IMPULSE RESPONSE MODELS FOR NOISY VIBROSEIS† DATA*

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ABSTRACT

A new method of Vibroseis deconvolution has been recently proposed by the authors. This discussion describes the effects of noise on the application of this method. The initial deconvolution step involves estimating the spectrum of the Vibroseis wavelet by homomorphic filtering. It is shown that noise causes problems with phase estimation. Hence, the Vibroseis wavelet is assumed to be zero phase. Examples demonstrate that zero phase cepstral filtering is a robust wavelet estimation approach for noisy data. The second step of the deconvolution method forms an impulse response model by a spectral extension method. Although this step can improve the resolution of seismic arrivals, it must be applied with caution in view of the deleterious effects of noise.

INTRODUCTION
In this paper we discuss the Vibroseis deconvolution method presented by Lines and Clayton (1977). The same method has been applied to the deconvolution of earthquake source functions by Clayton and Wiggins (1976). Clayton and Ulrych (1977) show that the technique can be used for resolving astronomical, Vibroseis, and teleseismic data.

The proposed Vibroseis deconvolution method involves two steps.
1. The spectrum of the wavelet is estimated by zero phase homomorphic deconvolution. The wavelet’s spectrum is then removed from the trace spectrum by frequency domain division.

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2. The band limited deconvolution is then used to produce an optimum model for the impulse response of the Vibroseis trace. This model is obtained by using autoregressive prediction in the frequency domain.

In particular we shall discuss the application of this method to noisy data. Past experience has shown that phase estimation is particularly difficult in noisy environments.

**Effects of Noise on Phase Estimation**

The problem of estimating phase curves for noisy data can be appreciated by considering two diagrams in the complex plane. The phase of a dipole or two term time sequence is considered. This example has previously been described by Claerbout (1976).

The \( z \) transform of a dipole sequence \((1, a)\) is given by:

\[
W(z) = 1 + az.
\]  
(1)

Here \( z = e^{-j\omega} \) where \( \omega \) is the angular frequency. Using polar coordinates such that \( a = re^{j\omega_0} \) we can express \( W \) as:

\[
W = 1 + re^{j(\omega_0 - \omega)}
\]

or

\[
W = 1 + r \cos(\omega_0 - \omega) + ir \sin(\omega_0 - \omega).
\]

The phase of \( W \) is given by:

\[
\theta = \tan^{-1}\left(\frac{\text{Im}(W)}{\text{Real}(W)}\right).
\]

Figs 1(a), (b) show the phase behaviour for cases where \( r < 1 \) and \( r > 1 \).

First consider fig. 1a. When \( r \) is less than unity, the vector describing \( W \) stays in the right half of the complex plane and the phase for \( \theta = 0 \) is the same as for \( \theta = 2\pi \). (For real \( a \) the phase has a value of zero when the angular frequency is zero).

The case \( r < 1 \) is termed minimum phase. When \( r > 1 \), we have the condition of maximum phase. Where \( r > 1 \), the vector describing \( W \) will encircle the origin. As the angular frequency increases from 0 to \( 2\pi \), the phase increases by \( 2\pi \). The behaviour of the phase for a maximum phase dipole is shown in fig. 1(b). Comprehensive discussions of minimum phase and maximum phase properties are given by Treitel and Robinson (1964) and Ulrych and Lasserre (1966). Any time sequence can be described as the convolution of minimum and non-minimum phase dipoles.
Problems with noise arise if \( r \) is almost unity, since the vector describing \( \hat{W} \) will pass very close to the origin as angular frequency increases. Consider a minimum delay signal where \( r \) is slightly less than 1. In this case, the addition of a small amount of noise can perturb the vector so that it will pass through the left half of the complex plane. Then \( \theta(2\pi) = \theta(0) + 2\pi \), and the noise would have made the minimum delay dipole into a maximum delay dipole. In cases where \( r \) is near one, small amounts of noise can cause phase curves to change by \( 2\pi \).

This is one of two phase problems pointed out by Clayton and Wiggins (1976). The other problem is that of having large phase deviations whenever signal amplitudes are small compared to the noise amplitudes.

The problems of phase estimation with noisy data have also been described by de Voogd (1976) and Lines and Ulrych (1977). In homomorphic
deconvolution, the phase is unwrapped by adding multiples of $2\pi$ wherever the phase curve is discontinuous. This unwrapping procedure is necessary in order to make the phase into a continuous analytic function.

**Zero Phase Homomorphic Deconvolution**

The use of homomorphic deconvolution has been outlined by several authors including Oppenheim, Schafer and Stockham (1969), Ulrych (1971), and Stoffa, Buhl and Bryan (1974). Also, a comparison of homomorphic deconvolution with predictive deconvolution has been given by Treitel and Robinson (1977).

Additive noise has posed one of the main problems with the homomorphic deconvolution method because of the phase computations. Hence, use of the zero phase assumption obviates many of the problems encountered when using homomorphic deconvolution on noisy data.

For the case of no additive noise, the Vibroseis trace is given by:

$$x(t) = w(t) \ast i(t)$$

where $w(t)$ is the Klauder wavelet, $i(t)$ is the impulse response of a layered earth, the symbol $\ast$ denotes convolution, and $t$ is time. Since the Klauder wavelet is the autocorrelation of the input Vibroseis sweep, $w(t)$ is zero phase. Our models of $w(t)$ will not be ideal Klauder wavelets, but will be wavelets whose spectra are modified by effects of "earth filtering" as described by Lines and Clayton (1977).

In homomorphic deconvolution, a function termed the *complex cepstrum* is computed. The complex cepstrum $\hat{x}(t)$ is defined as the inverse Fourier transform of the logarithm of the Fourier transform. That is,

$$\hat{x}(t) = \int_{-F_N}^{F_N} \log X(f) e^{2\pi ift} df$$

where $X(f)$ is the Fourier transform of $x(t)$ and $F_N$ is the Nyquist frequency.

Thus computation of the complex cepstrum transforms sequences that are convolved into sequences that are additive. Deconvolution can then be performed by subtracting a portion of the cepstrum. Consider the computation of the cepstrum: Taking the log of the Fourier transform of $x(t)$ gives:

$$\log X(f) = \log W(f) + \log I(f)$$

where $W(f)$ and $I(f)$ are Fourier transforms of the wavelet and impulse response sequences, respectively.

Since $w(t)$ is zero phase, $\log W(f) = \log |W(f)|$. Consequently, $\log W(f)$ is transformed into the real portion of the complex cepstrum. The
The real part of (5) is termed the zero phase cepstrum and is given by
\[ \hat{x}_0(t) = \int_{F_N}^{F} \log |X(f)| \, e^{2\pi i f t} \, df. \] (7)

In estimating \( W(f) \), the low "quefrencies" of \( \log |X(f)| \) are used ("quefrencies" describe the periodicities of a frequency domain function in the same way that frequencies describe the periodicities of a time domain function). In using the low quefrencies of \( \log |X(f)| \) we essentially assume that the wavelet spectrum contributes to the trend of the seismic trace's spectrum. Low quefrency information is extracted by low pass filtering of the cepstrum.

Examples given by Lines and Clayton (1977) show that filtering or "liftering" of the zero phase cepstrum can give a good estimate of \( W(f) \).

For the case where additive noise \( n(t) \) is present, the expression for \( x(t) \) is given by the following:
\[ x(t) = w(t) \ast i(t) + n(t) \] (8A)
or in the frequency domain
\[ X(f) = W(f)I(f) + N(f) \] (8B)
where \( W(f) \), \( I(f) \), and \( N(f) \) are the Fourier transforms of the wavelet, the impulse response, and the additive noise, respectively.

The problem of determining \( W(f) \) by homomorphic deconvolution is more complicated than the noiseless case since we deal with the logarithm of a sum when evaluating the cepstrum. This problem has also been discussed by Ray in a paper read at the 46th SEG-meeting.

In the presence of noise the logarithm of the trace's Fourier transform becomes:
\[ \log X(f) = \log W(f) + \log I(f) + \log \left[ 1 + \frac{N(f)}{W(f)I(f)} \right]. \] (9)

For high signal-to-noise ratios we can use a Laurent expansion to make the following approximation:
\[ \log \left( 1 + \frac{N(f)}{W(f)I(f)} \right) \approx \frac{N(f)}{W(f)I(f)} \quad \text{when} \quad |N(f)| \ll |W(f)I(f)|. \] (10)

To estimate \( w(t) \), we assume that \( \log W(f) \) contributes to the low quefrency part of \( \log X(f) \) where
\[ \log X(f) = \log |W(f)| + \log |I(f)| + i\theta + \frac{N(f)}{W(f)I(f)}. \] (11)
Here $\theta$ is the phase of the signal, where the signal is the convolution of $w(t)$ with $i(t)$.

We can obtain the zero phase cepstrum $\hat{x}_0(t)$, by transforming the real part of expression (6). In attempting to find $W(f)$ for the noisy case, we still assume that the low frequencies in $\hat{x}_0(t)$ are contributed by the log $W(f)$ term. For quasi-random noise this is not a bad assumption, as is shown by the set of examples in figs. 3 and 4.

Fig. 2 shows a synthetic trace and the wavelet spectrum used to produce $w(t)$. Different amounts of random noise were added to give signal-to-noise ratios of $\infty$, 5, 3, 1. Figs 3 and 4 show the deconvolutions and the estimates of the wavelet spectra for the noisy traces. Fig. 3 gives a bandpass filtered version of $i(t)$, which is the best band limited deconvolution that can be expected.

The deconvolution of the traces does not deteriorate badly with decreased signal-to-noise ratio. This suggests that zero phase cepstral filtering or "liftering" is robust for noisy data. Use of the zero-phase assumption has obviated phase estimation problems for noisy data.

Also, examination of fig. 4 indicates that noise does not severely damage estimates of the wavelet spectra. This is not surprising when one considers that true random noise affects only the dc level of the trace's power spectrum.

**THE OPTIMUM IMPULSE RESPONSE MODEL**

Having explored the first step of this deconvolution procedure in which we obtained a flattened band limited spectrum, we turn our attention to the following question. What is the best frequency domain model that can be used to resolve a series of arrivals in a Vibroseis trace?
Fig. 3. The desired deconvolution is given by band passing the impulse response spikes of fig. 2. Deconvolutions were performed on noisy traces derived from the trace of fig. 2. Signal-to-noise ratios are ∞, 5, 3, and 1.
Fig. 4. The spectra of the noisy traces are displayed. The estimated wavelet spectra are shown beside the corresponding trace spectra.
First, we consider the Fourier transform of a series of spikes and note that it is a sum of complex sinusoids. That is, if we denote the impulse response by

$$i(t) = \sum_{K} i(K \Delta t) \delta(t - K \Delta t)$$

its Fourier transform is given by

$$I(f) = \sum_{K} i(K \Delta t) e^{-2\pi ifK \Delta t}$$

(13)

where $\Delta t$ is the sample interval of $i(t)$.

As seen by (13), $I(f)$ is a sum of complex sinusoids. A good model for $I(f)$ can be obtained by using an autoregressive (AR) process. This has been demonstrated by Clayton and Wiggins (1976), Clayton and Ulrych (1977) and Lines and Clayton (1977).

An AR representation for $I(f)$ or $I(n \Delta f)$ is given by

$$I(n \Delta f) = \sum_{m=1}^{p} \alpha(m)I((n - m) \Delta f) + e(n \Delta f)$$

(14)

where $\alpha(m)$ are complex prediction filter coefficients determined by use of the Burg-algorithm (Burg 1975).

Values of $\alpha(m)$ are determined by using $I(f)$ values within the band width of the Vibroseis sweep. In (14), $e(n \Delta f)$ is the prediction error, $\Delta f$ is the sample interval in the frequency domain, and $p$ is the order of the AR process. The value of $p$—the length of the prediction filter—can be estimated by using the final prediction error criterion described by Akaike (1969) and Ulrych and Bishop (1975).

Use of (14) allows us to model $I(f)$ beyond the band limits of the Vibroseis sweep. This extension of $I(f)$ allows overlapping arrivals to be resolved. Examples taken from previous publications by the authors demonstrate this. Fig. 5 shows how the method was used to resolve arrivals for the trace of fig. 2.

Fig. 5. A deconvolution and spectrum for the trace of fig. 2 after using a prediction filter in the frequency domain.
Unfortunately, experience has shown that this step of spectral extension is not robust when applied to noisy data. This is due to the fact that noisy contributions to the spectrum are also predicted outside the band limits of the Vibroseis sweep. This effect can be minimized by choosing a very short prediction filter since the value of $p$ controls how much energy is predicted into other parts of the spectrum.

**Conclusions**

Our proposed Vibroseis deconvolution method has been analyzed for noisy data. In this method, zero phase cepstral filtering is robust in noisy environments. In the second deconvolution step a very short prediction filter should be used so that the noisy spectrum would not be greatly extended outside the band limits of the Vibroseis sweep.

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**References**


Burg J.P. 1975, Maximum entropy spectral analysis, PhD thesis, Stanford University, Stanford, California.


