Lecture 5

Radiation

5.1 What is radiation?

Last week we talked about the transfer of heat energy through conduction or diffusion, where heat moves because hotter and colder objects are touching and energy flows between the two. Today we will discuss *radiation*, where energy can directly propagate from one object to another even though they aren't touching. The most common experience that all of us have with radiation is the energy being transfered from nuclear reactions within the sun (which then diffuses outward to the solar surface, or *photosphere*) to the earth, with nothing but empty space in between. This radiated energy is sufficient to burn your skin just like a piece of hot metal would burn your skin through conduction.

How does this radiation work? For many centuries it was thought that, just like waves are carried on the ocean, radiation was carried on some sort of medium, termed the *aether*. Not until the classic Michelson-Morely experiment 110 years ago, which showed that the speed of light is independent of the direction of travel, did people begin to accept the earlier ideas of Maxwell, that radiation is a series of oscillating electric and magnetic fields that can travel through any or no medium.

As you learned in your basic physics classes, we now know that radiation can be thought of as an electromagnetic wave or as a series of particles (*photons*) traveling through space. The propagation of these waves is described by a set of 4 differential equations called the *Maxwell equations* which show that the source for an electric field is a changing magnetic field while the source of a magnetic field is a changing electric field. Thus once such a changing field begins it is self-sustaining and will propagate through space.

How does such an alternating field begin? Two everyday examples of sources of electromagnetic radiation are radio antennae and light bulbs. Radio antennae are conceptually easy to understand: an alternating current is set to run along a wire, causing an alternating electric field which, in turn causes an alternating magnetic field, and an electromagnetic wave is launched. In fact, any wire with an alternating current running through it emits electromagnetic radiation, which is why things like computers often make your radio sound staticy and also why the FAA makes you turn off your laptop during takeoff and landing: the wires and currents in your laptop are antanneae which launch electromagnetic waves, while the wires in the airplane and even the aluminum skin of the airplane itself are antennae which pick up the radiation, potentially causing spurious signals in the airplanes computers. But, of course, just like wires in your computer are antennae which emit radiation, they are also antennae which pick up radiation, so if they were emitting electromagnetic radiation that were capable of disrupting computers they would be disrupting themselves all the time. The FAA has never been known for having any understanding of physics. To their credit, they also ban the use of cellular phones. Why are these different?

Light bulbs are considerably more complicated and will be considered in the next section.

5.2 Blackbody radiation

Lightbulbs, like antennae, consist of wires through which an alternating current flows. But the visible light emitting is not directly caused by this flow (the current does cause a very tiny radio wave, though). Instead, the current heats the wire and causes *blackbody radiation.* (Actually it is only an approximation to blackbody radiation. We'll discuss these details later.)

Blackbody radiation is the spectrum of radiation emitted by a perfectly black object (the "blackbody") at a temperature T. The amount of radiation and the spectrum of the radiation are independent of the composition of the object, be it a black metal sphere or a black collection of Big Macs.

Throughout this class we've been trying to derive, either intuitively or at least formally, everything that we use. Unfortunately, no good way exists of deriving formulae for blackbody radiatiom without going through an exhaustive calculation in both quantum mechanics and statistical physics, so we're going to break down and simply make an assertion: the spectrum of radiation that is emitted by a blackbody is given by the *Planck formula:*

$$B(\lambda, T) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{\exp(hc/k\lambda T) - 1}, \tag{5.1}$$

where λ is the wavelength of the radiation, h is the Planck

constant, c is the speed of light, and k is the Boltzmann constant. In numeric terms,

$$B(\lambda, T) = \frac{c_1}{\lambda^5}\frac{1}{\exp(c_2/\lambda T) - 1)},$$

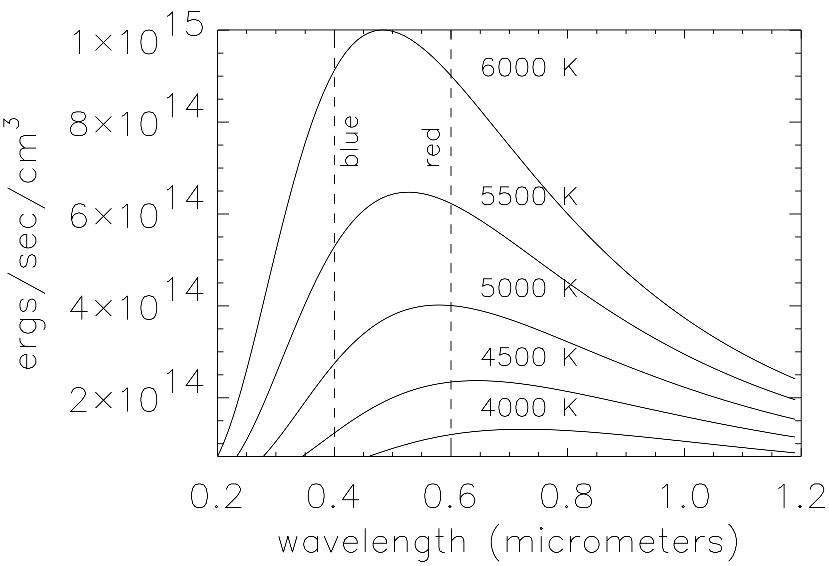

Figure 5.1: The Planck function for several temperatures. The dashed lines show the approximate wavelength range that can be discerned by the eye.

with $c_1 = 3.74185^{-5}$ erg cm^2 s^{-1} and $c_2 = 1.43883$ cm K (note I have given up on MKS units and have switched to cgs, since that what everyone always uses for this formula). A plot of the blackbody spectrum for a range of temperatures is shown in Figure 5.2

There are a two basic properties that you should remember about blackbody radiation. First, higher temperatures have peaks at lower wavelengths (blue hot) while lower temperatures have peaks at redder wavelenghts (red hot). Second, although cooler blackbodies peak at longer wavelenghts, hotter blackbodies are brighter than cooler blackbodies *at all wavelengths.*

Figure 5.2 shows that we can measure the temperature of an object simply by measuring the peak wavelenght of the blackbody emission. The surface of the sun is approximately 6000 K and it peaks around 0.5 μm. The earth, with a surface temperature of about 300 K, emits blackbody radiation that peaks at about 10 μm. The precise peak of the radiation can be found by differentiating the Planck function and setting the derivative equal to zero to see that

$$\lambda_{\max} = 2900/T \ \ \mu\text{m}.$$

If the blackbody peak is redward of about 0.6 μm (temperature cooler than about 4800 K, the object will look redish. If the peak is blueward of 0.4 μm (temperature greater than about 7250 K), the object will look blueish. This fact is the basis for our understanding that red-hot is hot, but blue-hot is even hotter. Something with a temperature between 4800 and 7250 K, like the sun, will appear neither blue nor red, but somewhere in between.

Knowing all of this, what is the approximate temperture of the filament in a light bulb? And why is the filament made of tungsten and placed in a vacuum seal?

Where does blackbody radiation come from, really? We are all familiar with the atomic emission at particular wavelenghts that occurs when an electron changes from one energy level to another, but this blackbody emission is a continuum, rather than a line, and doesn't depend on the properties of the emitting matter. Why?

The complete answer to this question again requires serious statistical mechanics to understand, but one way of thinking about it involves the vibrations within the solid latice of the object. Just like changes in the energy (orbital) of an electron leads to the emission of a photon, the change in energy of the vibration of the lattice also leads to the emission of a photon. And unlike electrons, which might only have a few allowed states, the vibrations can take a continuum of values, so the transition from one vibrational state to another leads to a continuum of radiation.

5.3 Frequency Units

The Planck function can be written in either *wavelength* or *frequency* units. Equation 5.1 gives the energy per unit area per unit time *per unit wavelength*, meaning it is in wavelength units. We could also instead write an equation for energy per unit area per unit time *per unit frequency* (which would end up being ergs/cm^2/sec/sec^{-1}, or simply ergs/cm^2!). To do this we use the fact that $\lambda = c/\nu$ and write

$$B_\lambda d\lambda = B_\nu d\nu$$

where B_λ is the Planck function in wavelength units and B_ν is in frequency units. We then know that

$$B_\nu = B_\lambda \frac{d\lambda}{d\nu}$$

or

$$B_\nu = -c/\nu^2 B_\lambda$$

or, explicityly,

$$B(\nu, T) = \frac{2\pi h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT) - 1}.$$

We could again differentiate this equation and find the frequency at which the blackbody emission peaks, but we would find something very suprising: the blackbody peaks at a different location when done in frequency units than when done in wavelength units. How can this be?

The reason is because wavelength and frequency are inversely proportional to one another, which means that a unit wavelenght and a unit frequency change as wavelength and frequency change. As an example, consider an iron bar with constant mass per unit length. Now let's decide to change units and instead of measuring mass per unit length, measure mass per unit inverse length. AT one side of the bar, where length is small, inverse length is large, so a unit inverse length is small, and the mass per unit inverse length is small. At the other end of the bar, inverse length is small, so unit inverse length is relatively large and mass per unit inverse length is large. The mass per unit inverse length is *not* constant even though it perfectly describes the mass distribution of the bar. Likewise, both wavelength and frequency unit Planck function perfectly describe the energy distribution of the radiation, they are just two ways of looking at the same thing. Is one more right than the other? In the example above, constant mass per unit length conveys more physical information more clearly than increasing mass per unit inverse length. For radiation, the natural unit to use is frequency rather than wavelength, as the energy of a photon is proportional to the frequency. Nonetheless both forms can be useful.

5.4 Total emission

Notice in Figure 5.2 that the total amount of emission from a blackbody (the integral of the spectrum over all wavelengths) increases rapidly with temperature. Temperatures below 4000 K essentially don't show up at all on the plot, and those above 6000 K are completely outside the range. Thus a room temperature metal bar, emitting blackbody radiation with a peak around 10μm doesn't feel like much, but increase the temperature to a few thousand degrees (red hot) and the radiation will begin to be felt from the bar. Increase to 10,000 degrees (blue hot) and the radiation will be quite intense.

We can do the integral of the Planck function and we find that the total blackbody intensity is given by

$$J = \sigma T^4 = 5.67 \times 10^{-8} \times T_{\mathrm{K}} \ \mathrm{W/m^2/K}$$

where σ is the Stefan-Boltzmann constant. This formula gives the total energy emitted per second into a single steradian for each square centimeter of the blackbody. An object with twice the surface area emits twice the total radiation. A 600 K object emits just as much as a 300 K object with 16 times the surface area (albeit the two spectra peak at different locations).

5.5 Thermal equilibrium

A blackbody (by definition) absorbs all of the radiation incident on it and then reemits it (as a blackbody). If the blackbody absorbs more radiation that it would emit at its temperature, it will heat up until it is emitting just as much radiation as it absorbs. Such a blackbody is said to be in *thermal equilibrium.*

We can calculate the thermal equilibrium blackbody temperature as a function of the distance from the sun. The sun has a surface temperature of 5800 K, a radius of 7×10^8m, and the earth-sun distance is 1.5×10^{11} m. In this case, the total power (energy/time) emitted by the sun is

$$P = \sigma T^4 A = 4 \times 10^{26}\mathrm{Watts}$$

so the power density (energy/time/area) at the distance of the earth is

$$J = 1400\mathrm{W/m}^2.$$

If a blackbody is both absorbing and emitting this amount of energy it must have a temperature of

$$T = (J/\sigma)^{1/4} = 396K.$$

Note that although the blackbody is in thermal equilibrium and is absorbing as much as it radiates, the spectrum of the absorbed and radiated energy is completely different. The solar spectrum peaks around 0.5 μm, while that of the blackbody peaks around 7.5 μm. In astronomy, the star light is said to have been *reprocessed* into the infrared.

This temperature is hotter than the temperature of the earth. Why is this? The main effect is that the earth is not a blackbody. A blackbody was defined as something that absorbed all energy radiant upon it which, by definition, would make it black. But the earth is not black (verify this for yourself by stepping outside and looking down). Instead of absorbing all of the light that is radiant on it, some of it is reflected. The light that is reflected does not go into heating, so the earth is cooler than a blackbody would be at this distance from the sun. To get to an average temperature of 300K, we need to absorb $(300/396)^{1/4} = 67\%$ of the incident light, which is approximately correct for the earth. The remaining 33% of the light, then, is reflected, and this is what we see when we look around at the scenery outside. (The real value of the earth's albedo is about 39%).

The amount of light reflected by a body is called the *albedo.* Objects in the solar system range in albedos from quite high for the cloudy planets (Venus, Jupiter, Saturn, Uranus, and Neptune are 72%, 70%, 90% and 82%, respectively), to medium for the icy satellites (55%, 50%, and 30% for Io, Europa, and Ganymede) to low for the rocky bodies (5.6%, 16%, and 14% for Mercury, Mars, and Pluto). Comets and asteroids are truly dark, with albedos somewhere near 4%. The surface temperatures of these bodies, then, will be related to the albedos. The objects with high albedos reflect most of the incoming sunlight, so will be anamolously cold, while the dark comets and asteroids will approach the blackbody temperature. Astronomers were initially concerned with reflected, rather than absorbed, light because reflected light is the light that you see when you look at a planet. The high albedo ones are much easier to see than the lower albedo ones.

The earth, in particular, changes albedo with changes in the weather and changes in the season. Clouds and ice are quite reflective, with albedos of something like 60%, while oceans and land surfaces are less reflective. During the northern winter, when the maximum amount of snow and ice is on the ground, the albedo of the earth is the highest, so the earth actually absorbs a little less of the available sunlight. This phenomenon leads to a potentially interesting global warming feedback. Imagine that the average temperature of the earth were to increase by a few degrees, melting some of the northern sea ice and causing smaller areas to be covered with snow. The total albedo of the earth would then decrease, leading to an increase in the energy absorption of the earth, and a subsequent increase in the temperature of the earth required to reradiate the higher amount of heat. But the higher temperature would lead to even less snow and ice and even higher temperatures. Effects like this one are almost impossible to predict and their existence is one of the reasons that the overall effects of global warming are so uncertain.