Ge/Ay 132

Problem set # 4

Due Tuesday, November 29th (let us know if you need a bit more time).

1. The isotopic distribution of water vapor in the Earth's troposphere and stratosphere can tell us a great deal about the transport of heat and the nature of climate. The figure below presents data from the NASA ATMOS experiment, which uses the Sun as a background source to take IR spectra of the Earth's statosphere. Using the HITRAN database, answer the following:

a. What water bands are the various H_2O transitions from, and what are the quantum numbers of the individual transitions shown?
b. For the given tangent heights listed, over what distance along the tangent must one travel before the pressure drops by a factor of 1/e?
c. Use the answer from b as a rough guess of the pathlength along with the absorption occurs, the quantitative intensities from HITRAN, and the spectra to calculate the coumn density of the H_2O isotopomers along the line-of-sight. What is the total column of air along this same path (needed to calculate the so-called mixing ratio of water in the stratosphere)? How do the ratios of these column densities compare with what you might expect for the same ratios at ground level? For more on this topic, see C.P. Rinsland et al. (1991), JGR 96, 1057; from which the figure was taken, or L. Moyer et al. (1996), GRL 23, 2385. The latest description of the HITRAN catalog may be found in L.S. Rothman et al. (1998), JQSRT 60, 665.

To do this, a good place to grab selected data from molecules and given isotopologues, generate plots, etc. using the HITRAN database is the HITRAN on the web environment, at `http://hitran.iao.ru/` The last page of this set presents a table overview of the HITRAN data format and catalog units, etc.

2. And now for something completely different! Even though CO rotational lines are optically thick, they can still be used as mass tracers of cold dense gas (within the context of Large Velocity Gradient models of molecular clouds). For the Milky Way, the standard H_2 mass-to-CO luminosity conversion factor is 4.6 $M_\odot$/ K km s^{-1} pc^2. Let's think about how hard it will be to detect CO from galaxies at various redshifts. For this you will need to know that the CO luminosity, in K km s^{-1} pc^2, can be written as

$$L'_{\rm CO} \quad = \quad 3.25 \times 10^7 S_{\rm CO} \Delta V \nu_{obs}^{-2} D_L^2 (1+z)^{-3} \quad ,$$

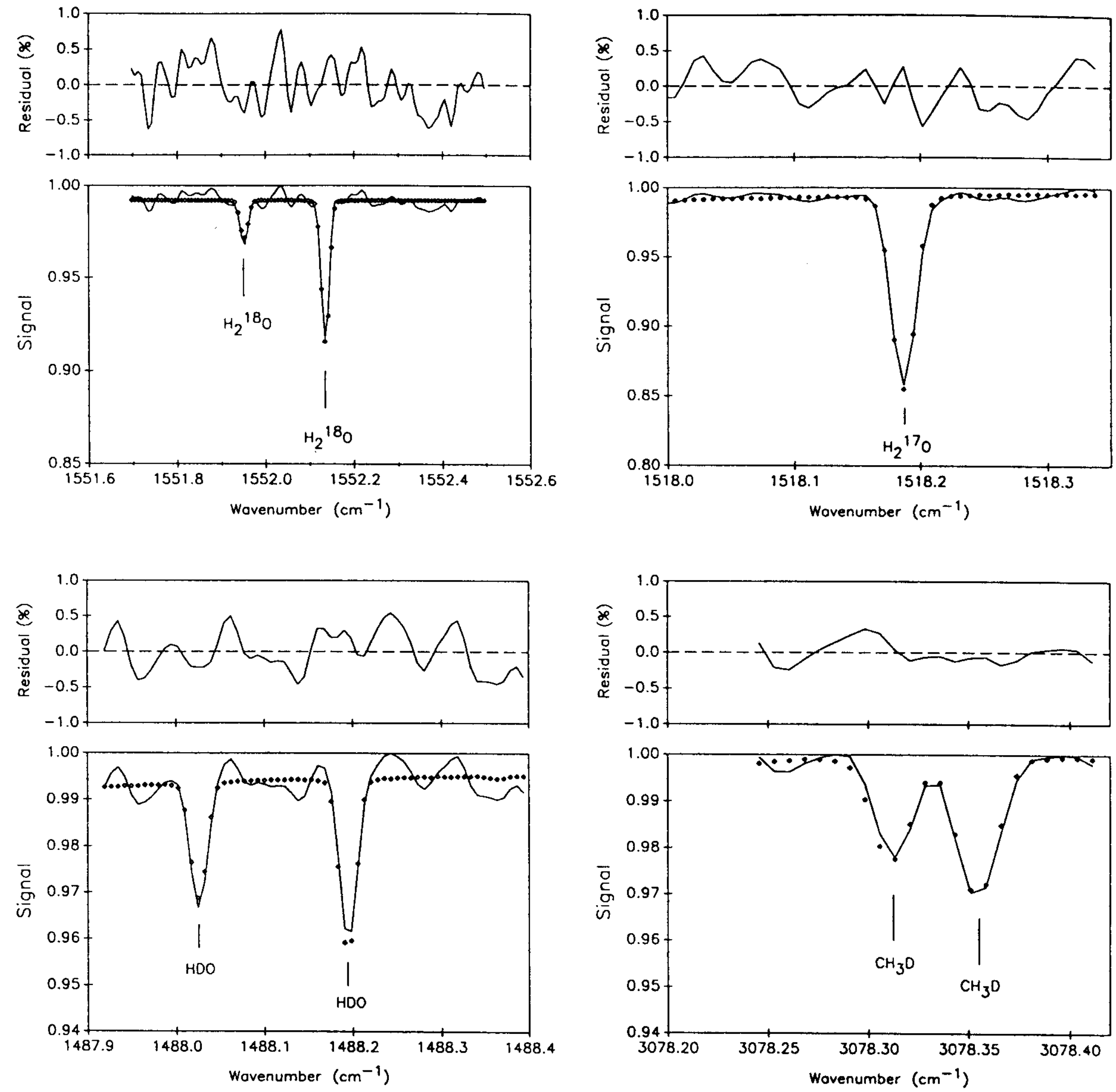
where the integrated CO line flux $S_{\rm CO}\Delta V$ is in Jy km s^{-1}, the observed frequency ν_{obs} is in GHz, and z is the redshift. D_L is called the *luminosity distance* (in Mpc, one of the many distances used in cosmology), and for the critical density universe cosmology consistent with WMAP and other microwave anisotropy results is given by

$$D_L \quad = \quad \frac{2c}{H_0} \left[1 + z \; - \; (1+z)^{1/2} \right] \quad .$$

The Hubble constant H_0 equals 71±4 km s^{-1} Mpc^{-1} from WMAP analyses, and c is the speed of light. Gas rich galaxies such as M51 contain ~10^{10} M$_\odot$ of H_2.

a. Consider the CO J=3→2 line. For M51 and galaxies with 20× and 400× M51's gas content, plot the integrated CO J=3→2 flux in Jy km s^{-1} for $z = 0.1 \rightarrow 10$.
b. Notice the factor of ν_{obs}^{-2} in the luminosity equation above. Since $\nu_{obs} = \nu_{rest}/(1+z)$, this dramatically decreases the flux for a given line at high redshift. Thus, another game to play is to keep ν_{obs}~constant, and use different transitions as they are redshifted into a given frequency window. Make the same calculations as in part a., but keep the frequency fixed at ~100 GHz. How much more flux do you get at $z \sim 5$? This approach only works, of course, as long as the temperature of the gas is high enough to excite the higher-J levels.
c. Given these numbers, what can be done? At the (now decommissioned) OVRO Millimeter Array, 50 hours of integration gives a 4σ RMS of ~1 Jy km s^{-1} in 500 km s^{-1} spectral channels. The Plateau de Bure interferometer has about twice the sensitivity (larger collecting area), and so reaches the same RMS levels in about 12 hours. For a galaxy with M(H_2)=10^{12} M$_\odot$, how long would these arrays need to integrate to achieve 4σ detections at $z = 5$ (at ~100 GHz) in velocity channels that are 55 km/s in width? The Atacama Large Millimeter Array (ALMA) can now get down to closer to ~10^{-3} Jy km s^{-1} in ten hours at 100 GHz (4σ). How long would it take ALMA to detect M51 at a redshift of 10 at the same velocity resolution? Assume in all cases that the lines are Gaussian in shape with a FWHM of 300 km/s.

As an aside, it turns out that galaxies at high redshift are often *lensed* by other galaxies or galaxy clusters that lie between us and the high redshift object. In favorable cases, the magnification factor can be substantially larger than 10. The flux from CO is boosted accordingly, and so the galaxies are easier to detect than the numerical estimates above would suggest. Indeed, nearly all of the high redshift detections in the pre-ALMA era were of lensed systems. In order to derive accurate estimates of the galaxy mass and the like, however, it is necessary to characterize the magnification accurately; and so this introduces some uncertainties into the analysis of luminous, gas rich systems at $z > 1$.



Examples of the ATMOS/Spacelab 3 and least squares best calculated fit (solid diamonds) for the infrared absorption limb sounding spectroscopy of isotopic water vapor and methane. The tangent heights and corresponding tangent pressures are 55.7 km and 0.42 mbar for $H_2^{18}O$; 29.8 km and 12.4 mbar for $H_2^{17}O$ and HDO. The CH_3D are for an extensive zonal average, the effective tangent height and corresponding tangent pressure 26.0 km and 21.8 mbar.

Example of HITRAN line-transition format.

Mol /Iso	$\nu_{\eta\eta'}$	$S_{\eta\eta'}$	$\mathcal{R}_{\eta\eta'}$	γ_{air}	γ_{self}	E''	n	δ	iv′	iv″	q′	q″	ierr	iref
21	800.451076	3.197E-26	6.579E-05	.0676	.0818	2481.5624	.78	.000000	14	6		P 37	465	2 2 1
291	800.454690	9.724E-22	1.896E-02	.0845	.1750	369.6303	.94	.000000	9	1	341619	331519	000	4 4 1
291	800.454690	3.242E-22	2.107E-03	.0845	.1750	369.6303	.94	.000000	9	1	341519	331419	000	4 4 1
121	800.455380	1.037E-22	1.657E-03	.1100	.0000	530.3300	.75	.000000	32	14	46 640	45 540	000	4 4 1
121	800.455380	1.037E-22	1.657E-03	.1100	.0000	530.3300	.75	.000000	32	14	46 740	45 640	000	4 4 1
101	800.456743	1.680E-23	1.659E-04	.0670	.0000	851.0494	.50	.000000	2	1	45 244 0-	44 143 0-	301	6 6 1
101	800.457045	1.710E-23	1.689E-04	.0670	.0000	851.0469	.50	.000000	2	1	45 244 1-	44 143 1-	301	6 6 1
101	800.457310	1.740E-23	1.718E-04	.0670	.0000	851.0442	.50	.000000	2	1	45 244 2-	44 143 2-	301	6 6 1
121	800.457760	4.726E-23	4.614E-03	.1100	.0000	920.0900	.75	.000000	32	14	502922	492822	000	4 4 1
121	800.457760	4.726E-23	4.614E-03	.1100	.0000	920.0900	.75	.000000	32	14	502822	492722	000	4 4 1
24	800.465942	9.792E-27	6.063E-04	.0754	.1043	1341.2052	.69	.000000	8	3		R 13	425	2 2 1
121	800.466160	1.061E-22	2.720E-03	.1100	.0000	632.1200	.75	.000000	32	14	471236	461136	000	4 4 1
121	800.466160	1.061E-22	2.720E-03	.1100	.0000	632.1200	.75	.000000	32	14	471136	461036	000	4 4 1
35	800.472900	3.878E-26	6.919E-04	.0686	.0871	629.0354	.76	.000000	2	1	1814 4	1713 5	455	5 5 1
101	800.473083	1.270E-23	1.254E-04	.0670	.0000	851.0095	.50	.000000	2	1	45 244 0+	44 143 0+	301	6 6 1
101	800.474860	1.210E-23	1.195E-04	.0670	.0000	851.0064	.50	.000000	2	1	45 244-1+	44 143-1+	301	6 6 1
31	800.475500	1.680E-24	3.617E-05	.0653	.0890	1092.4340	.76	.000000	2	1	51 547	50 248	002	1 1 2
291	800.476220	9.597E-22	6.010E-03	.0845	.1750	361.9747	.94	.000000	9	1	341420	331320	000	4 4 1
291	800.476220	3.199E-22	6.010E-03	.0845	.1750	361.9747	.94	.000000	9	1	341520	331420	000	4 4 1
101	800.476937	1.160E-23	1.145E-04	.0670	.0000	851.0037	.50	.000000	2	1	45 244-2+	44 143-2+	301	6 6 1
101	800.484334	1.740E-23	2.153E-05	.0670	.0000	106.0760	.50	.000000	2	1	8 4 4-1+	9 3 7-1+	301	6 6 1

FORTRAN Format (I2,I1,F12.6,1P2E10.3,0P2F5.4,F10.4,F4.2,F8.6,2I3,2A9,3I1,3I2) corresponding to:

Mol	I2-	Molecule number
Iso	I1-	Isotope number (1= most abundant, 2= second most abundant, etc.)
$\nu_{\eta\eta'}$	F12.6-	Frequency in cm^{-1}
$S_{\eta\eta'}$	E10.3-	Intensity in $cm^{-1}/(molecule \cdot cm^{-2})$ @ 296K
$\mathcal{R}_{\eta\eta'}$	E10.3-	Weighted transition moment-squared in $Debye^2$
γ_{air}	F5.4-	Air-broadened halfwidth (HWHM) in cm^{-1}/atm @ 296K
γ_{self}	F5.4-	Self-broadened halfwidth (HWHM) in cm^{-1}/atm @ 296K
E''	F10.4-	Lower state energy in cm^{-1}
n	F4.2-	Coefficient of temperature dependence of air-broadened halfwidth
δ	F8.6-	Air-broadened pressure shift of line transition in cm^{-1}/atm @ 296K
iv′,iv″	2I3-	Upper state global quanta index, lower state global quanta index
q′,q″	2A9-	Upper state local quanta, lower state local quanta
ierr	3I1-	Accuracy indices for frequency, intensity, and air-broadened halfwidth
iref	3I2-	Indices for table of references corresponding to frequency, intensity, and halfwidth