Problem set 8

Ge 108

December 2, 2007

1 Feel the pulse

A string with wave velocity v has a single sine-wave pulse of wavelength λ and height h centered on the origin at time 0. The string is not in motion at time 0.

(a) Write the function that describes the two initial conditions of the string, $u(x, 0)$, the displacement, and $\dot{u}(x, 0)$.

(b) Write down the general solution for time t.

We're now going to try to figure out the total energy of the wave. Because the string is not at motion initially, all of the energy is potential. One of our favorite ways to define potential is $U = \int F ds$, or the potential is the work done to bring the string to this position from its initial position. If we want to figure out the potential energy of a rock on top of a cliff, we would do it by slowing moving the rock from the bottom of the cliff to the top and keeping track of how much force was needed at each step. For the rock this is easy, since the force is always the same. For the string it's a bit more complicated because the forces are *not* always the same.

(c) What are the forces at $t = 0$ (as a function of distance x along the string) that are required to keep the string in its initial condition? (This is equivalent to asking what forces operate on the string immediately upon releasing it.) What is the total sum of the magnitude (i.e., absolute value) of all of the forces required to keep the string in its position?

Now, sometime before $t = 0$, we are going to bring the string from the equilibrium position to its initial position in small steps, ds, where s stars at zero and goes to h, the final wave height. In other words, if $h \times g(x)$ is the function that describes the initial pulse, we slowly bring the pulse to its initial condition by starting with a straight string and then moving to $s \times g(x)$ by small steps until we reach $h \times g(x)$.

(d) What is the total force required to keep the string at its diminished height, s, as a function of s?

Now you can integrate $F ds$.

(e) What is the potential of the string?

(f) You would like to think that the energy of the string is somehow related to the fact that you have a wave (or two) travelling at velocity v the is somehow carrying some mass (though it's really not moving any mass horizontally). In this conceptual view, what is the equivalent "mass" that is being moved by the pulse (remember that there are two pulses here)?