Ge/Ay 132

Problem set # 2

You are allowed to use the class notes and books to solve these problems. Collaboration is permitted, but please write up your solutions independently. Don't hesitate to contact us if there are ambiguities in the questions. The questions are worth 25, 15, 35 and 25 points. Due Friday, November 2^{nd}. Good luck!

1. The following table lists the energy differences, $\Delta E = E(J = 3/2) - E(J = 1/2)$ in cm^{-1} for the fine structure levels of the ground terms 3p $^2\mathrm{P}^o_J$ of several ions of the Aℓ I isoelectronic sequence.

Fine Structure Splittings

Ion	Z	ΔE (cm^{-1})
Ti X	22	7542
V XI	23	9696
Cr XII	24	12261
Mn XIII	25	15320

a. Find an expression of the form $\Delta E = AZ^B$ that represents these data fairly well.
b. During the solar eclipse in 1869, an emission line was observed in the solar corona at a wavelength of 5303 Å. Armed with the empirical relation found under a), suggest a possible identification for this emission line.
c. Estimate very roughly the temperature at which this ion would be the dominant (i.e., most abundant) form of its element in thermodynamic equilibrium. Assume that the electron density in the solar corona is $N(e) \approx 1.2 \times 10^{10}$ cm^{-3}.

2. Determine $n(\mathrm{C}^+)$ in diffuse clouds where the process

$$h\nu \ + \ \mathrm{C} \ \rightarrow \ \mathrm{C}^+ \ + \ e^- \qquad \Gamma = \ 10^{-10} \ s^{-1}$$

determines the production of C^+ and electrons (i.e. assume that C^+ carries all the ionization). Assume the major loss of C^+ to be

$$\mathrm{C} \ + \ e^- \ \rightarrow \ \mathrm{C} \ + \ h\nu \qquad k = \ 10^{-12} \ cm^3 s^{-1}$$

as well as steady state. The total carbon abundance is $n_\circ(\mathrm{C}) = 10^{-4} n(\mathrm{H})$, and you can assume the cloud has a physical density of 100 cm^{-3} (H + H_2). What is the fraction of ionized carbon relative to the total carbon, or $n(\mathrm{C}^+)/n_\circ(\mathrm{C})$ [you might want to start here]?

3. Consider the sulfur ion S^+ with the configuration $3s^2\ 3p^3$, and the corresponding terms ^{4}S, ^{2}D and ^{2}P. The energy levels are illustrated in the accompanying figure and the table below summarizes the relevant atomic data.

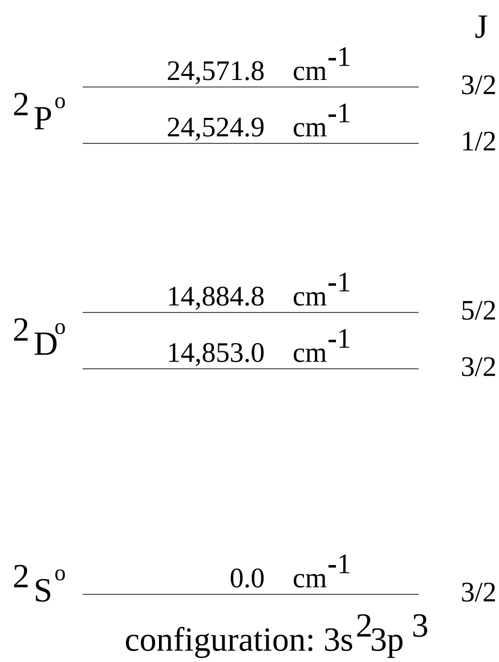
a. What is the critical density for each of the levels?

b. Set up the rate equations for the level populations for the 3–level system consisting of the ^{4}S, ^{2}D and ^{2}P terms (i.e., treat the ^2D_J & ^2P_J fine–structure levels as one level). Which terms in the equations are likely to be the dominant contributors to the populations of each of the levels?

c. Solve for the relative populations of the 3 levels assuming $T = 8 \times 10^3$ K and $n(e)$=100 cm^{-3}, taking only the most important terms into account. For extra credit, set up and solve the full statistical equilibrium equations numerically.

d. What is the ratio of the $^2D_{5/2}$–$^4S_{3/2}$ and $^2D_{3/2}$–$^4S_{3/2}$ line intensities at low densities? And at high densities? In what regime is this line ratio most sensitive to density?

Transition	Upper J	Lower J	Ω	A (s^{-1})
$^2D^o$-$^4S^o$	5/2	3/2	4.19	2.60×10^{-4}
	3/2	3/2	2.79	8.82×10^{-4}
$^2P^o$-$^4S^o$	3/2	3/2	1.52	0.225
	1/2	3/2	0.759	0.0906
$^2D^o$-$^2D^o$	5/2	3/2	7.59	3.35×10^{-7}
$^2P^o$-$^2P^o$	3/2	1/2	2.38	1.03×10^{-6}
$^2P^o$-$^2D^o$	3/2	5/2	4.79	0.179
	3/2	3/2	3.38	0.133
	1/2	5/2	2.56	0.0779
	1/2	3/2	1.52	0.163



SII Energy Levels

4. Consider the molecule thioformaldehyde, H_2CS.

a. Using the asymmetric rotor energy level formulas given in Table 8.4, calculate the first few (J=0, 1 and 2) rotational energy levels of the molecule. The inertial moments are A= 291291.641 MHz, B=17699.628 MHz, and C= 16651.83 MHz. Draw an energy level diagram.

b. The molecule can be classified as a near–prolate top. Using the approximate selection rules for this case, indicate in the diagram which transitions are electric dipole allowed.

c. Do the transitions lie in the centimeter, millimeter, submillimeter or far–infrared part of the spectrum?