# Problem set 4

Ge 108

Due 31 October 2001 (or 2 Nov if you have a field trip)

## 1 Limb darkening

If you look at the sun (wait, weren't you told never to do this?) you will notice that the edges of the sun are darker than the center of the sun. Explain this in terms of optical depths and temperatures. Assume that the optical depth of the sun at all wavelengths is very very very high so that you are seeing emission from very near the surface of the sun. Also assume that the sun's temperature gets uniformly hotter as you go deeper into the sun.

## 2 Runge-Kutta

**MATLAB:** Use the old program `numeric.m`, but before you run it type `profile on`. After you run `numeric`, simply type `profile report` to bring up a web page telling you how long your program took to run.

Change `numeric` from its original Euler integration scheme to a a second-order Runge-Kutta scheme to integrate the same equation. This will require modifying only two lines of the program!

(b) Compare the answer that you get with the Runge-Kutta scheme with that that you get with the old `numeric.pro` (which uses a *Euler* scheme) for the same step size (use `dt=0.1`). How much more accurate for the same step-size is the Runge-Kutta?

(c) Assume that you don't care to get more accurate, but you would like to be faster. Find out how large of a step size you can use for the RK scheme to get the same accuracy as the Euler scheme. How much computer time does this save you?

## 3 Reality

Now that you've actually done a Runge-Kutta integration by hand I'll let you know that in real life you will almost never have to do this. MATLAB has a built in integrator where you simply input a function and some parameters and you get out the solution. It is worth knowing how these sorts of things work so that you will be able to understand better cases in which they will *not* work, but from now on we'll just attempt to use these built in functions.

(a) Radiation from a 200 K blackbody encounters a slab of gas. The slab of gas is 1000 m thick and has a temperature structure where $T(x) = 300 + 700x$ where $x$ is the distance from the front of the slab in meters.

The absoption coefficient, as a function of frequency, is

$$\kappa_\nu = \nu/(3.0 \times 10^16\text{Hz})\text{m}^{-1}$$

(with $\nu$ in Hz).

What is the optical depth through the slab at wavelengths of 1, 5, 10, and 100 $\mu$m?

(b) For the 4 wavelengths above, if $\tau > 1$ at what distance into the slab does $\tau = 1$? What is the temperature at this distance? What is the intensity of a blackbody with these temperatures (at that wavelength)?

(c) Recalling the equation of radiative transfer:

$$\frac{dI_\nu}{ds} + k_\nu I_\nu = k_\nu B_\nu(T(x)),$$

numerically solve for the intensity of radiation from the slab at the 4 wavelengths above (do this by running the program separately 4 times with the wavelength as a parameter). See below for MATLAB instructions.

(d) Convert these 4 intensities to an equivalent blackbody temperature (i.e. how hot would a blackbody have to be to radiate this amount at this wavelength?). How do this compare with part (b)?

**MATLAB solution** MATLAB has several different ordinary differential equation integrators. You can see the list of them (with not very good explanations) by typing (in matlab) *help funfun*. Something like `ode45` is a good starting point for a 4th order integrator. Type `help ode45` for a somewhat cryptic explanation of how it works. Here is my summary. If you want to solve the ODE

$$\frac{dI}{ds} = f(I, s)$$

you must first create an M-file with the function. As an example of an M-file for $f(I, s) = s$ download `odefile.m`. To integrate the equation, $dI/ds = s$ over the range from $s = 0$ to $s = 100$ (to which we know the answer is 5000), type `s=[0,100]` to define the integration limits and then `ode45('f',s,0)` to do the integration. Admire the cool plot. The function `f` can now be replaced with anything that you would like to integrate.