual depth of the top surface is deeper on the western side of the Sunda trench and shallower on the eastern side compared to a flat earth model (Figures 6d and 6e). Therefore, predicted vertical displacements are underestimated and overestimated to the west and east sides, respectively, of the trench in the flat earth model. We do not observe...
significant systematic difference in horizontal displacements between these two models, which is possibly due to 3-D variations of coseismic slip along the trench resulting in a more complex horizontal displacement field compared to the vertical displacement. In summary, the maximum difference between two models is about 5% and mainly localized near the trench where the topographic gradient is highest (Figures 6b and 6e). While not large, these differences are sufficiently systematic that for all subsequent models, we use realistic estimates of topographic relief.

4.2. Surface Displacements Computed From Heterogeneous Elastic Models

To test the effect of 3-D elastic heterogeneity, we partition our model into six regions with values of density and Young’s modulus (Table 1 and Figure 4a) chosen in accordance with data from seismic refractions in the Sunda trench and nearby fore-arc basins [Kieckhefer et al., 1980, 1981]. Here, we use the ratio of Young’s modulus to the minimum value in the heterogeneous elastic model.
The material model $B$ is our preferred model (Figure 4b). We test three different heterogeneous models with increasing contrast between the upper accretionary wedge and the rest of the structural blocks (Figure 4b).

[17] To begin with, we use the same coseismic slip model [Hsu et al., 2006] and the fault geometry (fault $B$ in Figure 3) to estimate surface displacements. The directions of surface horizontal displacements in all heterogeneous elastic models (models $A$–$C$ in Figure 4b) are not much different from those in the homogeneous model (model $HS$ in Figure 4b). However, magnitudes in models $A$–$C$ are amplified due to soft unconsolidated sediments in accretionary wedge. The vertical surface displacements are less affected by soft layer at shallow depths. The differences in maximum surface displacements between model $HS$ and model $A$, model $HS$ and model $B$, and model $HS$ and model $C$ are about 15%, 35%, and 40%, respectively (Figure 7). Note that differences of surface displacements between homogeneous and heterogeneous models vary significantly depending on material properties.

4.3. Inverted Coseismic Slip and Fault Geometry in a Simplified Earth Model Using Synthetic Displacements From a Heterogeneous Elastic Model

[18] Elastic half-space and layered earth models are commonly used in studies of crustal deformation. The preceding tests beg the question of how well a half-space model and a layered earth model can fit surface displacements estimated from a complex 3-D elastic structure. We estimate the synthetic surface displacements on CGPS and coral sites using fault $B$ (Figure 3), material model $B$ (Figure 4b), realistic topography, and estimated coseismic slip distribution from Hsu et al. [2006]. The errors in synthetic surface displacements are assigned to be the observational errors of coseismic displacements. However, we reduce the amplitude of observational errors by a factor of 4 in coral sites to obtain a better fit in vertical displacements. After adjustments, the average standard deviation in CGPS and coral sites are 4.3 and 6 mm, respectively.

[19] We then use elastic half-space and layered earth models to invert for the optimal slip distribu-
tion and the fault geometry that can best fit synthetic displacements. The layered elastic model used here is a simplified version of model B (Figure 4b), which is composed of three layers. The reduced chi-square value ($\chi^2_r$) is used to evaluate the model fit. A value of 1 means that the model fits the data within uncertainties. By searching a wide range of fault geometries, we cannot find a satisfactory fit to the synthetic displacements using simplified earth models. The values of reduced chi-square in all testing fault modes are larger than 7 in an elastic half-space model and larger than 9 in a layered earth model. We define the optimal fault geometries as those resulting in reduced chi-square values ranging from 7 to 8 in an elastic half-space model (Figure 8) and 9 to 10 in a layered earth model (Figure 9). The modeling results show various fault geometries with similar fits to synthetic surface displacements.

Figure 8. Inverted coseismic slip and fault geometry in a half-space earth model using synthetic displacements from a heterogeneous elastic model (a) The input fault model (red curve) and a variety of testing fault models (black curve). The optimal models with values of reduced chi-square of 7–8 are shown in blue curves. (b) Slip distribution on the fault, $F_{HS}$, in Figure 8a is shown in color. The black and blue vectors show the observed and predicted horizontal GPS displacements. (c) Residuals of the vertical displacements. Red and blue circles indicate predicted values are larger and smaller than observations, respectively. (d) The difference in coseismic slip between Figure 8b and the input slip model.
models tend to underestimate the actual depth of the real fault.

We choose one fault model, $F_{HS}$ (blue curve indicated in Figure 8a), which is closest to the input fault model, and compare the slip distribution and predicted surface displacements with the input slip and surface displacements (Figures 8b–8d). We find predicted horizontal displacements from the elastic half-space model can reasonably fit synthetic displacements (Figure 8b). However, residuals in the vertical displacements show a systematic misfit (Figure 8c). On the other hand, predicted surface displacements using a fault model, $F_{layer}$ (Figure 9a) and a layered earth structure also show similar features as those in Figure 8.

We find that none of these inferred slip distributions are close to the input slip model. Models with simplified Green’s functions apparently map...
Table 3. Values of Weighted Root-Mean-Square Misfit, Maximum Coseismic Slip, Average Slip, and Coseismic Potency of All Models

<table>
<thead>
<tr>
<th>Fault A</th>
<th>Fault B</th>
<th>Fault C</th>
</tr>
</thead>
<tbody>
<tr>
<td>HS</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>S_{max} (m)</td>
<td>10.2</td>
<td>12.3</td>
</tr>
<tr>
<td>S_{mean} (m)</td>
<td>1.1</td>
<td>0.9</td>
</tr>
<tr>
<td>Potency (km^2 × m)</td>
<td>164,860</td>
<td>152,700</td>
</tr>
</tbody>
</table>

Optimal Model Determined by Cross Validation

<table>
<thead>
<tr>
<th>wrms (m)</th>
<th>0.0552</th>
<th>0.0691</th>
<th>0.0810</th>
<th>0.0895</th>
<th>0.0612</th>
<th>0.0649</th>
<th>0.0665</th>
<th>0.0678</th>
<th>0.0697</th>
<th>0.0668</th>
<th>0.0648</th>
<th>0.0629</th>
</tr>
</thead>
<tbody>
<tr>
<td>S_{max} (m)</td>
<td>10.7</td>
<td>12.0</td>
<td>11.7</td>
<td>11.7</td>
<td>11.5</td>
<td>10.6</td>
<td>9.9</td>
<td>9.9</td>
<td>12.0</td>
<td>11.2</td>
<td>10.5</td>
<td>10.2</td>
</tr>
<tr>
<td>S_{mean} (m)</td>
<td>1.0</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9</td>
<td>0.8</td>
<td>0.8</td>
<td>1.3</td>
<td>1.1</td>
<td>1.0</td>
<td>0.9</td>
</tr>
<tr>
<td>Potency (km^2 × m)</td>
<td>148,790</td>
<td>156,720</td>
<td>144,370</td>
<td>140,850</td>
<td>168,410</td>
<td>159,560</td>
<td>140,990</td>
<td>136,030</td>
<td>198,180</td>
<td>184,220</td>
<td>162,800</td>
<td>151,860</td>
</tr>
</tbody>
</table>

*Material models: HS, A, B, and C. Abbreviations: S_{max}, maximum coseismic slip; S_{mean}, average coseismic slip; potency, product of slip and slip area; wrms, weighted root-mean-square misfit.

4.4. Coseismic Slip Distributions Using FEM Derived Elastic Green’s Functions

As found previously [Masterlark et al., 2001; Masterlark, 2003] and as just demonstrated, surface displacements are significantly influenced by 3-D variations of material properties. To evaluate the effect of heterogeneous material properties on the inferred spatial distribution of coseismic slip, we use the fault geometry B in Figure 3 and realistic topography to compute 3-D Green’s functions at fault slip node pairs. The displacements on the surface resulting from a unit dislocation of a fault split node pair are used to interpolate displacements at all CGPS and coral sites. We then create Green’s functions composed of displacement components at each site. In this experiment, there are five versions of Green’s functions including a homogeneous model (model HS, Figure 4), a layered earth model from earth structure CRUST2.0 [Bassin et al., 2000], as well as three heterogeneous elastic models (models A, B, C; Figure 4b). We invert for coseismic slip using the same inversion scheme as Hsu et al. [2006], a weighted least squares algorithm with two forms of regularization: Laplacian smoothing and moment minimization. Because characteristics of the inferred models differ as a function of model roughness, data misfits and model solution length, we compare models with the same weighted root-mean-square misfit (wrms, listed in Table 3).

The differences in inferred coseismic slip for the layered model and the other heterogeneous models (Figure 4b) relative to model HS are shown in Figure 10. In general, the fits of horizontal displacements improve if the contrast between the upper accretionary wedge and the rest of the structural model is increased. In all models, we find systematic misfits in vertical displacements corresponding to the features observed in our synthetic tests (Figures 8c and 9c). These systematic misfits might imply that the elastic contrast in geological material models used in this study needs to be adjusted to obtain a better fit to the vertical displacements or more complex fault geometry is required. An alternative source of systematic vertical misfit is possibly associated with large observational errors in coral measurements compared to those in horizontal GPS displacements.

The maximum coseismic slip decreases with increases in rigidity contrast between the upper wedge and surrounding material blocks (Table 3 and Figure 10). For instance, the maximum coseismic slip of 10.1 m in model C is 10% less than 11.2 m in model HS. Figure 10 shows differences of coseismic slip in various elastic models relative to that in a homogeneous model (model HS). Discrepancies in fault slip between model HS and other material models get bigger when the elastic contrast between the two sides of the fault is increased. Compared to model HS, the coseismic slip derived from a layered earth structure, CRUST2.0 [Bassin et al., 2000], is less as illustrated in Figure 10a. The weak geological material on the top layer is likely to amplify surface displacements hence the layered
and heterogeneous elastic models require a small amount of slip on the fault (Figure 10 and Table 3). Coseismic slip close to the trench is generally small in heterogeneous material models, in contrast to significant updip slip from the coseismic rupture in model HS (Figure 10). We find heterogeneous models (models A, B, C) require less updip slip and downdip slip compared to the homogeneous model (model HS). Total coseismic potency in model C decreases by 20% compared to that in model HS (Table 3). This feature suggests that estimates on geodetic moment or stress variations at depth may be biased by the assumption of a uniform earth.

4.5. Coseismic Slip Distributions Derived From Various Fault Geometries

Finally, we compare optimal coseismic slip distributions inverted from different combinations of fault geometries (Figure 3) and heterogeneous elastic models (Figure 4b). All these models incorporate regional topographic variations. We choose damping parameters using cross validation and do not constrain values of \( 	ext{wrms} \) to be the same.

We use three different candidate fault geometries (faults A–C in Figure 3) to address the relationship between coseismic slip and 3-D elastic heterogeneity. Integrated coseismic potency decreases with the increasing rigidity contrast between the upper accretionary wedge and surrounding materials independent of fault geometry (Table 3). The rupture area is narrower in heterogeneous material models, models A–C, than in model HS, which is also a common feature regardless of fault geometry. However, the maximum coseismic slip does not always decrease with increasing rigidity contrast across the fault. For example, inferred maximum coseismic slip on fault A in model C is larger.
than that in model HS, which is opposite to the results using fault geometries of fault B or fault C (Table 3). The peak slip at depth differs between the various assumed fault geometries. Not surprisingly, a shallow fault requires less coseismic slip to fit surface displacements compared to a deep fault.

To find the optimal coseismic model from a combination of various fault geometry and 3-D elasticity, we choose the damping parameter according to cross validation. Details of optimal coseismic models, fault geometries, and material properties are listed in Table 3. We note that values of wrms in 3-D elastic structure models are similar to those in the homogeneous model except for fault A. The fit to surface observations in the 3-D FEM model is similar to that in a simple half-space model. This similarity possibly results from the fact that the faults considered in our study are constructed according to the fault model determined from joint inversions of geodetic and seismic data with the assumption of a layered elastic earth model [Hsu et al., 2006]. We find that none of the fault models listed in Table 3 can fit the observed data within uncertainties. Due to limitations in our knowledge of real fault geometry and 3-D elastic structure, it is not reasonable to evaluate modeling results only by the fit to the data. The coseismic model using the fault geometry B (Figure 3) and the material model B (Figure 4b), is constrained by data from seismicity and seismic refractions. Although the fit to surface observations is similar to the result from a simple half-space model, the resulting slip distribution may reflect the true fault slip behavior with more fidelity. Given these results, we believe that the best strategy is to constrain fault geometries and 3-D elastic structures based on a priori information. Then we can perturb fault geometries or material properties using inversions. However, this procedure is computationally challenging when we require a new finite element mesh for each fault geometry.

5. Conclusion

We explored the impact of adopting a realistic earth model on predictions of surface deformation using 3-D finite element modeling techniques. Surface displacements are more sensitive to variations in Young’s modulus than to variations in topography and Poisson’s ratio. The influence of 3-D variations in elastic properties and topography on surface displacements is significant if the fault slip (or more precisely, gradients in fault slip) occur close to gradients in rigidity or topography, with the free surface being an extreme example of this case.

If we generate synthetic displacements using a heterogeneous elastic model and then use an elastic half-space or layered earth model to estimate the slip distribution, we are unable to find any coseismic slip distribution that fits the synthetic surface GPS observations within the uncertainties. The coseismic potency and the centroid depth of the fault are underestimated, with systematic residuals in vertical displacements for the simplified earth models. The coseismic potencies resulted from slip models with the same misfits are less in all tested heterogeneous elastic models compared to that in a half-space model based on the same fault geometry.

Using our preferred 3-D heterogeneous elastic model to study the coseismic deformation of the 2005 Sumatra earthquake, we find that the fit to the surface displacements is similar to that of an elastic half-space model. However, our model takes into account available observations of elastic properties near the rupture area and may reflect a more realistic distribution of fault slip. We find that direct inversion of the fault geometry and slip distribution from GPS and coral data is generally a poorly constrained problem. Moreover, the remaining misfit implies that the true elastic structure and/or fault geometry might be more complex than our preferred model or that there are other deformation mechanisms taking place during earthquake ruptures, such as slip on splay faults or inelastic deformation.

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References


