1. Can the existence of diatomic molecules in the interstellar medium be explained only by disruption of larger grains (from stellar mass loss)?

Want to compare the observed mass input (from stellar mass loss) with the observed diatomic molecule abundance (predominantly H2):

\[
\text{INPUT} = 4 \times 10^{-10} \left[ \text{M}_{\odot} \text{yr}^{-1} \text{pc}^{-2} \right] \times 1.98 \times 10^{33} \left[ \text{g/M}_{\odot} \right] \times \frac{1}{30} \left[ \text{moles/g} \right] \times 6.022 \times 10^{23} \left[ \text{molec/mol} \right] \times \pi R_{\text{galaxy}}^2 \left[ \text{pc}^2 \right] 
\]

and with \( R_{\text{galaxy}} = 2 \times 10^4 \text{pc} \) ==> \( \text{INPUT} = 2.01 \times 10^{55} \left[ \text{molec/yr} \right] \)

Now we calculate the loss rate of observed diatomics:

\[
\text{observed # density of H atoms: } 1 \text{cm}^{-3} \\
\text{observed # density of diatomics: } 10^{-7} \times 1 \text{cm}^{-3} = 10^{-7} \text{cm}^3 
\]

So the total number of diatomics observed in the galaxy is:

\[
N_{\text{observed}} = 10^{-7} \left[ \text{cm}^{-3} \right] \times \pi R_{\text{galaxy}}^2 \left[ \text{pc}^2 \right] \times n_{\text{galaxy}} \left[ \text{pc} \right] \times 300 \text{pc} \\
\times 3.09 \times 10^{18} \left[ \text{cm/pc} \right] \\
= 1.11 \times 10^{60} \text{ molecules in galaxy (observed)} 
\]

Dissociation rate of \( 1.11 \times 10^{-10} \text{ sec}^{-1} = \Gamma \) ==> \( \tau = 1/\Gamma \sim 300 \text{ yrs} \)

so the average lifetime of a diatomic is \( \sim 300 \text{ yrs} \)

\[
\Rightarrow \text{equilibrium destruction rate} = \text{OBSERVED LOSS} = \frac{N_{\text{observed}}}{\tau_{\text{destruction}}} = 3.69 \times 10^{57} \text{ molec/yr} 
\]

To balance this observed loss, we would have to input molecules at the same rate. But the observed input rate from stellar mass loss is only:

\[
\frac{\text{OBSERVED INPUT}}{\text{REQUIRED INPUT}} = \frac{2.01 \times 10^{55} \left[ \text{molec/yr} \right]}{3.69 \times 10^{57} \left[ \text{molec/yr} \right]} = 0.005 \text{ (0.5\% of the required amt.)} 
\]

So other sources are necessary to produce molecules in the interstellar medium, ie we must make bonds.
2. We are given: 
\[ d\tau = n_{\text{inf}} \frac{\pi e^2}{m_e c} \phi(v) * g_{\text{af}} * dL \] 
(Note that this expression does not include the oscillator strength. This parameter depends on the particular atom or molecule involved, and for HI it is \(\sim 0.41\). The above equation will give numbers \(\sim 2\) times reality because of this emission.)

The total optical depth is 
\[ \tau = \int_{L}^{1} d\tau \]

-if we assume all factors are constant over the pathlength:
\[ \tau = 1 = n_{\text{inf}} \frac{\pi e^2}{m_e c} \phi(v) * g_{\text{af}} * L \quad \Rightarrow \quad L = \frac{m_e c}{n_{\text{inf}} \frac{\pi e^2}{m_e c} \phi(v) * g_{\text{af}}} \]

This curve drops mostly due to \(\phi(v)\) (a little due to \(g_{\text{af}}\)).

What is the path length at 912 Å and 180 Å?

912 Å 
\[ \nu = 3.29 \times 10^{15} \text{ Hz} \]
\[ g_{\text{af}} = 0.797 \quad (\text{Spitzer, p.106}) \]

\[ \Rightarrow L (912 \text{ Å}) = \frac{9.1 \times 10^{-28} * 3 \times 10^{10} * 3.29 \times 10^{15}}{1 * 2\pi (4.8 \times 10^{-10})^2 * 0.797} = 7.778 \times 10^{16} \text{ cm} \]

using 1 pc = 3.08 \times 10^{18} \text{ cm}

\[ \Rightarrow L(912 \text{ Å}, \tau=1) = 0.025 \text{ pc} \]

150 Å 
\[ \nu = 1.67 \times 10^{16} \text{ Hz} \]
\[ g_{\text{af}} = 0.994 \quad (\text{Spitzer, p.106}) \]

\[ \Rightarrow L (150 \text{ Å}) = \frac{9.1 \times 10^{-28} * 3 \times 10^{10} * 1.67 \times 10^{16}}{1 * 2\pi (4.8 \times 10^{-10})^2 * 0.994} = 8.12 \times 10^{18} \text{ cm} \]

\[ \Rightarrow L(150 \text{ Å}, \tau=1) = 2.63 \text{ pc} \]

Dense clouds are on the order of several parsecs in diameter. The typical pathlength for a
912 Å photon is << cloud size so this transition is very sharp. Viewing a dense cloud at 912 Å, we would only see the outer surface. At 180 Å, L is on the order of the cloud size so viewing a could at 180 Å, we could see deeper into the cloud.

3. The rate at which atoms stick to a grain surface is just the product of the collision rate and the sticking probability f(x).

The collision rate is given by:

\[
\text{Rate} = (n_{\text{grain}} \pi a^2) \ast v \ast n_{\text{atom}}
\]

where \( v \) = thermal velocity \( \epsilon = n_{\text{grain}} \pi a^2 \) = total grain crosssectional area

-so the sticking rate is:

\[
\text{Rate} = dn_{\text{atom}}/dt = - \epsilon v n_{\text{atom}} f_{\text{atom}}
\]

\[
\Rightarrow \frac{dn_{\text{atom}}}{n_{\text{atom}}} = - \epsilon v n_{\text{atom}} f_{\text{atom}} dt \quad \Rightarrow \quad n_{\text{atom}}/n_{\text{initial}} = e^{-\epsilon v f t}
\]

So we have an expression for the depletion as a function of time:

Depletion is defined as:

\[
D = \log_{10} \left( \frac{n}{n_{\text{cosmic}}} \right) \quad \text{(Class notes, p. 2 caption in table)}
\]

\[
= \log \left[ \left( \frac{n}{n_0} \right) \ast \left( \frac{n_{\text{cosmic}}}{n_0} \right) \right]
\]

where \( n_0/n_{\text{cosmic}} = 0.1 \) (given)

for \( \text{Ca}^{++} \):

\[
n(\text{Ca}^{++})/n_0(\text{Ca}^{++}) =
\]

where \( \epsilon = 1.8 \times 10^{-21} \ast 90 \)

\[
v = \text{thermal velocity} = 1.29 \times 10^4 \ (T/M)^{1/2} \quad \text{(with M in amu)}
\]

\[
= 1.29 \times 10^4 \ (95/40)^{1/2} = 1.99 \times 10^4 \text{ cm/sec}
\]

\[
f = 1.0
\]

\[
t = 10^{15} \text{ sec}
\]

\[
\Rightarrow D(\text{Ca}^{++}) = \log [3.98 \times 10^{-2} (0.1)]
\]

\[
= -2.40
\]

for \( \text{Si}^{++} \):

\[
n(\text{Si}^{+})/n_0(\text{Si}^{+}) =
\]

where \( \epsilon = 1.8 \times 10^{-21} \ast 90 \)

\[
v = 1.99 \times 10^4 \text{ cm/sec}
\]

\[
f = 0.25
\]

\[
t = 10^{15} \text{ sec}
\]

\[
\Rightarrow D(\text{Ca}^{++}) = \log [4.47 \times 10^{-1} (0.1)]
\]
This helps illustrate how important the sticking probability is.

4.
   a) crossectional area of the grain:
      \[ r = 1000 \text{ Å} = 1 \times 10^{-5} \text{ cm} = \text{grain radius} \]
      \[ \pi r^2 = 3.14 \times 10^{-10} \text{ cm}^2 \]

   total area that \( 10^{37} \text{ ergs/sec} \) passes through at \( 400 \text{AU} \):
      \[ 1 \text{ AU} = 1.5 \times 10^{13} \text{ cm} \]
      \[ 4 \pi R^2 = 4 \pi (400 \times 1.5 \times 10^{13})^2 \]
      \[ \text{R}=\text{distance from star to grain} \]
      \[ = 4.5 \times 10^{32} \text{ cm}^2 \]

   the grain receives: \[ \frac{3.14 \times 10^{-10} \times (10^{37})}{4.5 \times 10^{32}} = 6.98 \times 10^{-6} \text{ ergs/sec} \]

radiates as a blackbody:
   \[ F_{\text{out}} = (Te^4) \times \text{surface area} \]
   \[ = Te^4 (5.67 \times 10^{-5}) 4 \pi (1 \times 10^{-5}) \]
   \[ = Te^4 (7.13 \times 10^{-14}) \]

At equilibrium, Power in = Power out

\[ \implies 6.98 \times 10^{-6} \text{ ergs/sec} = Te^4 (7.13 \times 10^{-14}) \]

\[ \implies Te = 99.5 \text{ K} \]

\((\text{H}_2\text{O begins subliming in ultra high vacuum at } \sim 130\text{K})\)

More generally: Power in = Power out

\[ P_{\text{in}} = L_{\text{sun}} \left( \frac{\pi r^2}{4\pi R^2} \right), \text{ where } r = \text{grain radius}, R = \text{distance from star to grain} \]

\[ P_{\text{out}} = Te^4 4\pi r^2 \]

\[ L_{\text{sun}} \left( \frac{\pi r^2}{4\pi R^2} \right) = Te^4 4\pi r^2 \]

\[ T = \left( \frac{L_{\text{sun}}}{16 \pi R^2} \right)^{\frac{1}{4}} \text{ or } T \left( \frac{1}{R} \right)^{\frac{1}{2}} \]

- also note that the temperature does not depend on grain size!
4. b) If the grain is silicate in composition, \( c \approx 3 \text{cal/gmK} \) and \( \approx 3 \text{ gm/cm}^3 \)

The temperature rise is just: \( T = \frac{E}{cM} \)
where \( E = \text{energy input} \)
\( c = \text{heat capacity} \)
\( m = \text{mass} \)

\[ T = \frac{E}{c \left(\frac{4}{3} \pi r^3\right)} \]

for a 2000 Å grain:
\[ T = \frac{10 \text{ eV} \times 3.83 \times 10^{-20} \text{ cal/eV}}{3 \text{ cal/gmK} \times 3 \text{ gm/cm}^3 \times 4/3 \pi (1000 \times 10^{-8} \text{ cm})^3} = \frac{1.02 \times 10^{-20}}{(1000 \times 10^{-8} \text{ cm})^3} = 1.02 \times 10^{-5} \text{ K} \]

for a 200 Å grain:
\[ T = \frac{1.02 \times 10^{-20}}{(100 \times 10^{-8} \text{ cm})^3} = 1.02 \times 10^{-2} \text{ K} \]

for a 20 Å grain:
\[ T = \frac{1.02 \times 10^{-20}}{(10 \times 10^{-8} \text{ cm})^3} = 10.2 \text{ K} \]

So you can see the heating is not significant for "typical" photons (< 13.6 eV) absorbing, even for the smallest grains. Cosmic rays (with energies often \( 10^5 \) - \( 10^6 \) ev) can do a lot more damage.

5. The goal of this problem is to show that atomic carbon is predominantly ionized in diffuse clouds. Simplifying the chemistry:

Production: \( h + \text{C} \rightarrow \text{C}^+ + e^- \quad = 10^{10} \text{ s}^{-1} \)
Loss: \( \text{C}^+ + e^- \rightarrow h + \text{C} \quad k = 10^{-12} \text{ cm}^3 / \text{s} \)

\[ \frac{dn (\text{C}^+)}{dt} = n(C) - k \ n(\text{C}^+) \ n(e^-) = 0 \quad \text{(steady state)} \]

-since \( \text{C}^+ \) carries all the ionization \( \Rightarrow n(\text{C}^+) = n(e^-) \)

\[ n(C) = k \ n^2(\text{C}^+) \]
Now if we let \( n_o(C) = \) total carbon = \( n(C^+) + n(C) \) (no molecules)

\[
\begin{align*}
&\Rightarrow \quad (n_o(C) - n(C^+)) = k n^2(C^+) \\
&\Rightarrow \quad k n^2(C^+) + \frac{1}{k} n(C^+) - \frac{1}{k} n_o(C) = 0
\end{align*}
\]

Solving the quadratic equation, we get:

\[
\begin{align*}
n(C^+) &= -\frac{1}{2k} \left[ \left( \frac{1}{k} \right)^2 + 4 \frac{1}{k} n_o(C) \right]^{\frac{1}{2}} \\
n(C^+) &= -\frac{1}{2k} \left[ 2k \left( 1 + 4 \frac{k}{n_o(C)} \right) \right]^{\frac{1}{2}}
\end{align*}
\]

The negative root gives a non-physical result. Plugging in the values,

\[
\begin{align*}
n(H) &\sim 100 \text{ cm}^{-3} \quad \Rightarrow \quad n_o(C) \sim 10^{-2} \text{ cm}^{-3} \\
n(C^+) &= -10^{-10} / (2\times10^{-12}) + 10^{-10} / (2\times10^{-12}) \left[ 1 + 4(10^{-2}) \ast (10^{-12} / 10^{-10}) \right] \\
n(C^+) &= 9.999\times10^{-7} / (2\times10^{-12}) \quad \text{and} \quad n_o(C) \sim 10^{-2} \text{ cm}^{-3}
\end{align*}
\]

\[
n(C^+)/n_o(C) = 0.9999 = 99.99 \% \quad \text{(of carbon is ionized in diffuse clouds)}
\]

6. \( \text{OH}^+ + \text{CO} \rightarrow \text{HCO}^+ + \text{O} \)

Table 3.1 of Duley & Williams: \( \text{(CO)} = 1.95\times10^{-24} \text{ cm}^3 \)

\[
\begin{align*}
M_{\text{OH}} &= 17/6.02\times10^{-23} = 2.82\times10^{-27} \text{ g/molecule} \\
M_{\text{CO}} &= 28/6.02\times10^{-23} = 4.52\times10^{-23} \text{ g/molecule} \\
\Rightarrow &= M_{\text{OH}} M_{\text{CO}} / (M_{\text{OH}} + M_{\text{CO}}) = 1.75\times10^{-23} \text{ g}
\end{align*}
\]

so: \( k = 2\pi \frac{q}{\lambda} \left( \frac{1}{l} \right)^{\frac{1}{2}} = 2\pi \left( 4.8\times10^{-10} \right) / \left( 1.95\times10^{-24} / 1.75\times10^{-23} \right)^{\frac{1}{2}} = 1.01\times10^9 \text{ cm}^3/\text{sec} \)

Eqn 3.10 of Duley & Williams gives: \( k_{\text{ADO}} = k_L \left[ 1 + (c/1.95^{\frac{3}{2}}) \right] \)

Given that \( c = 0.1 \text{ Debye}, \quad c/1.95^{\frac{3}{2}} = 0.1 \text{ Debye} / 0.7 \text{ [Debye/Å}^{3/2}] \)

-consulting figure 3.2, at reasonable temperatures \( (150 < T < 650 \text{ K}) \), \( c < 0.01 \).

The ADO correction term \( \left( 2^{nd} \text{ term in } k_{\text{ADO}} \text{ expression} \right) \) is thus:
using 1Debye = 10^{18} \text{ (erg-cm}^3)^{1/2}

correction term = (c \frac{d}{\sqrt{kT}})^{1/2} \sim c^*0.1x10^{-18}/(10^{-24}) (2/\pi 10^{-16}*10^2)^{1/2}

\sim c^*0.1x10^{-18}/(10^{-12} * 10^{-7}) - c

and as c < 0.01 << 1, the correction factor to the Langevin rate is very small and can be neglected.

\[ k_{ADO} \sim k_L \]

\[ H_3^+ + HCN -> HCNH^+ + H_2 \]

Table 3.1 of Duley & Williams : (HCN) = 2.59x10^{-24} \text{ cm}^3

\[ M_{H_3} = 3.02/6.02x10^{-23} = 5.02x10^{-24} \text{ g/molecule} \]

\[ M_{HCN} = (1+12+14) / 6.02x10^{-23} = 4.49x10^{-23} \text{ g/molecule} \]

\[ = M_{OH} M_{CO} / (M_{OH} + M_{CO}) = 4.52x10^{-24} \text{ g} \]

so: \[ k = 2\pi q / (c/\sqrt{kT})^{1/2} = 2\pi (4.8x10^{-10}) (2.59x10^{-24}/4.52x10^{-24})^{1/2} = 2.28x10^{-9} \text{ cm}^3/\text{sec} \]

Eqn 3.10 of Duley & Williams gives: \[ k_{ADO} = k_L [ 1 + (c \frac{d}{\sqrt{kT}})^{1/2} ] \]

Given that \[ d = 3 \text{ Debye}, \quad \frac{d}{\sqrt{kT}} = 3 \text{ Debye} / (2.59 \text{ A}^3)^{1/2} = 1.86 \text{ [Debye/A}^{3/2}] \]

figure 3.2 indicates

\[ c \sim 0.26 \text{ at 150K} \]
\[ c \sim 0.23 \text{ at 650K} \]

The ADO correction term (2nd term in \( k_{ADO} \) expression) is thus:

using 1Debye = 10^{18} \text{ (erg-cm}^3)^{1/2}

correction term = (c \frac{d}{\sqrt{kT}})^{1/2} \sim c^*3x10^{-18}/(2.59x10^{-24})^{1/2} (2/\pi 10^{-16}*10^2)^{1/2}

so:

\[ \text{at 150 K, } k_{ADO} = 3.7 \text{ } k_L \]
\[ \text{at 650 K, } k_{ADO} = 2.1 \text{ } k_L \]

So the correction factor cannot be neglected - ie, the Langevin rate is not adequate to describe such a reaction.

7. Need to find the rate of change of [HD]:

\[ \frac{d[HD]}{dt} = k [D][H_2] - [HD] \]
where
\[ k \text{[D][H}_2\text{]} = \text{the production via a given reaction} \]
\[ [\text{HD}] = \text{the loss via photodissociation} \]

Assuming steady state,
\[ k = \frac{[\text{HD}]}{k \text{[D][H}_2\text{]}} \]

As we know
\[ \frac{[\text{HD}]}{[\text{H}_2]} = 10^{-6} \text{ and } \frac{1}{10^{-10} \text{ s}^{-1}} \]
we only need to find [D] to determine k.

If we assume that all deuterium is either in D or HD in diffuse clouds (a legitimate estimate), and knowing that

\[ (\text{D/H})_{\text{cosmic}} \sim 2 \times 10^{-5} \]

since [HD] / [H] is only \(10^{-6}\), almost all deuterium is in the form of D. Thus,

\[ [\text{D}] / [\text{H}] \sim 2 \times 10^{-5} \], and as [H] \(\sim n_{\text{total}}\)

\[ \Rightarrow [\text{D}] \sim (2 \times 10^{-5}) \times (100) = 2 \times 10^{-3} \text{ cm}^3 \]

\[ k \sim 10^{-10} \times 10^{-6} / 2 \times 10^{-3} = 5 \times 10^{-14} \text{ cm}^3 \text{ s}^{-1} \]

then
\[ 5 \times 10^{-14} = 10^{11} e^{-E/kT} \]

\[ \Rightarrow E = 5.3 \text{ kT} \]

and, for \(T = 100\text{K}\) in a diffuse cloud:

\[ E \sim 530 \text{ K} \]