Fluvial fan deltas: Linking channel processes with large-scale morphodynamics

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[1] Alluvial fan deltas are built by aggrading and avulsing river channels. A numerical model of flow and sediment transport in channels is combined with a condition for channel avulsion to describe evolution of the fan surface. The model combines elements of two recent approaches: (1) diffusional models of depositional basin filling, which describe the evolution of overall morphology, and (2) cellular models of channel evolution, which provide simplified descriptions of channel dynamics. Here the cellular formulation is used simply as a template for discretizing a standard set of equations describing flow and sediment transport within channels. The basic formulation is thus grid-invariant, and the results are in actual rather than arbitrary time and length scales. The model captures the interaction between microscale (channel) morphodynamics, including converging and diverging flow and avulsion, and morphodynamics at the macroscale (fan). It provides a starting point for the study of a wide range of topics, including the three-dimensional stratigraphy of fan delta deposits and the response of fan deltas to engineering activities. INDEX TERMS: 1824 Hydrology: Geomorphology (1625); 1860 Hydrology: Runoff and streamflow; 1815 Hydrology: Erosion and sedimentation; 1821 Hydrology: Floods; KEYWORDS: fans, fan deltas, avulsion, sedimentation, rivers

1. Introduction

[2] Fluvial fan deltas are fan-shaped zones of sediment accumulation ending in standing water. The fan delta of the Yellow River, China, is shown in Figure 1 [Pang and Si, 1980]. Sediment from upstream causes channels to aggrade and then migrate or avulse to lower elevations. A repeat of these events allows the surface to develop a large-scale morphology in the shape of a fan, a slope that declines toward standing water and a prograding front. Progradation can be arrested by tectonic subsidence or rising base (e.g., sea) level.

[3] Quantitative modeling of fans and fan deltas can be traced back to the work of Price [1972], Kenyon and Turcotte [1985], and Tetzlaff and Harbaugh [1989]. Here two recent techniques for morphodynamics are joined to develop a numerical model of fluvial fan deltas that links small-scale channel dynamics and large-scale fan morphodynamics. The first of these is embodied in diffusional basin-filling models [e.g., Paola, 1989; Paola et al., 1992]. Such models describe the flow and sediment transport in channels with descriptions of mass and momentum balance combined with standard formulations for sediment transport and resistance. When combined with (1) the approximation of normal flow to describe momentum balance within channels and (2) a criterion for bank-full channel geometry in terms of a dimensionless Shields stress, the Exner equation of sediment continuity in such models inevitably reduces to the form of a linear or nonlinear diffusion equation with a sink term for subsidence and a moving boundary at the distal end [e.g., Swenson et al., 2000]. The details of the processes of channel avulsion and migration are, however, replaced with long-term averaging whereby channel deposits are spread uniformly across basin width. The linkage between small-and large-scale morphodynamics is thus rather broad brush. Such a model has been adapted to axisymmetric fan deltas [Parker et al., 1998a] and applied to a mine-tailing basin [Parker et al., 1998b]. A somewhat similar model has also been presented by De Chant et al. [1999].

[4] There are, however, questions concerning fan evolution that cannot be answered by such models but are amenable to the second technique, i.e., cellular models of flows in channel networks. First proposed in morphodynamics to study drainage basin evolution [e.g., Willgoose et al., 1991; Howard, 1994; Sun et al., 1994], they describe the evolution of large-scale morphology using rules allowing small-scale cells to interact so as to describe channel processes. Barzini and Ball [1993] applied the cellular format to a study of landscape evolution in flood, and Murray and Paola [1994] used it to achieve the first model of braided channels in which the pattern is both self-forming and self-maintaining. An early work on fans which serves as a precursor to cellular formulations is that of Price [1972]. In some cellular models, however, the “rules” describing cell interaction are simplifications involving arbitrary measures of time and space and

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“tuned” parameters. Such implementations, while useful in exploratory studies, can suffer from one or more of the following limitations: the rules (1) are not dimensionally homogeneous, (2) do not provide rigorous scaling, and (3) are grid specific.

[5] The present approach combines elements of both diffusional and cellular approaches. It provides a simplified, scale-independent description of fan deltas that overcomes the above limitations. The resulting model is used in a first study of the effect of changing water discharge, changing base level, and tectonics on fan morphology at the scale of the fan as well as the scale of the channels which construct it.

[6] For the purpose of the present analysis, a fan delta is defined to be any deltaic deposit that develops a fan-like shape due to avulsion. It should be pointed out in advance, however, that the present form of the model does not account for overbank deposition of sediment. As a result, it is not applicable to deltas within which most of the sediment has been deposited by overbank flow.

2. Model for Flow and Sediment Transport in Channels

[7] Here the formulation is simplified as much as possible in order to study the essential features of fluvial fan deltas. Many of these simplifications can be relatively easily relaxed in a more detailed study.

[8] The computational domain is a square of size $L_x$ initially containing water ponded to elevation $z_p$ above a horizontal bed. Water and sediment are introduced at rates $Q_w$ and $Q_{sf}$ at one vertex of the basin, allowing a fan to prograde into the domain. Base level $z_p$ is either set constant or allowed to vary in a prescribed way that is independent of the growth of the fan itself. The sediment supplied to the fan is assumed to be uniform with size $D$. Compaction is neglected so that the deposit porosity $\lambda$ remains constant. Groundwater losses are neglected.

[9] Flow in channels is approximated as steady and uniform, i.e., satisfying the normal flow approximation. A justification of this assumption is given by Cui and Parker [1997], who demonstrate that the normal flow approximation becomes asingly justified as the length scale of the morphology in question (the fan delta in the present case) becomes increasing large compared to a characteristic flow adaptation length (known as the backwater length in the case of subcritical flow). The approximation holds for subcritical as well as supercritical flow as long as the Froude number is not too small. It does, however, break down in a zone near the water’s edge of a fan delta, where the present model may not be accurate.

[10] The hypothesis of steady, uniform flow is here assumed to apply only intermittently, i.e., during flood events [Paola, 1989]. That is, the fan is considered to be morphologically inactive for long periods of time but occasionally subjected to flood flows, each of which is considered to be a single sustained event during which water and sediment discharge are temporally constant. Here the intermittency is not included explicitly, so that a calculation time span of, say, 100 years may correspond to thousands or tens of thousands of actual years. At any given time, the fan may contain more than one active channel. During floods the flow condition in the channel(s) on the fan is approximated as bank-full. The model includes tectonic effects; the computational domain may be subject to subsidence, uplift, or tilt.

[11] All the water and sediment are carried in channels which aggrade and avulse across the fan. The relations governing flow and sediment transport within channels are the physically based, dimensionally homogeneous relations given by Parker et al. [1998a]. Let $Q_w$, $Q_s$, $B$, $H$, $U$, $u_*$, $q_s$, and $S$ denote the volume water and sediment discharges within a channel, channel width, depth, velocity, shear velocity, volume sediment transport per unit width, and down channel bed slope, respectively. Three conservation relations apply to flow within the channels. The equation of water continuity takes the form

$$Q_w = BHU. \tag{1}$$

The equation of conservation of sediment mass in the flow takes the volumetric form

$$Q_s = Bq_s. \tag{2}$$

The normal flow approximation of the one-dimensional (1-D) equation of momentum conservation of the flow is

$$u_* = gHS, \tag{3}$$

where $g$ denotes gravitational acceleration.

[12] In addition to conservation relations, it is necessary to specify constitutive relations describing the dynamics of river flow. Flow resistance can be described in terms of a generalized dimensionless Manning-Strickler relation [e.g., Parker et al., 1998a]:

$$U \over u_* = \alpha_r \left( {H \over D} \right)^p, \tag{4}$$

where $\alpha_r$ is a dimensionless resistance coefficient and $p$ is a dimensionless exponent. The Chezy relation can be recovered from equation (4) by setting $p = 0$, whereas the
Manning relation is recovered for \( p = 1/6 \). A generalized sediment transport relation can be specified as

\[
q_s = \frac{\alpha_s \alpha_{so}}{\sqrt{RgD}} (\tau^* - \tau^*)^n. \tag{5}
\]

In equation (5), \( R \) denotes the submerged specific gravity of the sediment, given by the relation

\[
R = \frac{\rho_w}{\rho_s} - 1, \tag{6}
\]

where \( \rho \) denotes the density of water and \( \rho_s \) denotes the density of sediment. In the present implementation, \( R \) is set equal to 1.65. In addition, \( \tau^* \) denotes the dimensionless Shields stress, given by the relation

\[
\tau^* = \frac{u_{*}^2}{RgD}, \tag{7}
\]

and \( \tau^*_b \) denotes a critical Shields stress below which sediment transport does not occur. The coefficient \( \alpha_{so} \) refers to sediment transport in a wide rectangular flume, and \( \alpha_{so} > 1 \) denotes an order one correction for natural channels [Brownlie, 1981; Paola, 1996, Paola et al., 1999]. Finally, the term on the left-hand side of equation (5) is the Einstein number describing dimensionless sediment transport rate.

[13] Sediment transport relations that can be placed directly in the form of equation (5) include that of Meyer-Peter and Muller [1948] for gravel bed streams and that of Engeland and Hansen [1972] for sand bed streams. In addition, many other relations, such as that of van Rijn [1984], can be locally approximated in the form of equation (5).

[14] The final relation needed to close the problem is here formulated as an approximate relation for Shields stress at bank-full flow, here called a relation for channel definition. The relation is formulated as

\[
\tau^* = \frac{\tau^*_b}{R}, \tag{8}
\]

where \( \tau^*_b \) and \( \alpha_b \) are appropriately defined parameters that are approximated as constants [Parker et al., 1998a; Dade and Friend, 1998]. In the numerical calculations presented below, \( \tau^*_b \) is set equal to 1.8, i.e., the average value recommended for sand bed streams with self-formed channels of Parker et al. [1998a].

[15] Relations (1)–(5) and (8) can be used to obtain the following solutions for \( B, H \) and \( Q_s \), as functions of \( S \) and \( Q_{sw} \):

\[
B = \alpha_b^{-(3+2p)/2} \alpha_{so}^{-1} S^{1+p} \left( \frac{Q_s}{\sqrt{gD}} \right), \tag{9a}
\]

\[
H = \frac{\alpha_b}{S}, \tag{9b}
\]

\[
\frac{Q_s}{\sqrt{RgD D^2}} = \alpha_{so} \alpha_{so}^{-3+2p/2} \alpha_{so}^{-1} \left( \frac{\alpha_b}{R} - \tau^* \right)^{n-p} \left( \frac{Q_s}{\sqrt{gD D^2}} \right). \tag{9c}
\]

The above relations are (1) physically based and (2) dimensionally homoeous. They are used here as a subset of the “rules” of the cellular model. Their derivation is described in more detail by Parker et al. [1998a]. In essence, however, they describe generalized, physically based relations for channel geometry and sediment transport.

### 3. Cellular Discretization: Channel Interaction and Avulsion

[16] The square computational domain of size \( L_b \) is discretized into \( N \times N \) square cells, each of size \( a = L_b/N \). The elevation of the fan surface is represented by the heights associated with each of the cells. Every active channel within this domain is represented by marked bonds between neighboring (nearest neighbor or next nearest neighbor) cells. A marked lattice bond becomes unmarked when the flow abandons it, and an unmarked bond become marked as flow avulses into it. Water and sediment may flow from one cell into any of the four nearest neighbors and four diagonal neighbors that are marked. Slope \( S_{ij} \) between cell \( i \) and its neighboring cell \( j \) is equal to \((\eta_i - \eta_j)/L_{ij} \), where \( \eta \) is bed elevation and \( L_{ij} = a \) for nearest neighbors and = \( \sqrt{2} \) \( a \) for diagonal neighbors. The magnitude of the discharge from cell \( i \) to cell \( j \) is \( Q_{wij} \). Water conservation requires that

\[
\sum_j \text{sgn}(i, j) Q_{wij} = 0 \tag{10}
\]

where \( \text{sgn}(i, j) = 1 \) for flow from \( i \) to \( j \) and \(-1 \) otherwise.

[17] As channels aggrade, they avulse and branch. At each branching cell \( i \) the discharge flowing out of the cell into a branch is assumed to be proportional to \( S_{ij}^\gamma \) from point \( i \) to neighboring point \( j \) of the branch where \( \gamma = 0.5 \):

\[
Q_{wij} = \left( \sum_{k \text{ with flow}} Q_{wki} \right) \frac{S_{ij}^\gamma}{\sum_{k \text{ with flow}} S_{ik}^\gamma}. \tag{11}
\]

where the admissible values of \( j \) and \( k \) pertain to neighboring cells that receive flow from cell \( i \). Equation (11) is adapted from Murray and Paola [1994]; the choice of \( \gamma = 0.5 \) is motivated by the corresponding power in the Manning-Strickler or Chezy resistance relations.

[18] Once the discharge \( Q_{wij} \) and bed slope \( S_{ij} \) have been obtained for channels connecting all cells containing flow, channel width \( B_j (<a) \), depth \( H_j \) and sediment transport rate \( Q_{swj} \) of the channel between cells \( i \) and \( j \) are computed from equations (9a)–(9c). The bed elevation of the \( i \)th cell is then changed in accordance with the Exner equation of conservation of bed sediment, which takes the discretized form

\[
\Delta \eta_i = -\sum_j \text{sgn}(i, j) Q_{wij} \Delta t, \tag{12}
\]

where \( \lambda \) denotes bed porosity and \( \Delta t \) denotes the time increment.

[19] In the present simplified model, channels are allowed to avulse but not shift gradually. In accordance with experimental results of Bryant et al. [1995] that show that avulsion frequency increases with sedimentation rate, the following formulation is used. An avulsion originating from
cell $i$ out of neighboring cell $j$ and into neighboring cell $k$ is initiated if the following inequality is satisfied:

$$\frac{(\eta_i - \beta H_j) - \eta_k}{L_{ik}} > S_{ij}. \quad (13)$$

Here $\beta$ is an order one constant. This relation is perhaps best explained in the context of avulsion from one nearest neighbor $j$ to another nearest neighbor $k$, in which case it takes the form

$$\eta_k < \eta_j - \beta H_j. \quad (14)$$

That is, avulsion from $j$ to $k$ occurs if $\eta_k$ is lower than $\eta_j$ by an amount scaling with (bank-full) depth. While equation (13) is empirical in nature, it is dimensionally homogeneous. In addition, the observational evidence of Mohrig et al. [2000] on avulsion of sand bed paleochannels in Spain support the concept that the condition of incipient avulsion should be formulated in terms of a channel bed that has aggraded to be some order one fraction of bank-full depth above the surrounding topography. In particular, the data of Mohrig et al. [2000] suggest a value of $\beta$ between 0.5 and 1.

[20] Once avulsion occurs, a new channel is generated from the point of avulsion. In the simplest possible implementation, this new channel would follow the path of steepest descent. Here a random choice is made that weights the new channel toward this path but allows for statistical deviation according to a normal function. In particular, the direction $j$ in which the new channel will extend from $i$ is chosen randomly with a probability $p_{ij}$ given by

$$p_{ij} = \frac{S_{ij} \exp\left(-\frac{(\delta \theta_j - \theta_j)^2}{2\sigma_j^2}\right)}{\sum_j S_{ij} \exp\left(-\frac{(\delta \theta_j - \theta_j)^2}{2\sigma_j^2}\right)}. \quad (15)$$

In the above equation the permissible values of $j$ correspond to cells neighboring $i$ which have bed elevation $\eta_j < \eta_i$; the same limitation holds on the summation in $j$. The parameter $\delta \theta_j$ denotes the angle of deviation of the straight line from $i$ to $j$ from the straight line from cell $i$ to the cell immediately downstream before avulsion. In the present cellular implementation, $\delta \theta_{ij}$ may only take the values $\pm 0$, $\pm \pi/4$, $\pm 2\pi/4$, and $\pm 3\pi/4$. Finally, the parameter $\theta_0$ determines the standard deviation of the probability function. The gross morphology of the fan itself, however, is not too sensitive to the choice of $\theta_0$. The extension of a new channel is continued until it (1) meets another existing channel, (2) reaches the standing water at the distal end of the fan, or (3) runs into a local minimum and forms a lake.

[21] Equation (15) is applied when the space between the avulsion site and the channel convergence site consists of more than one cell. If the space between the avulsion site and the channel convergence site is one cell, equation (15) is no longer needed, as a direct link can then be formed from the avulsion site to the channel convergence site.

[22] The choice of the functional form of equation (15) is based on a simple Gaussian assumption. It is interesting to note, however, that in the relatively flat, for

| Table 1. Dimensionless Parameters Used in the Model |
|----------------|----------------|
| Parameter | Value |
| $\omega_{sa}$ | 11.25 |
| $\omega_{sr}$ | 1 |
| $\omega_s$ | 15 |
| $\tau_a$ | 2.97 |
| $\tau_b$ | 0 |
| $n$ | 2.5 |
| $p$ | 0 |
| $R$ | 1.65 |
| $\gamma$ | 0.5 |
| $\lambda$ | 0.4 |
| $\beta$ | 1 |

which the values of $S_{ij}$ vary little among all possible choices of $j$, equation (15) becomes identical with the probability density function associated with “most probable path” theory [von Shelling, 1950]. The curve geometry predicted from a simplified version of the most probable path is the well-known sine-generated curve [Leopold and Wolman, 1957; Langbein and Leopold, 1966]. Here the parameter $\theta_0$ is set equal to $\pi/(2\sqrt{2})$ so as to correspond to the family of sine-generated curves studied by Sun [1998]. As noted above, however, the results of the predictions for fan morphology are not strongly dependent on the choice of $\theta_0$.

[23] Avulsion does not imply immediate abandonment of the original channel. Both continue to flow, although one may be eventually abandoned. After each erosion or sedimentation step, the network configuration is examined to determine if any marked bond representing a segment of channel no longer carries water flow and/or if any channel segment satisfies the condition for initiation of a new avulsion. The corresponding bonds on the lattice are then relabeled. This picture is consistent with the field observations of Smith et al. [1989] and Smith and Perez-Arlucea [1994]. The delta front advances into standing water by filling at channel termini.

4. Model Implementation

[24] The model was implemented using dimensionless parameters that for the most part correspond to the case of the sand fan in the diffusional basin-filling model of Parker et al. [1998a]. The dimensionless parameters used in the model are specified in Table 1. The parameters $\omega_{sa}$, $\omega_s$, $\tau_a$, $\tau_b$, $n$, and $p$ specified in Table 1 correspond precisely to those used in the sand fan of Parker et al. [1998a]. In particular, the choices of $\omega_{sa}$, $\omega_s$, $\tau_a$, $\tau_b$, $n$, and $p$ correspond to a Chezy resistance relation and the sand transport relation of Engelund and Hansen [1972]. The parameter $\omega_{sr}$, which is set equal to 1.5 in the work of Parker et al. [1998a], is here set equal to unity for the sake of simplicity. The submerged specific gravity $R$ of the sediment is assumed to be equal to 1.65, and the porosity $\lambda$ of the sediment deposit is assumed to be 0.4; both values are identical to that used by Parker et al. [1998a]. As noted previously, the parameter $\gamma$ is set equal to 0.5. Finally, owing to the lack of more precise information the parameter $\beta$ associated with avulsion is set equal to 1.

[25] The dimensioned input parameters used in this implementation are given in Table 2. In the base case defined there, the length of the calculational domain $L_b$ is...
assumed to be 10 km, and cell size \( a \) is assumed to be 100 m. The value of \( a \) must be chosen to be larger than the width of the widest channel in the calculation. The basement is assumed to be flat and horizontal, and the depth of ponded water \( x_p \) is held constant at 2 m. In the base case the water discharge \( Q_w \) of 20 m\(^3\)/s, the volume sediment feed rate \( Q_{sf} \) of 0.04 m\(^3\)/s, and the grain size \( D \) of 0.3 mm all take values identical to those for the sand fan of Parker et al. [1998a].

In addition to the base case, several variant cases are considered as well, including increased water discharges \( Q_w \) of 40 and 80 m\(^3\)/s and an increased depth of ponded water \( x_p \) of 7 m. In addition, tectonic tilting is considered as well, as described below.

It should be recalled that no flow intermittency is used for the calculations presented here. As can be seen from the above description, however, wherever possible (except in the case of \( a_{sa} \)), dimensionless parameters have been equated to those of the sand fan of Parker et al. [1998a], and the base case again corresponds to the sand fan of Parker et al. In Parker et al. a flow intermittency, i.e., fraction of time for which the fan delta is morphologically active, was set equal to 0.05. With this value a time span of 100 years of (morphologically active) time in the present model translates to the elapse of 2000 years of actual time.

Figure 2 shows the state of the fan delta under the conditions of the base state after an elapsed time of 168 years from the time when water and sediment are first introduced into one vertex of the basin. Figure 2a shows the distributary network of channels that are active at that time. The widths of the line segments denoting the channels are proportional to channel width. The color bar provides a representation of the downstream variation in volume sediment concentration in each channel. In Figure 2b the overall topography of the fan delta is illustrated in terms of a contour map. The deviation of surface elevation from the average surface elevation along a line of constant radial coordinate is illustrated in terms of the color bar. The overall resemblance between Figures 1 and 2b is strong, although it is clear that the model is too simple to provide an accurate description of the evolution of channel termini in standing water.

Figure 3 shows the evolution of channels on a fan in which the water discharge alone has been altered to 40 m\(^3\)/s. The higher discharge results in fewer, wider channels. The channel configurations at six different times from the commencement of water and sediment supply to one vertex are shown. The channels labeled 1, 2, 3, 4, 5, and 6 correspond to years 138, 147, 148, 158, 167, and 168, respectively. Figure 3 provides a snapshot of channel activity as the fan progrades. At year 168, only a single channel, i.e., that labeled 6, is active.

In Figure 4 the average long profile of the cellular model is compared with implementation of the diffusional basin-filling model of Parker et al. [1998a] under identical conditions, i.e., that of the base case and that for which \( Q_w \) only is augmented to 40 m\(^3\)/s. The two models are in reasonably close agreement for both cases \( Q_w = 20 \) m\(^3\)/s and \( Q_w = 40 \) m\(^3\)/s, illustrating (1) a concave upward elevation profile and (2) bed slope that tends to decrease with increasing water discharge when sediment feed rate and grain size are held constant. In the case of the lower water discharge the proximal bed slopes tend to be somewhat lower in the cellular model. The overall agreement is

Table 2. Dimensioned Input Parameters Used in the Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( Q_w )</th>
<th>( Q_{sf} )</th>
<th>( D )</th>
<th>( x_p )</th>
<th>( l_b )</th>
<th>( a )</th>
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<tr>
<td>Dimensions</td>
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<td>m(^3)/s</td>
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<td>m</td>
<td>km</td>
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<td>0.3</td>
<td>2</td>
<td>10</td>
<td>100</td>
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<tr>
<td>Variant</td>
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<td></td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 2. (a) Active channel network at 168 years for the base case, with channel width proportional to the line widths and the color bar scaling volume sediment concentration in the channels. (b) Fan topography at 168 years for the base case. The contour lines denote 2.5 m intervals; standing water is indicated in blue. The coloring denotes deviation from axially averaged bed elevation, with red denoting locally high regions and green denoting locally low regions.
of significance because the diffusional basin-filling model has been found to generally agree with data for at least one field site [Parker et al., 1998b].

The effect of various geologic controls are studied in Figures 5, 6, and 7. In Figure 5 the base case has been continued for 75 years, after which the water discharge \( Q_w \) was impulsively raised from 20 to 80 m\(^3\)/s, holding all other parameters constant. (b) Active channel network at year 100 corresponding to the topography of Figure 5a.

Figure 3. Active channel network at various times for a simulation corresponding to the base case, except that water discharge \( Q_w \) is increased from 20 to 40 m\(^3\)/s. The channels labeled 1, 2, 3, 4, 5, and 6 correspond to years 138, 147, 148, 158, 167, and 168.

Figure 4. Comparison of predicted average long profiles of the cellular model (open symbols) with the predictions of a corresponding diffusional basin-filling model (lines) at 72 and 168 years. The cases represented include the base case, for which water discharge \( Q_w = 20 \) m\(^3\)/s, and a case for which \( Q_w \) alone has been increased to 40 m\(^3\)/s.

Figure 5. (a) Fan topography at year 100. The base case was continued until year 75, after which water discharge \( Q_w \) was impulsively raised from 20 to 80 m\(^3\)/s, holding all other parameters constant. (b) Active channel network at year 100 corresponding to the topography of Figure 5a.

Although the fan morphology described by the model is driven by sediment deposition, the model also describes upstream-driven erosion due to increased water...
discharge, as shown in Figure 5, and downstream-driven erosion due to dropping base level, as shown in Figure 6. The model also encompasses upstream-driven erosion caused by a reduction in sediment supply.

Figure 7 shows the base case but with a slow rotational tilting of the basement about an axis extending from the vertex of the fan to the opposite corner of the basin. The basement is initially horizontal and rotates with an angular speed of 0.001 rad/yr. Water surface elevation is kept constant relative to the initial horizontal basement during this rotation. The snapshot of Figure 7a shows fan topography at year 175, and the corresponding channel network is shown in Figure 7b. The left-hand side of the fan in Figure 7a is thus subject to uplift to the point that most of the basement becomes emergent. The right-hand side, on the other hand, undergoes subsidence, so that the depth of standing water becomes per. Uplift on the left side clearly slows fan progradation and deflects the channels to the right. Subsidence on the right side also slows progradation due to the necessity of filling deeper water with sediment in order for the fan delta to build outward. Thus progradation is maximized along the axis of rotation.

These several examples illustrate that the cellular model is (1) able to reproduce the results of a diffusional basin model which has been found to agree with field data for at least one site [Parker et al., 1998b] and (2) provides a reasonable representation of the response of a fan delta to such changing controls as water discharge, base level, and tectonism.

5. Sensitivity of the Model to the Avulsion Parameter $\beta$

Table 1 describes most of the dimensionless parameters that must be set in order to implement the model. It is thus of value to inquire about the sensitivity of the model to these parameters.

All of the parameters except one in Table 1 can be determined from direct measurements of flow in river discharge, as shown in Figure 5, and downstream-driven erosion due to dropping base level, as shown in Figure 6.

Figure 6. (a) Fan topography at year 225. The base case has been continued until year 100, after which water surface elevation $\xi_b$ alone has been raised impulsively from 2 to 7 m and held there. In year 200 the base level was impulsively lowered back to 2 m and held there. (b) Active channel network at year 225 corresponding to Figure 6a.

Figure 7. (a) Fan topography at year 175. The conditions are that of the base case, except for a slow, constant rotation of the basement about an axis passing through the fan delta vertex and the opposite vertex at the rate of 0.001 rad/yr. Water surface elevation has been held constant during this rotation. (b) Active channel network at year 175, corresponding to the topography of Figure 7a.
channels without specific reference to fans. The exception is $\beta$, which characterizes channel avulsion through equation (13). It is seen from equation (13) that as $\beta$ increases, the height of aggradation needed for avulsion increases. Although the evidence of Mohrig et al. [2000] indicates that the parameter is of the order of unity, it is as yet not definitively constrained. With this in mind, the sensitivity of the model to this parameter is studied in Figure 8.

Figure 8 describes the active channel network and shoreline at day 274 of year 105 for the four cases $\beta = 2, 1, 0.5$, and 0.25. The flow conditions are those of the base state, with the sole exception that $Q_w = 40$ m$^3$/s. The connected arrows denote the active channels, with arrow width proportional to channel width. The thin shaded line denotes the shoreline of the fan delta.

6. Deficiencies of the Model: Future Extensions and Applications

While the authors believe that the model presented here represents a major advance over previous numerical models of alluvial fans, it without drawbacks. The model does not include multiple grain sizes and compaction, features that could easily be included. The model considers only constant flood flows and does not explicitly account for overbank deposition of sediment. These features could be added, although not as easily as the previous two. The model is too simple to describe in any detail the formation of submerged in-channel bars which may later become emergent. A more complex flow model would be needed to allow for a study of bar-channel interaction. Channels move only by avulsion and not by, for example, bend migration. Although bend migration could be added [e.g., Sun et al., 1996], the cellular discretization is not particularly well suited to a description of bends. The description of the termini of channels in standing water is not particularly accurate. Correction of this feature would necessitate the inclusion of backwater effects from ponded water as well as at least some form of subaqueous sedimentation so as to allow an incipient channel to build to emergence. When channels incise into a fan, they are not allowed to regrade the fan by side erosion of terraces. The inclusion of this feature represents a major challenge for future research. Finally, the model does not describe the effect of vegetation in preventing avulsion. This problem may be rectified by requiring a boundary shear stress sufficient to break the vegetation and expose the noncohesive sediment below in order to allow an avulsion to propagate.

Extensions of the model suggest avenues of significant new research on basic and applied topics. A model with multiple grain sizes could be used to study channel architecture and channel-scale stratigraphy in subsiding depositional basins. The addition of compaction would allow for a study of the subsidence of fan deltas under their own weight. This feature combined with the addition of levees on some or all of the channels could be used in an applied study of, for example, the fate of the Mississippi fan delta with and without engineering works. The issue is of interest because the present-day levees on the Mississippi River within its fan delta are causing considerable in-channel aggradation. They also exacerbate subsidence because of compaction on the surface of the fan itself by blocking the supply of replenishing sand that would be deposited overbank by floods.

7. Conclusions

The numerical model of fan delta presented here provides an explicit link between processes at the level of the channel(s) and the overall morphodynamics of the fan delta. It builds on earlier diffusional basin-filling models by the addition of a description of flow and sediment transport in individual channels and avulsion of those channels. The governing relations of the model are dimensionally homogeneous and physically based. The cellular approach is used only as a convenient discretization technique.

Within limits, the model is capable of describing the evolution of channel(s) on the fan delta, fan delta progradation, and the response of the fan delta to, for example, varying water discharge, varying base level, and tectonic tilt. The overall results are in general agreement with the diffusional basin-filling model of Parker et al. [1998a, 1998b], but the present analysis provides a much richer description of fan morphology and evolution.
The model suggests future avenues for a variety of basic and applied research topics, including fan stratigraphy and the fate of fan deltas, such as the Mississippi River when subjected to engineering works such as levees that prevent the replenishment of sediment on surfaces that are subsiding due to compaction.

Notation

- \( a \): cell size.
- \( B \): channel width.
- \( D \): grain size.
- \( g \): gravitational acceleration.
- \( H \): cross-sectionally averaged flow depth in channel.
- \( i \): index denoting \( i \)th cell.
- \( j \): index denoting \( j \)th cell.
- \( L_b \): length of square basin.
- \( L_{ij} \): distance from the \( i \)th cell to the adjacent \( j \)th cell.
- \( N = L_{ij}/a \): exponent in sediment transport relation.
- \( p \): exponent in flow resistance relation.
- \( p_{ij} \): probability that an avulsed channel flow leads from a given \( i \)th cell to a new adjacent \( j \)th cell.
- \( Q_w \): water discharge.
- \( Q_s \): volume sediment discharge in a channel.
- \( Q_{sf} \): volume sediment feed rate to the fan.
- \( q_s \): volume sediment discharge per channel width.
- \( R \): submerged specific gravity of sand.
- \( S \): streamwise bed slope of channel.
- \( U \): average flow velocity in channel.
- \( u* \): shear velocity.

Dimensionless coefficients in equations (4), (5), and (8):
- \( \alpha_{3b} \), \( \alpha_{ps} \), \( \alpha_{sps} \), \( \alpha_{p} \)
- \( \beta \): dimensionless parameter in equation (13).
- \( \Delta t \): time increment in equation (12).
- \( \eta \Delta \): increment of bed elevation in equation (12).
- \( \theta_{ij} \): angle of deflection of the line from the wet \( i \)th cell to an adjacent dry \( j \)th cell relative to the line of flow from the \( i \)th cell to the next wet cell immediately downstream.
- \( \gamma \): exponent in equation (11).
- \( \gamma \): bed elevation.
- \( \lambda \): bed porosity.
- \( \theta_o \): dimensionless parameter in equation (15).
- \( \tau_i^* \): bank-full Shields stress of an active channel.
- \( \tau_s^* \): critical Shields stress for the onset of sediment motion.
- \( \xi_n \): elevation of ponded water.

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