Steeples and Steeples (1996) performed a statistical treatment of events triggered by the 1906 earthquake and estimated the likelihood that the so-called triggered events happened by chance alone. They divided the area into four regions: California north of latitude 37°N, California south of 37°N, Oregon, and Arizona. They then used the catalog of McAdie (1907) to determine how often earthquakes were felt in each of those regions during the period 1897 through 17 April 1906, and they used that information to estimate the probability that each of those regions would independently have at least one felt earthquake during a random 24-hour period. They argued that the probability was at most 1.39×10^{-6}, or that a similar series of earthquakes could be expected at most about once every 1978 years.

In the present paper, we improve on the argument of Steeples and Steeples (1996) by considering the magnitude of each triggered event in the context of the frequency of earthquakes which are at or above that magnitude, as tabulated from modern catalogs. We adopted an approach that somewhat parallels that of Steeples and Steeples: we used the "ball-in-the-box" analogy from probability theory, as did Steeples and Steeples, to assess the probability that the events were unrelated. Our method, however, differs from theirs in several important ways: in addition to considering the magnitudes of the earthquakes, we divided the area into regions differently than Steeples and Steeples, we expanded the period of study from the day following the 1906 mainshock to a 2-day period following the mainshock, and we corrected erroneous data used by Steeples and Steeples (1996).

Our method involved several procedures, which are outlined below:

1. The West Coast area was subdivided into a number of regions whose boundaries are loosely guided by tectonic provinces. The regions include: southern California, which is defined for this purpose as the area north of 32°N, south of 36°N, east of 123°W, and west of 114°W; Arizona, defined as the area north of 31.5°N, south of 36.5°N, east of 114°W, and west of 109°W; Oregon, defined as the area north of 42°N, south of 45°N, east of 125°W, and west of 117°W; Nevada, including California east of the Sierras, defined as the area within the polygon with vertices at 42°N, 122°W, 39°N, 120°W, 36°N, 118°W, 36°N, 114°W, and 42°N, 114°W; and the 1906 San Francisco earthquake fault rupture and its near-field aftershock zone (hereafter, the northern California region), defined as the area north of 36°N, south of 42°N, and west of the region previously grouped with Nevada.

2. We made the following assumptions:

- Earthquakes in each region are independent of earthquakes in other regions.
- Earthquakes are random in time.
- Earthquake swarms count as one event.
- Clearly defined events and their aftershock sequences count as one event.

3. We listed every triggered event from our catalog which occurred within 48 hours of the mainshock. This list included all events within the southern California, southern Arizona, southern Oregon, and Nevada regions as defined in Step 1 above, even those events near Reno, NV which occurred less than 400 km from the 1906 mainshock rupture. For some of these events (*i.e.,* the Imperial Valley and Santa Monica Bay events), the magnitude was already determined; for the rest, we estimated the minimum magnitude which would be consistent with the felt reports. A brief summary of the reasoning behind our choice of magnitude for each event is given in Table 1. For earthquake swarms or mainshock-aftershock sequences, we listed only the largest event.

   Note that one event listed in Lawson *et al.* (1908) and discussed in Steeples and Steeples (1996)—an event supposed to take place at 12:31 p.m. on 18 Apr 1906 in Los Angeles—was not confirmed by a single newspaper or diary in southern California; rather, it appears to be a misdated report of the earthquake which was widely documented to have hit Los Angeles at 12:31 p.m. on 19 Apr 1906.

4. For each event on the triggered earthquake list, we determined the frequency of earthquakes of equal or greater magnitude at any point within the appropriate region. For example, for the M 6.1 Brawley earthquake, we determined how often an earthquake of M 6.1 or greater occurs anywhere in the southern California region. To do this, we searched the Council of the National Seismic System (CNSS) Worldwide Earthquake Catalog (online), which is a compilation of catalogs contributed by CNSS member networks. We restricted the search to the 17-year period 1 Jan 1984, 00:00:00, to 31 Dec 2000, 23:59:59, which is the period during which the overall catalog is believed to be the most complete. We also assumed an average recurrence rate of 100 years for events on the northern San Andreas fault to establish a frequency for $M \approx 7.8$ or larger events in the northern California region. Our assumption arises from the fact that the true recurrence rate is very poorly constrained for the northern San Andreas, but the shortest known average recurrence interval anywhere on the San Andreas is 100 years at Wrightwood (K. Sieh, Caltech, personal communication, 2001; Fumal *et al.*, 1993); hence, 100 years is a conservative estimate for the northern San Andreas.

5. Based on the recurrence frequency of each event on the list from Step 3, we used probability theory to calculate the likelihood that all the events which occurred within a given region would occur independently during a random 48-hr period. Then, using combinatorial theory, we analyzed the probability that all the triggered events were independently random events.

Each triggered event as well as the mainshock is listed in Table 1, along with the event's frequency. A few of the most common events (such as M 2.7 events in southern California) occur frequently enough that they would not significantly affect the outcome of the probability calculation, so they are ignored for the calculation and their frequencies are given in Table 1 only as "common."

The probability ($P_{i,m}$) of one or more earthquakes of magnitude m or greater occurring in any one of these regions (i) during a random 2-day period during the sample time should be given by

$$P_{i,m} = 1 - \left(\frac{k-1}{k}\right)^{n_{i,m}} \tag{1}$$

where k is the number of 2-day periods in the sample and $n_{i,m}$ is the number of earthquakes of magnitude m or greater which occurred in region i during the sample time.

This situation is probabilistically equivalent to the scheme of randomly placing n balls into k cells. If there is only one ball (n=1), the probability that it is in the jth cell is $1/k$ and the probability that it is not in the jth cell is $(k-1)/k$. For $n>1$, the probability that *none* of the balls are in the jth cell is $[(k-1)/k]^n$. Here, the probability that there is no earthquake of magnitude m or greater during a given two day period in region i is $\left[(k-1)/k\right]^{n_{i,m}}$, and one minus this value is the probability that there will be one or more such earthquakes.

Using the fact that there are 6210 days or 3105 two-day periods during the 17-year period from 1984 to 2000, we calculated $P_{i,m}$ for earthquakes of various magnitudes in the different regions. Table 2 gives the estimated probability of at least one earthquake of the type indicated occurring the stated region.

We could also determine the probability of two or more earthquakes above a certain magnitude in one particular two-day period. That would be given by 1 – [(probability of no eqs of $M \geq m$) + (probability of exactly one eq of $M \geq m$)], or

$$1 - \left[\left(\frac{k-1}{k} \right)^{n_{i,m}} + n_{i,m} \left(\frac{1}{k} \right) \left(\frac{k-1}{k} \right)^{n_{i,m}-1} \right] \tag{2}$$

We wanted to know the probability of southern California experiencing both a M 6.1 earthquake and a M 5.0 earthquake that were independent of one another (*i.e.,* in markedly different locations) within a given 2-day period. We can break this down into two mutually exclusive cases, the sum of which gives our overall probability:

$$P_{1,\,(6.1 \text{ and } 5.0)} = P_1\,(\text{2 or more earthquakes of } M \geq 5.0\text{, at least one of which is } M \geq 6.1)$$

$$= P_1\,(\text{2 or more } M \geq 6.1\text{, any number } M\text{ 5.0-6.0}) + P_1\,(\text{exactly 1 } M \geq 6.1\text{, 1 or more } M\text{ 5.0-6.0}) \tag{3}$$

$$= P_1\,(\text{2 or more } M \geq 6.1) \cdot 1 + P_1\,(\text{exactly 1 } M \geq 6.1) \cdot P_1\,(\text{1 or more } M\text{ 5.0-6.0}) \tag{4}$$

$$= \left\{ 1 - \left[\left(\frac{k-1}{k} \right)^{n_{1,6.1}} + n_{1,6.1} \left(\frac{1}{k} \right) \left(\frac{k-1}{k} \right)^{n_{1,6.1}-1} \right] \right\} + \left\{ \left[n_{1,6.1} \left(\frac{1}{k} \right) \left(\frac{k-1}{k} \right)^{n_{1,6.1}-1} \right] \left[1 - \left(\frac{k-1}{k} \right)^{n_{1,5.0}-n_{1,6.1}} \right] \right\} \tag{5}$$

$$= 1 - \left(\frac{k-1}{k} \right)^{n_{1,6.1}} - n_{1,6.1} \left(\frac{1}{k} \right) \left(\frac{k-1}{k} \right)^{n_{1,5.0}-1} \tag{6}$$

This probability has been computed and is also listed in Table 2.

Assuming independence from one region to another, the probability of the occurrence of more than one “hypothetical” earthquake scenario listed in Table 2, within a random 48-hr period, is the product of the probabilities for each individual region. For example, let P^* be the probability that southern California experienced independent M 6.1 and M 5.0 events, that Arizona (as defined in this paper) experienced a M 3.5 event, and that Oregon (again, as we defined it) experienced a M 3.5 event, all in a specific 2-day period. Then P^* is given by

$$P^* = P_{1,\,(6.1 \text{ and } 5.0)} \cdot P_{2,(3.5)} \cdot P_{3,(3.5)} = 2.7 \times 10^{-10} \tag{7}$$

To the extent that our assumptions are valid, the probability that the events in the Imperial Valley, Santa Monica Bay, Arizona, and Oregon all occurred by chance during the 48-hr period immediately following the 18 Apr 1906 San Francisco earthquake is 2.7×10^{-10}, or about 1 in 3.7 billion. Note that all of these events were more than 400 km away from the 1906 mainshock rupture, so these events would not be considered aftershocks by any conventional definition. Had we included the Nevada event in our calculations as well, the probability that all the events coincided by chance would be two orders of magnitude lower.

Note also that the time period selected for comparison out of the modern catalogs (1984-2000) was a time of elevated seismicity, at least for earthquakes of $M > 6$ in southern California. Had we looked instead at a larger sample space—say the years 1933-2000, during which there were 22 earthquakes of M 6.1 or larger, with 19 of them not being aftershocks of larger events—we would have had an average of 0.28 events of M 6.1 or larger per year, instead of 0.41. The lower rate of M 6.1 and larger events would imply a lower probability that all the events in question occurred by chance.

Another interesting question is how often we can expect a M 6.1 or larger earthquake in southern California, an unrelated M 5.0 or larger earthquake in some other part southern California, a M 3.5 or larger earthquake in Arizona, and a M 3.5 or larger earthquake in Oregon, to all occur within the same 48-hr period, provided that they only occur together by chance. If these events coincide once every 3.7 billion two-day periods, that would be about once every 20 million years. It would be even more unusual for these events to coincide, by chance, with a M 7.8 or larger earthquake in northern California.

For the purpose of completeness, we determined how unusual such a situation would be. The probability that a M 7.8 northern California event, independent M 6.1 and M 5.0 southern California events, a M 3.5 Arizona event, and a M 3.5 Oregon event will happen over a particular two-day period is given by

$$\begin{aligned} P^{**} &= P_{1,\,(6.1 \text{ and } 5.0)} \cdot P_{2,(3.5)} \cdot P_{3,(3.5)} \cdot P_{5,(7.8)} \\ &= 1.5 \times 10^{-14} \end{aligned} \tag{8}$$

The reader should be cautioned that this last number is not very meaningful, as it is the probability that all the events occur over a *specific* two-day period, but we do not care which two-day period it turns out to be. More meaningful, however, is the time-scale implication of this number. An event with probability 1.5×10^{-14} can be expected to recur once every 6.7×10^{13} cycles. In our case, that is once every

6.7×10^{13} two-day periods, or equivalently once every 370 billion years. Of course, given the relatively young age of the Earth—not to mention the even younger age of the San Andreas fault—this entire sequence of events would never be expected to occur unless some or all of the events were related.

TABLE 1

Triggered Events within 48 hours of the 1906 San Francisco mainshock

Date (1906)	Time	Location	Lat (° N)	Lon (° W)	Mag	Reasoning behind Magnitude	No. of similar events *
18 Apr	05:12	San Francisco Mainshock			7.8	from ... (need a reference)	1 per 100 yrs
18 Apr	05:48	west of Phoenix, AZ	33.6	113.2	≥ 3.5	MMI IV-V near Salome (150 km west of Phoenix), MMI II in Phoenix; at both locations, the motion was described as "distinct," strongly suggesting that these were not poorly timed accounts of the mainshock, which would have been characterized by long-period motion	16
18 Apr	16:30	near Brawley, CA	33.14	115.59	~ 6.1	see this paper, Figure _ preceded by a series of foreshocks beginning at 13:30, and followed by a sequence of aftershocks	7
18 Apr	21:10	near Pomona, CA	34.1	117.8	≥ 3.0	"severe" shock in La Verne, "light" in Glendora, "slight" in Chino 2 other light shocks felt at Glendora only, at 20:45 and 22:30	common
18 Apr	22:55	near Avila Beach, CA	35.2	120.7	very small	"heavy and distinct" at Avila Beach; no other felt reports	common
19 Apr	01:30	near Paisley, OR	42.69	120.55	≥ 3.5	MMI V at Paisley followed by at least 3 aftershocks during the next hour and a half	12
19 Apr	12:31	near Santa Monica Bay, CA	33.90	118.50	~ 5.0	see this paper, Figure _	23
19 Apr	14:05	near Reno, NV	39.6	119.7	very small	MMI II-III at Reno, noted most strongly in NE part of town	common
19 Apr	16:30	near Santa Barbara, CA	34.4	119.7	very small	"mild" in Santa Barbara; no other felt reports	common
19 Apr	20:20	east of Reno, NV	39.56	119.20	≥ 4.5	MMI IV-V at Fernley and Hazen, MMI IV at Wadsworth and Olinghouse; not reported from Reno	50

** Number of events of equal or greater magnitude, in the same region, 1984-2000, unless otherwise noted; "common" indicates that the event is rather common, and it would not significantly affect the calculation of probabilities.*

TABLE 2

Probability of Occurrence of Earthquake Scenarios within a Random 48-hr Period

Region (i)	Region Name	Hypothetical Magnitude (m)	Probability of occurrence of an earthquake of $M \geq m$ within a random 48-hr period ($P_{i,m}$)
1	Southern California	5.0	$P_{1,(5.0)} = 1 - \left(\frac{3104}{3105}\right)^{23} = 0.00738$
1	Southern California	6.1	$P_{1,(6.1)} = 1 - \left(\frac{3104}{3105}\right)^{7} = 0.00225$
2	Arizona	3.5	$P_{3,(3.5)} = 1 - \left(\frac{3104}{3105}\right)^{16} = 0.00514$
3	Oregon	3.5	$P_{3,(3.5)} = 1 - \left(\frac{3104}{3105}\right)^{12} = 0.00386$
4	Nevada	4.5	$P_{4,(4.5)} = 1 - \left(\frac{3104}{3105}\right)^{50} = 0.01598$
5	Northern California	7.8	$P_{5,(7.8)} = 1 - \left(\frac{18261}{18262}\right)^{1} = \frac{1}{18262} = 5.48 \times 10^{-5}$
1	Southern California	6.1 and 5.0 (two events)	$P_{1,\,(6.1 \text{ and } 5.0)} = 1 - \left(\frac{3104}{3105}\right)^{7} - 7\left(\frac{1}{3105}\right)\left(\frac{3104}{3105}\right)^{22} = 1.37 \times 10^{-5}$