1 Double your pleasure

Consider a system, as illustrated below, that has two pendula of length $l$, with a mass $m$ at the bottom of each, that are connected by a spring with spring constant $k$.

(a) What are the equations of motion for the two masses, assuming that the pendulum motions are small (so $\theta = \sin \theta$) and that the spring can be approximated to be completely horizontal at all times (this would be impossible, of course, but what we are really assuming is that the vertical motions of the springs are much much smaller than the length of the spring, so that we need only consider the horizontal motion of the masses for calculating the stretching of the spring).
(b) What are the frequencies of this oscillation?
(c) What are the modes?

2 L.A. Strings

For an infinite string with mass per unit length \( p \) and tension \( T \), we found the wave equation to be

\[
\frac{\partial^2 u}{\partial t^2} = \frac{v^2}{\partial x^2}
\]

and we found that one solution to the equation was

\[
u(x, t) = u_1 \exp(ikx + ikvt) + u_2 \exp(ikx - ikvt).
\]

Show that, in fact, any function of the form

\[
u(x, t) = f(x + vt) + g(x - vt)
\]

is a solution to the wave equation.

Consider now a function \( h(x) \) where, for all values of \( x \) less than zero or greater than one, the function is equal to zero. Between 0 and 1, though \( h(x) \) is a silhouette of the downtown LA skyline (hmmm... is there such a thing?).

If \( u(0) = h(x) \) and \( \dot{u}(0) = v \frac{dh}{dx} \), what does the solution to the wave equation look like at a later time? Sketch the solution at a few times in the future. What if \( \dot{u}(0) = -v \frac{dh}{dx} \)? If \( \dot{u}(0) = 0 \)?

3 Seriously coupled springs

Program \texttt{springs.m} calculates the position of a set of \( n \) masses connected by \( n + 1 \) springs which are eventually connected to the wall. As the program is currently set up, the number of masses is 19, the number of springs is 20, all masses are the same, all springs are the same, and the system is initially perturbed by moving the first mass away from the wall.

The program works by calculating the acceleration felt by each mass, first from the spring on the left, then from the spring on the right, and keeping track of the changing velocity and position of the mass.

(a) Figure out how the program works (or, alternatively, if you prefer, write your own program from scratch).

(b) Run the program. The initial perturbation eventually travels all the way to the right wall. How long does it take for this signal to reach the right wall? Modify the program so that all of the spring constants are 10 times higher. How long does the signal take to reach the right wall now? What do you think the velocity of signal propagation is, in terms of \( k \), \( m \), and \( l \) (just use \( m \) and \( l \) to make the units come out correctly).
(c) Modify the program so that the motions are now damped. Use a damping constant of 1, in the units of the problem. How long does it take for most of the motion to die away? What if the damping constant is 0.1?