# Lecture 7

# Applications of radiative transfer

## 7.1 What we see at the ground

Let's do a real calculation of what we see from the ground when we look up at the sky at the sun in the infrared portion of the spectrum. Last time we talked in detail about optical depth and things that are optically thick and optically thin. In Figure 7.1 we see optical depth through the earth's atmosphere depth as a function of frequency. In the optical region of the spectra there is almost no absorption of sunlight and all of the light gets through (hmmm... interesting coincidence: the optical region of the spectrum is about the only place where there is not significant molecular absorption in the spectrum. It also happens to be the peak of the solar spectrum. Hmmmmmmm.) What about the infrared portion?

The peak of solar emission is around 0.6 $\mu$m, so in the infrared we are on the Rayleigh-Jeans section of the spectrum and it is falling as $\nu^2$. The atmosphere is much cooler – let's say 300 K – so in the regions where it's optically thick it will radiate as a 300 K blackbody. Using the optical depths from Figure 7.1, the radiative transfer equation (equation 6.1, where $\tau = k_\nu l$), and a 6000 K and 300 K blackbody for the sun and the sky, respectively, we get:

$$I_\nu = B_\nu(6000K)\exp(-\tau) + B_\nu(300K)(1-\exp(-\tau)), \qquad (7.1)$$

where all of the parameters above are functions of frequency. We plot the results of this calculation in Figure 7.1.

What if, instead of looking at the sun, we look at blank night sky? Then $I_n u = 0$, so all we see is thermal emission from the optically thick portions of the sky, as shown in Figure 7.1, and nothing at all from the optically thin portions.

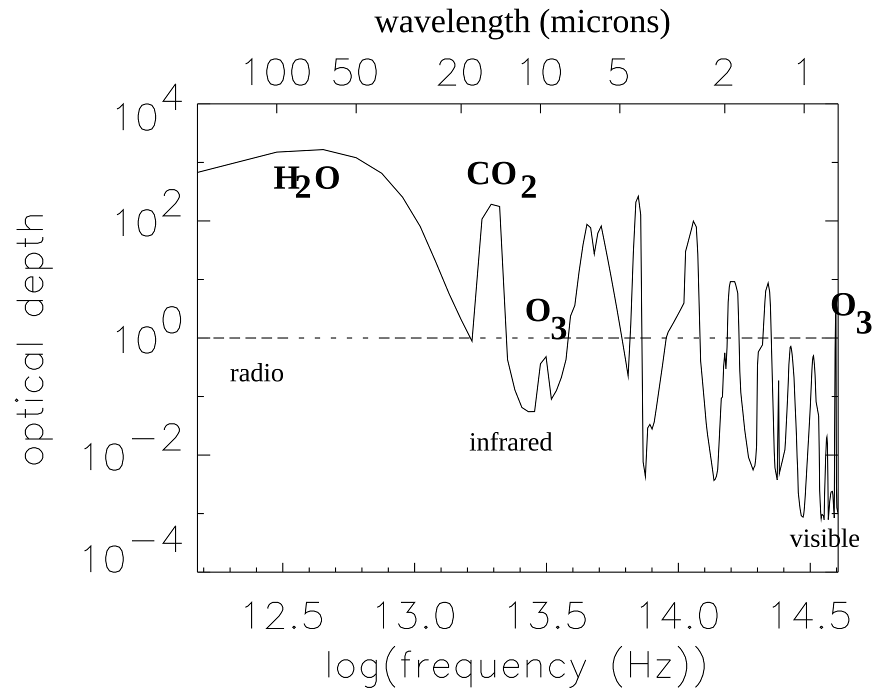

Figure 7.1: Optical depth of the earth's atmosphere as a function of frequency (and infrared wavelengths on top).

## 7.2 What they see from space

We've calculated what the sun and the sky look like from the surface of the earth, but what does the earth look like from space? The calculation is identical to that done from the sun from the surface, except that now $I_\nu = B_\nu(300K)$. If we substitute this into equation 7.1, we find simply that

$$I_\nu = B_\nu(300K).$$

All we see is a 300 K blackbody! In the optically thin regions, we see the 300 K blackbody of the surface, and in the optically thick regions, we see the 300 K blackbody of the atmosphere.

In real life, the earth does not look like this from space, because the atmosphere is not the same temperature as the surface, nor is its temperature uniform. What we really find for the full radiative transfer equations (and in this week's problem set) is that at each frequency the atmosphere radiates as a blackbody at the temperature at which the line becomes optically thick. Thus,

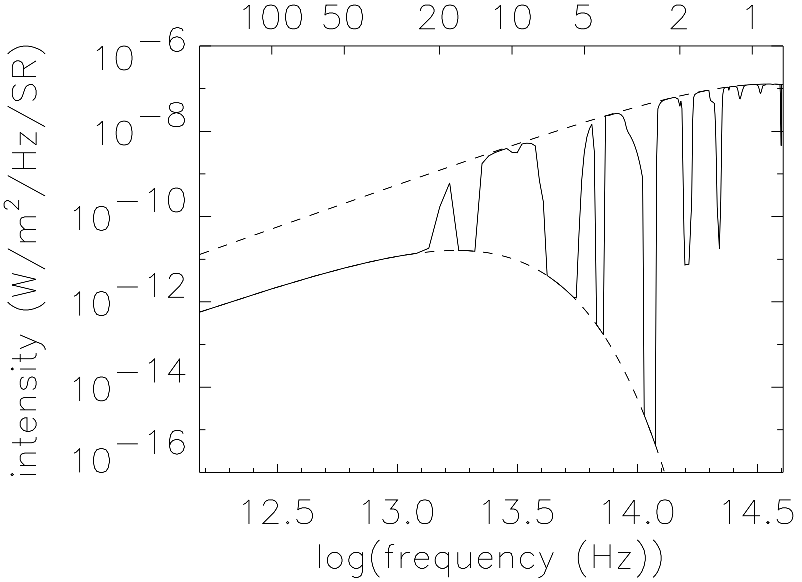

Figure 7.2: Spectrum see when looking at the sun through the earth's atmosphere, assuming that the sun is a 6000 K blackbody and that the atmosphere is at 300 K. The dashed lines show 300 and 6000 K blackbodies.

for example, from Figure 7.1, we see that around 100 $\mu$m the optical depth of the atmosphere is huge, so that means that it becomes optically thick very quickly. So from above it radiates like a cold blackbody at the temperature high in the atmosphere (note that from below, it radiates like a *hot* blackbody because, when looking from the surface, the optical depth approaches unity very quickly). At about 1.8 $\mu$m, on the other hand, the atmosphere is only barely optically thick, so from space the blackbody temperature appears as that of the atmosphere close to the ground which, in practice, is indistiguishable from the surface. Assuming that the atmosphere is uniformly 150 K, the emission from space appears as in Figure 7.2. The optically thick portions of the spectrum now appear as *absorption lines.* In fact, the tiny dip in the spectrum that appears right at 10$\mu$m (due to $O_3$) is one of the signatures that might signal that an extra-solar planet has has earth-life life on it.

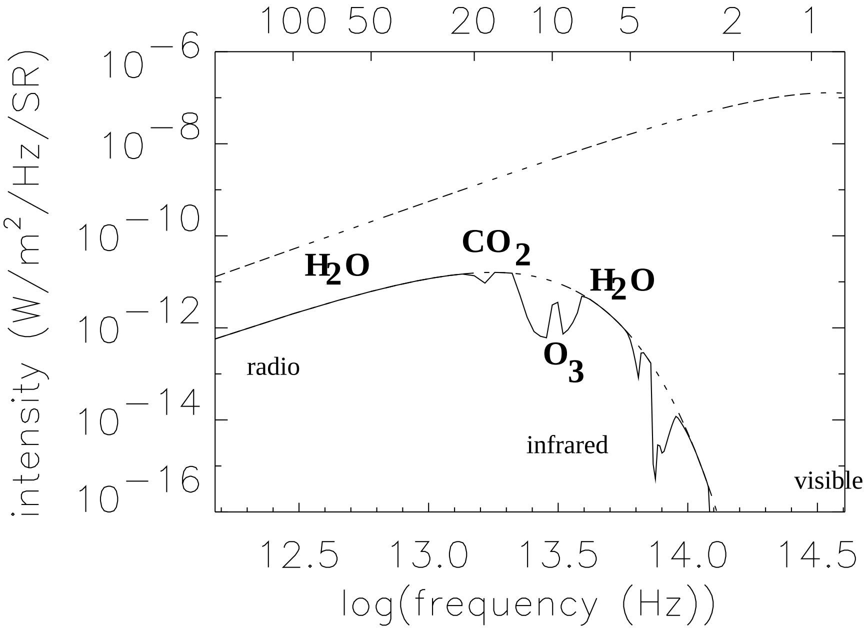

Figure 7.3: Spectrum of sky emission when looking at a blank portion of the sky, assuming that the sky is at a uniform 300 K.

## 7.3 What other things look like

### 7.3.1 A brown dwarf

The fact that the emission at any frequency tells us essentially what the temperature is at the point at which that frequency is optically thick allows us to use sepctra to probe the temperature structure of atmospheres (if we know the composition structure and therefore absorption efficiency and where different frequencies are optically thick) or the compositional structure (if we know the temperature structure). As an example of how this works, look at Figure 7.3.1, which is an infrared spectrum of Gl229b, currently the only know *brown dwarf* in the universe (a brown dwarf is an object somewhere midway between a planet and a star, in fact, the spectrum of this brown dwarf looks remarkably similar to the spectrum of Jupiter). Let's look at this spectrum in detail. Based on the discussion in the previous section, we can now see what this spectrum means. First off, unlike the earth, there is no surface underneath that is radiating. *All* of the emission becomes optically thick at some point (consider that on the earth the surface itself is optically thick at all wavelengths, so really it's sort of the

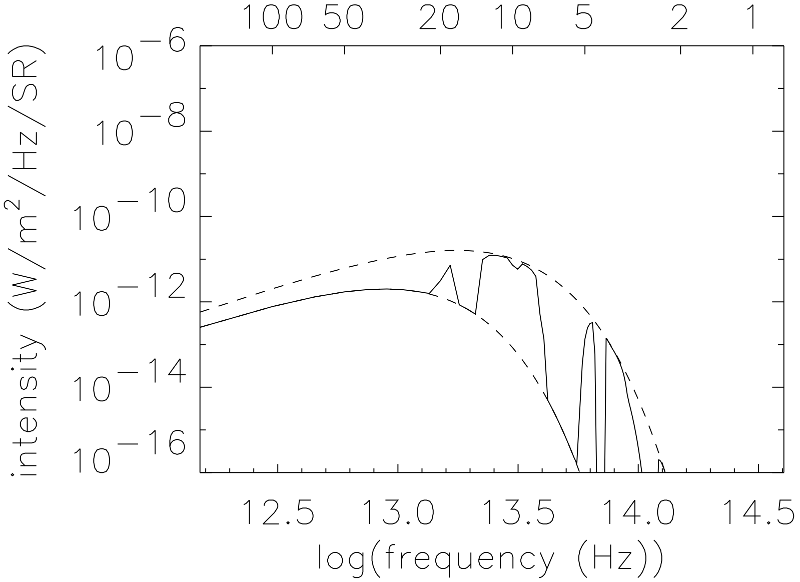

Figure 7.4: Emission from the earth as seen from space, assuming that the atmosphere is uniformly 150 K. The dashed lines are now 300 and 150 K blackbodies.

same). So what we see at every point is the temperature at the depth at which the frequency becomes optically thick. For example, in the deep absorption at $\log \nu = 14.35$ (which is due to methane absorption or opacity) the bottom of the absorption line is cold, with a temperature of $\sim 700K$. What does this mean? At this frequency, the optical depth is high, so the brown dwarf becomes optically thick very high in the atmosphere, where the temperature is still cold. At the nearby frequency of $\log \nu = 14.4$, on the other hand, the optical depths are low, so the brown dwarf does not become optically thick until deep in the atmosphere where the temperatures are high. The same analysis can be done for all of the frequencies in the spectrum, and we can learn what's there, how much of it is where, and how hot it is. In real life, what is done is that the spectrum is matched to a model, where everything that we think we know about brown dwarf chemistry, structure, and temperature is input, and the model is tuned to see how well it can be made to match the spectrum.

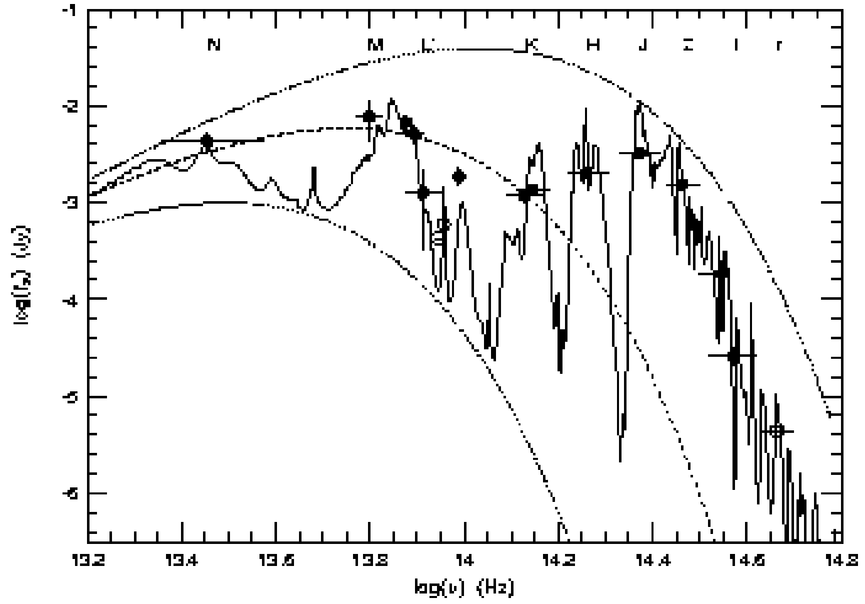

Figure 7.5: A spectrum of Gl229b, a nearby brown dwarf which looks remarkably similar to Jupiter. The three dashed lines show blackbodies at temperatures of 500, 900, and 1200 K.

### 7.3.2 Jupiter with spots

Some of the most dramatic images of Jupiter during the Shoemaker-Levy 9 bombardment came from telescopes operating at a wavelength of about $2\mu$m (Figure 7.3.2). Let's explore what was going on at the time. First off, Jupiter, like Gl229b, is full of methane that absorbs strongly at $2\mu$. The top of the atmosphere is cool, so at 2 $\mu$ a deep absorption line apppears. If you take an image of Jupiter *only at that wavelength*, the planet will appear very dark (except for the poles – we'll discuss this part in a minute). Now bombard the atmosphere with a comet and put a layer of fine dust and soot in the very top of the atmosphere, higher even than the spot at which the methane band becomes optically thick. This dust now *reflects* light back to the earth, rather than letting it pass through the atmosphere. So instead of a large absorption, we see the full component of $2\mu$m sunlight, and the spot where dust is appears quite bright. This process is exactly that that is going on in Figure 7.3.2 at the comet impact sights, and also at the poles. At the poles, the atmosphere

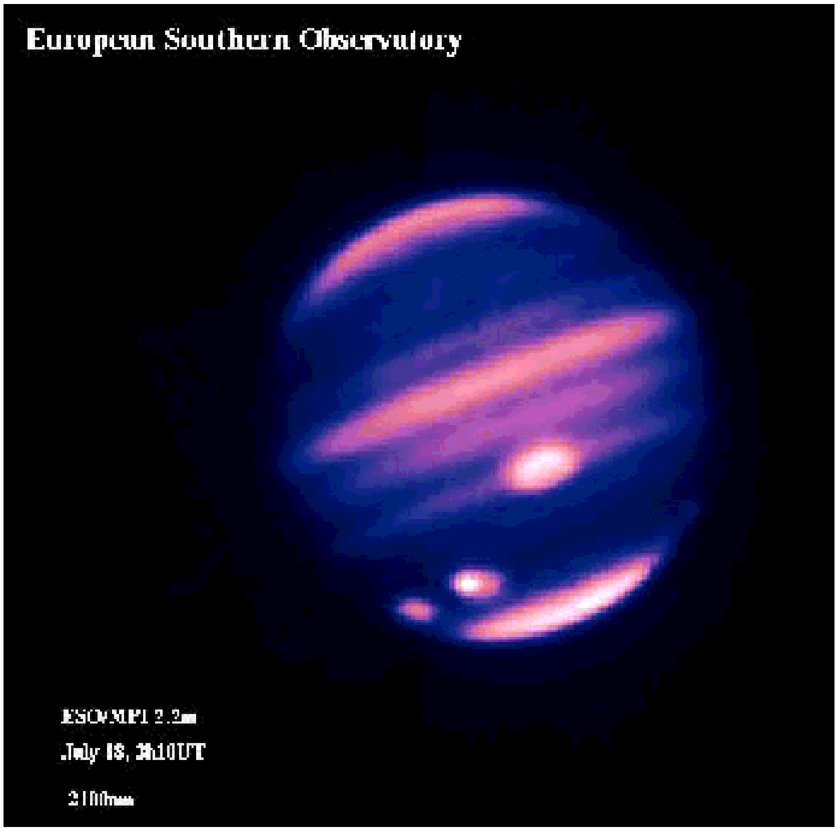

Figure 7.6: An image of Jupiter during the SL9 impacts obtained through a $2\mu$m filter. Jupiter is mostly dark in this absorption band, but the dust created at the impact sights sits high above the methane absorption regions and reflects sunlight back to the earth.

is continuously being bombarded by electrons and ions that are traveling along magnetic field lines and making aurora (just like the northern and southern lights on the earth). When these ions and electrons hit the atmosphere, they chemically combine and make little haze particles that also reflect the sunlight, just like the comet particles.