Source Finiteness and Rupture Propagation Using Higher-Degree Moment Tensors

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Abstract Higher-degree moment tensor representation of seismic sources is obtained for a horizontally layered homogeneous medium. A Taylor series expansion of Green’s function around a reference source position and time is made, and this enables us to approximate the seismic radiation in both the regional and teleseismic distances through a sequence of terms representing increasingly detailed aspects of the source behavior. Source coefficients and orientation factors of the first- and second-degree moment tensors are obtained. The representation is applied to a unilateral rupture Haskell fault model, and the synthetic seismograms of different models calculated by the higher-degree moment tensors are compared with the theoretical solutions for a propagating source. Our results show that, the representation of higher-degree moment tensors up to degree 2 can describe the response of a moving source well enough, and it’s possible to use the moment series as a tool for calculating seismograms from finite and propagating faults in the forward sense. The computation takes much less time than the method of summing point sources over the fault surface. The information yielded by the higher-degree moments may solve problems such as the fault-plane ambiguity and the space–time evolution of the rupture propagation of an earthquake.

Introduction

The moment tensor representation has been applied extensively to study the properties of seismic sources, and the zero-degree moment tensor representation for a point source is especially widely used, both for the purpose of modeling the seismic wave field and for obtaining source parameters. It’s possible to approximate the effect of a moment tensor density by a limited number of moment tensors of degree 0 when the source region is relatively small compared with the wavelengths. The information, for example, moment centroid location and time provided by the inversion of five independent moment tensor components, has become widely available and the basic procedures have been standardized (e.g. Dziewonski et al., 1995). The zero-degree moment tensor, however, represents averages over the fault in which only the long-wavelength part of the seismograms can be used, and it does not contain the spatial information of the source, and kinematic source parameters, like the geometry and rupture velocity of the fault.

By far one of the most difficult parts in computing the synthetic seismogram is solving the propagation effects through realistic velocity structures. Only for a few grossly simplified models (e.g., whole space or half-space) are analytical solutions possible. Backus (1977a,b) attempted to give the source representation using a glut moment tensor density by a limited number of moment tensors of degree 0 and higher. Doornbos (1982) considered the cases in the long-wave approximation and gave the formulation of seismic response represented in terms of 20 source parameters for a homogeneous half-space model, and estimated that for sources with $M_s > 6$, the relative contribution of second-degree moments may be of the order of 10% or more, even in long-period seismograms. Stump and Johnson (1982) studied the importance of higher-degree moments in modeling finite propagating faults in a half-space structure. McGuire et al. (2001) tried to get the space–time distribution from the information provided by the second-degree moments for a teleseismic event. However, the propagation effects are more prominent in the regional or near-field distances, and no detailed investigation of the contribution of higher-degree moments have been made for a more realistic layered earth model.

In this article, the higher-degree moment tensor representation of a propagating source for a horizontally layered homogeneous medium was given. The source coefficients and orientation factors of the first- and second-degree moment tensors for the calculation of Green’s functions were obtained. We applied the representation to a simple unilateral rupture Haskell fault model. The synthetic seismograms
of higher-degree moment tensors propagating in a homogeneous or a horizontally layered medium were compared with the seismograms generated by using a summation of point sources over the fault plane, and the effects of different model parameters, for example, the characteristic length of fault plane and the average rupture velocity, were investigated.

Representation of Higher-Degree Moment Tensors

Given the equivalent force system \( \mathbf{f}(\mathbf{x}, t) \), the response of the Earth \( \mathbf{u}(\mathbf{x}, t) \) can be expressed as an integral over the source region \( V \) (e.g., Aki and Richards, 1980):

\[
\mathbf{u}(\mathbf{x}, t) = \int G_j^i(\mathbf{x}, \zeta; t, \tau) \ast f_j(\zeta; \tau) dV,
\]

where \( G_j^i(\mathbf{x}, \zeta; t, \tau) \) is the Green’s function, representing the \( i \)th component of displacement at position \( \mathbf{x} \) and time \( t \) for single source excitations in the \( j \)th direction point \( \zeta \) and time \( \tau \), and the asterisk indicates temporal convolution.

To adapt to our present purposes, defining a moment tensor density distribution \( \mathbf{m}(\zeta, \tau) \), which is nonzero only within the source volume \( V \), and the displacement vector \( \mathbf{u}(\mathbf{x}, t) \) can be given by:

\[
\mathbf{u}(\mathbf{x}, t) = \int G_{jk}^i(\mathbf{x}, \zeta; t, \tau) \ast m_{jk} dV,
\]

where \( G_{jk}^i(\mathbf{x}, \zeta; t, \tau) \) is the Green’s function, representing the \( i \)th component of displacement at position \( \mathbf{x} \) and time \( t \) for single source excitations in the \( j \)th direction point \( \zeta \) and time \( \tau \), and the asterisk indicates temporal convolution.

Expanding the Green’s function \( G_{jk}^i \) directly around a suitably chosen reference source position \( \zeta^0 \):

\[
G_{jk}^i(\mathbf{x}, \zeta; t, \tau) = \begin{cases} 
G_{jk}^i(\mathbf{x}, \zeta^0; t, \tau) \\
+ G_{jk}^i(\mathbf{x}, \zeta^0; t, \tau)(\zeta - \zeta^0) \\
+ G_{jk}^i(\mathbf{x}, \zeta^0; t, \tau)(\zeta - \zeta^0)(\zeta^m - \zeta^m) \\
+ \ldots,
\end{cases}
\]

the later terms in the preceding equation involve higher-degree moment tensors, which contain information about the spatial distribution of failure in an earthquake and have great potential value for studying source finiteness and rupture propagation (Stump and Johnson, 1982; Julian, 1998). The displacement can now be written as:

\[
\mathbf{u}(\mathbf{x}, t) = \begin{cases} 
G_{jk}^i(\mathbf{x}, \zeta^0; t, \tau) \ast m_{jk}(\zeta); \tau \\
+ G_{jk}^i(\mathbf{x}, \zeta^0; t, \tau) \ast M_{jk}(\zeta); \tau \\
+ \ldots,
\end{cases}
\]

where \( M_{jk} \ldots \) are spatial moment tensors of the stress glut of order \( s \) (i.e., having \( s \) spatial indices \( k, l, \ldots \)).

By expanding \( G_j^i \) about a suitably chosen reference source point and replacing the convolutions by summations with the aid of temporal moments, Doornbos (1982) and Dahm and Krüger (1999) obtained an equation with 90 source parameters (moment tensors up to degree 2) for the asymptotic wave functions in the far field. With the “smoothing” assumption that the six components of the moment tensor density have a common space and time history, the number of source parameters will be greatly reduced to 20. Here the same basic procedure is adapted and the smoothing source term is assumed through this article. However, note that we do not apply the asymptotic wave functions in the far field as used in Doornbos (1982) to evaluate the spatial derivatives in equation (4). Instead, we use the exact wave function, which can be used in the calculation of a finite propagating source for both teleseismic and local distances. In the time domain, the complete representation of higher-degree moment tensor can then be obtained as the following:

\[
\mathbf{u}(\mathbf{x}, t) = \hat{M}_{jk} \ast \left[ G_{jk}(\tau) - \Delta(\tau)G_{jk} + \frac{1}{2}(\tau^2)G_{jk} \right.
\]

\[
+ \frac{1}{2}(\tau^2)G_{jk} + \frac{1}{2}(\tau^2)G_{jk} + \ldots \right]
\]

with

\[
M_{jk} = \int m_{jk} dV
\]

\[
\hat{M}_{jk} = \int \hat{M}_{jk} d\tau
\]

\[
\hat{M}_{jk}(\tau) = \left(\frac{\tau - \tau^0}{\tau^0}\right) \hat{M}_{jk} d\tau
\]

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\]

The preceding complete expansion is made around an ap-
propriate source position $\xi^0$ and a reference source time $t^0$ both in spatial and temporal spaces, and it enables us to approximate the seismic radiation $u$ through a sequence of terms that represent increasingly detailed aspects of the source behavior (Kennett, 2001). As shown in the following sections, this transform of the representation of displacement enables us to calculate the Green’s function, its first and second differentiation with respect to the Cartesian spatial coordinate $\xi$, and the different integration terms in equation (7), which are related to $M$ for a simple propagating model, for example, uni- or bilateral propagating Haskell model analytically.

For a simplified point source, only the first term is retained, and the displacement for an arbitrary fault with strike $\phi$, dip $\delta$, and rake angle $\lambda$ or equivalently moment tensor solution $M_d$ can be calculated using algorithms like the reflectivity method and the generalized rays theory. Here a generalized reflection-transmission coefficient matrix with integration of wavenumber $k$ is applied to calculate the Green’s function, and Bouchon’s discreet wavenumber summation scheme is adapted to calculate the wavenumber integration in the frequency domain (Yao and Harkrider, 1983; He et al., 2003).

Calculation of Green’s Functions

We give a brief description of how to derive source coefficients and orientation factors for a given zero-degree moment tensor following the methods used in wave-motion problems (Yao and Harkrider, 1983). The same procedure will be applied to derive the source coefficients of the differentiation of Green’s functions with respect to the spatial coordinate $\xi$, which will be used in the summation of higher-degree moment tensor displacement calculation.

When a single concentrated force in direction $n$ with time function $f(t)$ acts on the source point, in the frequency domain the solution of elastodynamic equation has the following expression (e.g., Ben-Menahem and Singh, 1981):

$$ u_i = \frac{F(\omega)}{4\pi \rho \alpha^2} \left[ \delta_{ij} k^2 \frac{e^{-ik_d R}}{R} + \frac{\partial^2}{\partial x_i \partial x_j} \left( \frac{e^{-ik_d R} - e^{-ik_s R}}{R} \right) \right], \quad (8) $$

where the subscript $i$ denotes the displacement component with respect to some convenient reference system, $j$ is the direction of the single body force, $\rho$ is density, $F(\omega)$ is the Fourier transform of the force $f(t)$, $\delta_{ij}$ is the Kronecker function, $k_d = \omega \alpha$ and $k_s = \omega \beta$; and $R$ is the distance between the source and receiver.

Physically, a seismic source can be expressed by the $P$-, $SV$-, and $SH$-wave motion radiated from the source. Adapting the concepts of displacement potentials for a rectangular point-shear dislocation in an infinite elastic medium, the seismic function, or the displacement potentials of the source can be written in terms of the wave-number $k$, and circular frequency $\omega$ for an arbitrarily oriented dislocation within the coordinate system shown in Figure 1:

$$
\begin{align*}
\varphi_x &= \frac{M_0}{4\pi \rho} \left( \frac{D(\omega)}{\omega^2} \right) \sum_{m=0}^{\infty} A_m e^{-ik_d z} J_m(k R) dk \\
\psi_x &= \frac{M_0}{4\pi \rho} \left( \frac{D(\omega)}{\omega^2} \right) \sum_{m=0}^{\infty} SV_m e^{-ik_d z} J_m(k R) dk \\
\chi_x &= \frac{M_0}{4\pi \rho} \left( \frac{D(\omega)}{\omega^2} \right) \sum_{m=0}^{\infty} B_m e^{-ik_d z} J_m(k R) dk
\end{align*}
\quad (9)
$$

where, $\varphi_x$, $\psi_x$, and $\chi_x$ are the $P$-, $SV$-, and $SH$-displacement potentials in frequency domain. $P_m$, $SV_m$, and $SH_m$ are called source coefficients, $A_m$ and $B_m$ are orientation factors, which are functions of the fault geometry, for example, $\lambda$, $\delta$, $\theta$ for a simple shear dislocation model; $z_s$ is the $z$ coordinate of source; $a = (k^2 - k^2_0)^{1/2}$, $b = (k^2 - k^2_0)^{1/2}$; $J_m(k)$ is the ordinary Bessel function of order $m$; $M_0$ is the seismic moment and $D(\omega)$ is the Fourier transform of the source time history. For a shear dislocation point source, the orders $m = 0, 1, 2$ and only 5 items of orientation factors with corresponding source coefficients are retained, which will be shown in the following section.

The introduction of the basic solution of elastodynamic equation (equation 8) and the displacement potentials (equation 9) enables us to follow the same deduction procedure to obtain the source coefficients and orientation factors for the calculation of higher-degree moment tensors $G_{ijkl}^j$ and $G_{ijklm}^{ijkl}$ by completing the partial derivatives with respect to coordinate $\xi_j$ and $\xi_m$. As shown in the later sections, no formal discrepancy of the displacement field exists for different orders of moment tensors. The radiation patterns, however,

![Figure 1. Coordinate system for the dislocation formulation. $z$ is positive downward.](image)
become much more complex as more lobes are introduced as the degree of the moment tensor increases.

Calculation of \( G'_{j,k} \) for a Layered Medium

Considering a model with \( n = 1 \) horizontal, homogeneous, and isotropic layers overlaying a semi-infinite medium (Fig. 2), the vertical, radial, and tangential displacement fields on the free surface for a buried point source can be expressed as (He et al., 2003):

\[
\begin{align*}
 w_j(t) &= \frac{1}{4\pi\rho} \frac{d}{dt} \left[ \dot{D}(t) \star \sum_{m=0}^{2} A_m W_m(t) \right] \\
 q_j(t) &= \frac{1}{4\pi\rho} \frac{d}{dt} \left[ \dot{D}(t) \star \sum_{m=0}^{2} A_m Q_m(t) \right] \\
 v_j(t) &= \frac{1}{4\pi\rho} \frac{d}{dt} \left[ \dot{D}(t) \star \sum_{m=1}^{2} B_m V_m(t) \right]
\end{align*}
\]  

(10)

with

\[
\begin{align*}
 A_0 W_0 &= A_0^{(1)} W_0^{(1)} + A_0^{(2)} W_0^{(2)} \\
 A_0 Q_0 &= A_0^{(1)} Q_0^{(1)} + A_0^{(2)} Q_0^{(2)}
\end{align*}
\]  

(11)

\[
\begin{align*}
 A_0^{(1)} &= -\frac{1}{2} (M_{11} + M_{22}) \\
 A_0^{(2)} &= -\frac{1}{2} M_{33} \\
 A_1 &= M_{13} \cos z_j + M_{23} \sin z_j \\
 A_2 &= \frac{1}{2} (M_{11} - M_{22}) \cos 2A_z + M_{12} \sin 2A_z \\
 B_1 &= -M_{13} \sin A_z + M_{23} \cos A_z \\
 B_2 &= -\frac{1}{2} (M_{11} - M_{22}) \sin 2A_z + M_{12} \cos 2A_z,
\end{align*}
\]  

(12)

where \( M_{ij} \) is the six independent components of moment tensor in the geographic coordinate; \( D(t) \) is the time history and it should be emphasized here that the smoothing assumption—the moment tensor has the same time history—is applied. \( A_m \) is the orientation factor indicating the radiation pattern of the seismic energy. \( W_m(t), Q_m(t), \) and \( V_m(t) \) are step responses that correspond to the vertical, radial, and tangential displacements of three fundamental shear dislocations: 45 dip-slip fault \((m = 0, 45^\circ \text{DS})\), dip-slip fault \((m = 1, \text{DS})\) and strike-slip fault \((m = 2, \text{SS})\), respectively. In the frequency domain, they have the following expressions:

\[
\begin{align*}
 W_m(\omega) &= \int_0^\infty w_m J_m(\kappa) d\kappa \\
 Q_m(\omega) &= \int_0^\infty [q_m J_m(\kappa) - v_m m \frac{J_m(\kappa)}{kr}] d\kappa \\
 V_m(\omega) &= \int_0^\infty [q_m m \frac{J_m(\kappa)}{kr} - v_m J_m(\kappa)] d\kappa
\end{align*}
\]  

(13)

where

\[
J_m(x) = \frac{d J_m}{dx}
\]  

(14)
where, $\varepsilon = 1$ for the plus superscript and $-1$ for the minus superscript.

Calculation of $G_{i,kl}$ and $G_{i,klm}^j$ for a Layered Medium

Following the basic deduction procedure described in the preceding section, we can obtain the source coefficients and orientation factors for the higher-degree moment tensors $G_{i,kl}$ and $G_{i,klm}^j$. It can be expected that with the increasing of the degree of moment tensors, more coefficients will be introduced into the equation. In the frequency domain, $\varphi_i^1$, $\psi_i^1$ and $\chi_i^1$, the displacement potentials contributed by the first-degree moment tensor can finally be described as:

$$\begin{align*}
\varphi_i^1 &= \frac{M_0}{4\pi \rho} \left( \frac{D(\omega)}{-\omega^2} \right) \sum_{m=0}^{3} A_m \int_0^\infty P_m e^{-\alpha z - \omega t} J_m(\omega r) dk d
\\
\psi_i^1 &= \frac{M_0}{4\pi \rho} \left( \frac{D(\omega)}{-\omega^2} \right) \sum_{m=0}^{3} A_m \int_0^\infty SV_m e^{-\beta z - \omega t} J_m(\omega r) dk d
\\
\chi_i^1 &= \frac{M_0}{4\pi \rho} \left( \frac{D(\omega)}{-\omega^2} \right) \sum_{m=0}^{3} B_m \int_0^\infty SH_m e^{-\gamma z - \omega t} J_m(\omega r) dk d
\end{align*}$$

(17)

the superscript 1 on the left term of equation (17) denotes specially the contribution by the first-degree moment tensor, and the source coefficients and orientation factors of the first-degree moment tensors $G_{i,kl}$ are as follows:

$$\begin{align*}
m = 0 & \quad P_0^{(1)} = -k^2/a \quad SV_0^{(1)} = k(2k^2 - k_0^2)/2b \quad SH_0^{(1)} = \varepsilon k_0^2/2 \\
P_0^{(2)} = -2ka \quad SV_0^{(2)} = k/b \quad SH_0^{(2)} = 0 \\
P_0^{(3)} = -\varepsilon a^2 \quad SV_0^{(3)} = -k/b \quad SH_0^{(3)} = 0 \\
m = 1 & \quad P_1^{(1)} = \frac{1}{4} k_0^2/a \quad SV_1^{(1)} = \varepsilon k_0^2/4 \quad SH_1^{(1)} = -\frac{1}{4} k_0^2 k/b \\
P_1^{(2)} = -2ka \quad SV_1^{(2)} = \varepsilon(2k^2 - k_0^2) \quad SH_1^{(2)} = b k_0^2/k \\
P_1^{(3)} = -ka \quad SV_1^{(3)} = -\varepsilon k^2 \quad SH_1^{(3)} = 0 \\
P_1^{(4)} = 0 \quad SV_1^{(4)} = 0 \quad SH_1^{(4)} = -\frac{1}{4} k_0^2 k/b \\
m = 2 & \quad P_2^{(1)} = -\frac{1}{4} k^2/a \quad SV_2^{(1)} = -k/b \quad SH_2^{(1)} = \varepsilon k_0^2/2 \\
P_2^{(2)} = -\varepsilon k^2 \quad SV_2^{(2)} = -k(2k^2 - k_0^2)/2b \quad SH_2^{(2)} = \varepsilon k_0^2/2 \\
m = 3 & \quad P_3 = -\frac{1}{4} k^3/a \quad SV_3 = -\varepsilon k_0^3/4 \quad SH_3 = \frac{1}{4} k_0^3 k/b
\end{align*}$$
\[ A_0^{(1)} = M_{13} \Delta(\xi_1) + M_{23} \Delta(\xi_2) \]
\[ A_0^{(2)} = (M_{11} + M_{22}) \Delta(\xi_3) \]
\[ A_0^{(3)} = M_{33} \Delta(\xi_3) \]
\[ A_1^{(1)} = [(3M_{11} + M_{22}) \Delta(\xi_1) + 2M_{12} \Delta(\xi_2)] \cos A_z + [(M_{11} + 3M_{22}) \Delta(\xi_2) + 2M_{12} \Delta(\xi_1)] \sin A_z \]
\[ A_1^{(2)} = (M_{13} \cos A_z + M_{23} \sin A_z) \Delta(\xi_3) \]
\[ A_1^{(3)} = M_{33} [\Delta(\xi_1) \cos A_z + \Delta(\xi_2) \sin A_z] \]
\[ A_1^{(4)} = -2(M_{11} + M_{22}) [\Delta(\xi_1) \cos A_z + \Delta(\xi_2) \sin A_z] \]
\[ A_2^{(1)} = [(M_{11} - M_{22}) \cos 2A_z + 2M_{12} \sin 2A_z] \Delta(\xi_3) \]
\[ A_2^{(2)} = [M_{13} \Delta(\xi_1) - M_{23} \Delta(\xi_2)] \sin 2A_z + [M_{13} \Delta(\xi_2) + M_{23} \Delta(\xi_1)] \sin 2A_z \]
\[ A_3 = [(M_{11} - M_{22}) \Delta(\xi_1) - 2M_{12} \Delta(\xi_2)] \cos 3A_z + [(M_{11} - M_{22}) \Delta(\xi_2) + 2M_{12} \Delta(\xi_1)] \sin 3A_z \]
\[ B_0^{(1)} = M_{13} \Delta(\xi_2) - M_{23} \Delta(\xi_1) \]
\[ B_1^{(1)} = -[(3M_{11} + M_{22}) \Delta(\xi_1) + 2M_{12} \Delta(\xi_2)] \sin A_z + [(M_{11} + 3M_{22}) \Delta(\xi_2) + 2M_{12} \Delta(\xi_1)] \cos A_z \]
\[ B_1^{(2)} = (-M_{13} \sin A_z + M_{23} \cos A_z) \Delta(\xi_3) \]
\[ B_1^{(3)} = M_{33} [-\Delta(\xi_1) \sin A_z + \Delta(\xi_2) \cos A_z] \]
\[ B_1^{(4)} = 2(M_{11} + M_{22}) [\Delta(\xi_1) \sin A_z - \Delta(\xi_2) \cos A_z] \]
\[ B_2^{(1)} = [-(M_{11} - M_{22}) \sin 2A_z + 2M_{12} \cos 2A_z] \Delta(\xi_3) \]
\[ B_2^{(2)} = -[M_{13} \Delta(\xi_1) - M_{23} \Delta(\xi_2)] \sin 2A_z + [M_{13} \Delta(\xi_2) + M_{23} \Delta(\xi_1)] \cos 2A_z \]
\[ B_3 = -[(M_{11} - M_{22}) \Delta(\xi_1) - 2M_{12} \Delta(\xi_2)] \sin 3A_z + [(M_{11} - M_{22}) \Delta(\xi_2) + 2M_{12} \Delta(\xi_1)] \cos 3A_z \]

We can see that for the first-degree moment tensor, the order \(m\) varies from 0 to 3, and the orientation factors appear more complex, which breaks the normal symmetry radiation pattern of \(P-SV\) waves generated by a point-source model, and this property may be further applied to distinguish the fault plane from the auxiliary plane.

Appendix gives source coefficients and orientation factors of the second-degree moment tensors \(G_{ijklm}^l\), in which the order \(m\) reaches to 4, meaning that Bessel functions with order 4 should be involved in the integration to get the surface displacement contributed by the higher-degree moment tensors.

Application to Haskell Fault Model

Assuming a unilateral rupture Haskell fault model (e.g., Lay and Wallace, 1995) with fault length \(L\), width \(W\), rupture velocity \(v\) (Fig. 3); the final slip \(\Delta D\) is uniform, then the rupture duration \(T_{rup} = L/v\). If the rupture plane is in the \((x, y)\) plane, the stress glut rate is defined by:

\[ \dot{M}_{jk} = \mu \Delta D \delta(t - x/v) \left[ H(x) - H(x - L) \right] \frac{[H(y + W/2) - H(y - W/2)] \delta(z)}{[H(y + L/2) - H(y - L/2)] \delta(z)}, \]

where \(H\) is the Heaviside step function, \(\delta\) is Dirac's delta function, and \(\mu\) is the shear modulus; the stress glut rate is nonzero for \(-W/2 \leq y \leq W/2, 0 \leq x \leq L\), and \(0 \leq t \leq L/v\).

For a unilateral rupture Haskell model, we can get the integration in equation (7) analytically:

\[ \dot{M}_{jk} = m_{jk} LW, \quad \dot{M}_{jk} \Delta(t) = \dot{M}_{jk} \left( \frac{2L}{v} - \tau_0 \right) \]

with

\[ m_{jk} = \mu \Delta D f_{jk}, \quad M_0 = \mu \Delta D LW, \quad \dot{M}_{jk} = M_0 f_{jk}. \]

A vector with rake angle \(\lambda\) on a fault plane (Fig. 4) can be expressed in the geophysical coordinate by its three components \(r = r \cdot (f_{11}^{(1)}, f_{11}^{(2)}, f_{11}^{(3)})\), where:

\[ f_{11}^{(1)}(\lambda) = \cos \lambda \cos A_z + \sin \lambda \cos \delta \sin A_z \]
\[ f_{11}^{(2)}(\lambda) = \cos \lambda \sin A_z - \sin \lambda \cos \delta \cos A_z \]
\[ f_{11}^{(3)}(\lambda) = -\sin \lambda \sin \delta. \]

If the rupture propagates along the \(x\) direction, and the angle between \(x^\prime\) and rupture direction is \(\lambda_{r^\prime}\), then we can use \(\lambda_{r^\prime}\) to express vector \(r\):
Radiation Pattern

The plot at the top of Figure 5 illustrates the radiation patterns of various moment tensor terms for the radial components in the \((x, z)\) plane, and the bottom one is for the tangential components. To give an example, only radiation patterns of terms \(M_x, M_z, \Delta(z, z)\) and \(M_y, \Delta(z, z)\) are shown in the figure. The response of zero-degree moment tensor is represented by a typical quadrupole radiation pattern for a point-source shear fault. With the increasing of the degree of moment tensors, directivity effects break the symmetry between the rupture and auxiliary plane. Complexity of the radiation patterns increases significantly and more lobes are introduced into the radiation patterns. It is expected that the relatively small propagation velocity makes this effect more obvious for 5 waves. It is possible that, if observational data from a variety of azimuths were available, the information in the higher-degree moment radiation patterns could be used to stack the seismograms, and with the enhancing of the effects of the higher-degree moments, the fault finiteness and the real fault plane could be resolved.

Seismograms Generated by Higher-Degree Moment Tensor Representation

First, seismograms from finite propagating sources in homogeneous media will be generated using the preceding equations of higher-degree moment tensor representation. The seismograms are compared with the theoretical ones for a propagating source. The seismograms for a propagating source are calculated by the commonly used approach: superposing seismograms from several hundred densely spaced point sources, which approximates a rupturing source with finite size (Anderson, 1976; Hartzell et al., 1978). While summing the synthetic seismograms for numerous points on the fault surface, the spacing of the point sources is selected to be close enough to avoid spatial aliasing, and this process is especially computationally expensive.

To study properties of the seismograms generated by higher-degree moment tensor, several different fault model cases (Table 1) in the semi-infinite homogeneous medium with parameters \(v_p = 6.2\) km/sec, \(v_s = 3.5\) km/sec, \(\rho = 2.7\) g/cm\(^3\), \(\lambda = 90^\circ\), \(\lambda = 90^\circ\), \(\delta = 90^\circ\), and \(A_1 = 90^\circ\) are selected. The characteristic length of the fault plane, the average rupture velocity \(v_{\text{avg}}\), the epicentral distance between the source and receiver are allowed to vary in these cases.

The propagating source model is shown in Figure 3. For simplicity, we assume that the width \(W\) is much shorter than the length of the fault \(L\), and thus the effect of the width can be neglected. The width of the fault plane is chosen as 0.1 km. A simple ramp source function with a finite rise time \(T = 1\) sec (Fig. 6) for each propagating point source on the fault plane is chosen.

The vertical, radial, and tangential components of ground displacement for case 1 at azimuths of 30\(^\circ\), 210\(^\circ\), and
Some basic characteristics of the higher-degree moment summation can be deduced from Figure 7. In this case, the length of the fault is 1 km, and the average rupture velocity on the fault plane is 3 km/sec, generating a rupture time $T_{\text{rup}}$ of 0.33 sec, which is much smaller than the rise time 1 sec. We can see from the comparison at azimuths of 30° and 210° that, with the degree of the moment tensor increasing, the frequency content of the contribution of the higher moment tensor also increases, and the total contribution including the

90° are given in Figure 7. Here no instrument or attenuation operator have been convolved with the responses. The displacements are plotted to the same scale for those at azimuths of 30° and 210° but to a different scale for radial and vertical responses at the azimuth of 90°. The lowest dotted line represents responses of the zero-degree moment tensor—the commonly used point-source model. The seismogram of the middle solid line is the total contribution of the zero-, first-, and second-degree moment tensors; and the thick solid line above is the theoretical seismograms calculated by the summing of densely propagating point sources on the fault.

<table>
<thead>
<tr>
<th>Model Parameters</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$ (epicentral distance) (km)</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>160</td>
</tr>
<tr>
<td>$H$ (depth) (km)</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>$L$ (km)</td>
<td>1.0</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>$W$ (km)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$v_{\text{rup}}$ (rupture velocity) (km/sec)</td>
<td>3.0</td>
<td>3.0</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>$T$ (rise time) (sec)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$T_{\text{rup}}$ (rupture time) (sec)</td>
<td>0.33</td>
<td>1.67</td>
<td>3.33</td>
<td>3.33</td>
</tr>
</tbody>
</table>
is still acceptable. This is especially true for the tangential component response. With the increasing of the degree of moment tensors, the frequency content of the seismograms of the summed moments also increases. Comparing the seismograms at azimuths of $30^\circ$, $210^\circ$, and $90^\circ$, we can see that the responses of higher-degree moment tensors converge to the theoretical seismograms of finite fault well, whereas the contribution of zero-degree moment tensor is quite different from the theoretical one. At an azimuth of $30^\circ$, the fault is rupturing toward the observation point, and the frequency content of the propagating fault response is clearly higher than that of a static point source model and the peak of the displacement amplitude moves forward clearly. It has an opposite effect for the case at $210^\circ$. The propagation effect becomes evident when the rupture duration is larger than the rise time.

Synthetic seismograms for case 3 are shown in Figure 9, in which the rupture velocity is changed to 1.5 km/sec compared with 3.0 km/sec of case 2. With the decreasing of rupture velocity, the rupture time increases a lot, and the propagation effect is expected to be much more obvious, and this can be clearly seen from the dotted line and thick line in Figures 8 and 9. The little fluctuation in the seismograms is contribution of the higher-degree moment tensors.

Case 4 is almost the same as case 3, except that the observation is made at a regional distance of 160 km. The three components of displacement for this model at azimuths
Figure 8. Comparisons of synthetic seismograms generated by the higher-degree moment tensor representation and those calculated by the summation of densely propagating points on the Fault. Comparisons are shown for seismograms recorded at varying azimuths at $30^\circ$ (a) and $210^\circ$ (b), respectively. The bottom dotted line, the middle thin line, and the top thick line represent the contribution of zero-degree moment tensor, the higher-moment summation, and the theoretical one. The seismograms are calculated for case 2.

Figure 9. Same as in Figure 8, except it's for case 3 and recordings at $A\zeta = 30^\circ$ (a) and $A\zeta = 210^\circ$ (b).
of 30° and 210° are shown in Figure 10. Clearly, the effects of the first- and second-degree moments are now important on both the amplitude and frequency content. For the vertical and radial components, the contribution of the higher-degree moment tensors can be seen mainly in the shear and Rayleigh waves. At an azimuth of 30°, the fault ruptures toward the receiver, and the responses by the higher-degree moment tensors add constructively and yield an amplitude of the shear or Rayleigh wave much higher than the amplitude of the zero-degree term alone. For the cases at an azimuth of 210°, the responses generated by the higher-degree terms add destructively, leading to a more decreased amplitude than the point-source model.

Viewing the moments in the frequency domain and comparing them with the point-source model, we can gain further information provided by the moment tensor series. Figure 11 compares the three different displacement components at azimuths of 30° and 210° of the point-source model, the higher-degree moment tensor, and the propagating point summation for case 4 in the frequency domain, respectively. At very long periods, we can see that all the records generated by different models display almost the same frequency content, which means that the zero-degree moment tensor dominates the record and the use of point-source approximation in studying the source is adequate. Because the zero-degree moment tensor dominates the record at long periods, the scalar moment determined from point-source spectra will not be affected by propagation.

With the increasing of frequency, however, the frequency property of different synthetic records begin to diverge. The synthetic propagating seismograms and the responses generated by moment summation shown as the solid and dashed lines bifurcate at about 0.4 Hz, which is a good example of the propagating effect at different azimuths. This also means that at 0.1–0.4 Hz, the first- and second-degree moment tensor components become comparable in size to the zero-degree term, and if one wishes to do some inversion problems in studying the source finiteness using the higher-degree moment terms, the appropriate frequency windows for study should be chosen with care. The spectra for the seismogram at 30° holds much more energy than that at 210° in this frequency band, yielding an apparent higher corner frequency at 30° than at 210°. The spectra amplitude of the zero-degree moment tensor, however, lies between the two solid lines at azimuths of 30° and 210°, showing no effect of the propagation.

At frequencies above 1 Hz, however, all the moment tensor series for three components begins to diverge. It is expected that if more accurate seismograms in higher-frequency bands are desired, then the moment tensors of degree 3 and greater should be included. Nevertheless, with the increasing of the degree of moment tensors, the more spatial derivatives of Green’s function should be considered, leading to greatly increased number of necessary source coefficients and orientation factors, and the introduction of numerical noise will become a serious problem in this case.

All the previous synthetic seismograms can be made for a new horizontally layered homogeneous media, because the
Table 2

<table>
<thead>
<tr>
<th>(\alpha) (km/sec)</th>
<th>(\beta) (km/sec)</th>
<th>(\rho) (g/cm(^3))</th>
<th>Layer Thickness (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.2</td>
<td>3.5</td>
<td>2.7</td>
<td>32</td>
</tr>
<tr>
<td>8.2</td>
<td>4.5</td>
<td>3.4</td>
<td>/</td>
</tr>
</tbody>
</table>

Discussions and Conclusions

While studying the source finiteness and rupture propagation using models in Table 1, the important parameters are the fault size \(L\) and \(W\), the rise time \(T\) at each point on the fault, and the rupture velocity of the propagating rupture front \(v_{rup}\). For simplicity, the width of the fault plane is assumed to be very small compared with the length of the fault, and thus the characteristic size of the fault could be represented by the length \(L\) alone. In this problem, two characteristic times are involved, and they are the rise time \(T\) and the rupture time \(T_{rup} = L/v_{rup}\).

Comparing the effect of the two characteristic times, two important cases should be considered. (1) The rupture time is shorter than the rise time: \(T_{rup} < T\). In this case, the rupture front expands quickly, and covers the entire fault plane before the healing of the fault surface, and thus the entire fault is moving during an interval equal to \(T - T_{rup}\).

In case 1, we have \(T = 1\) sec and \(T_{rup} = 0.33\) sec. The displacements at azimuths of 30° and 210° (Fig. 7) show that the responses of the first- and second-degree moment tensors

Figure 11. Comparisons of the displacement spectra for case 4. The solid, dashed, and dotted lines represent the spectra of displacements of propagating source model, higher moment summation, and point-source model, respectively. The upper panel is for model at an azimuth of 30° and the lower for 210°. (a) Vertical component. (b) Radial component. (c) Tangential component.
Source Finiteness and Rupture Propagation Using Higher-Degree Moment Tensors

components add constructively and yield shear-wave amplitude, which is almost one and a half times that generated by a point-source model. At an azimuth of 210°, the responses add destructively, and the squeezed amplitude is only 60–70% of that generated by a point source. The large variation of azimuthal amplitude changes can be observed in many earthquakes and may help to explain the role of propagation effect in the azimuthal magnitude changes as well as in predicting ground motions from finite propagating faults.

At an azimuth of 90°, the point-source model alone predicts no P-SV motions at all, whereas the match between the displacements generated by the higher-degree moment tensors and the summed propagating point-source model is good enough (Fig. 7c). It could be deduced that in the synthetic data, one can observe the effects of finite faults and rupture propagation quite clearly near the nodal plane, in which no motion will be generated for a traditional point source. The little constructive interference makes the displacement amplitudes observed very small—almost a factor of 100 below the observations at 30° and 210°, but with the improved signal-to-noise ratio of observed broadband waveform data, or if the azimuthal coverage is good enough, or if some stacking could be done the use of nodal seismograms may be helpful in solving the fault-plane ambiguity for seismic data.

For some earthquakes, unilateral rupture is a sufficient model of the faulting process, but there are lots of earth-

Figure 12. Comparison of displacement responses for case 4 in a horizontally layered homogeneous medium (Table 2). The recordings are at \( \text{Az} = 30° \) (a) and \( \text{Az} = 210° \) (b).
quakes nucleating in the center of the fault and spreading in both directions. The source finiteness and rupture propagation effect of this bilateral rupture can also be modeled by using higher-degree moment tensor representation. The integration terms in equation (7) can be obtained analytically as done in the Application to Haskell Fault Model section and then the coefficients can be applied to calculate the contribution of different-degree moment tensors. The propagation effect of the bilateral rupture model, however, is much less than that of a unilateral model, and the source time function varies much less with azimuth. It is often impossible to distinguish a bilateral rupture from a point-source model (Lay and Wallace, 1995). So in this article, the representation of higher-degree moment tensors is only applied to the unilateral rupture Haskell model as an illustration.

In summary, the moment tensor representation can be useful in modeling source finiteness and rupture propagation on seismograms. Higher-degree moment tensor representation of seismic sources is obtained for a horizontally layered homogeneous medium in this study. The representation is applied to a simplified unilateral-rupture Haskell model, and regional synthetic seismograms contributed by the higher-degree moment tensors are compared with the theoretical solutions for a propagating source. Our results show that it’s possible to use the moment series as a tool for calculating seismograms from finite and propagating faults in the forward sense, which will save more computation time than the traditional summation of densely spaced points along the fault. The convergence of series summation to the theoretical propagation responses enables one to generate the synthetic seismograms at relatively higher frequencies. The brake in the symmetry of the radiation patterns of the higher terms may help us to solve the ambiguity problem between the rupture and auxiliary plane. The limit of source parameters and the efficiency in calculating the synthetic seismograms make it possible to solve the worthy inverse problem of the space–time evolution of the rupture propagation for a finite-fault model.

Acknowledgments

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Appendix on following two pages.
Appendix

Source coefficients and orientation factors of the second-degree moment tensors $G_{jkm}$ can be obtained as in the following, and the order $m$ reaches to 4.

\[ m = 0 \begin{align*} P_0^{(1)} &= -\frac{1}{8} k^3/a \quad SV_0^{(1)} = -\frac{1}{8} ek^3 \quad SH_0^{(1)} = \frac{1}{4} k^2 b^2/lb \\ P_0^{(2)} &= 2k^2/a \quad SV_0^{(2)} = ek(2k^2 - k_0^2) \quad SH_0^{(2)} = k_0^2 b \\ P_0^{(3)} &= \frac{1}{8} k^3/a \quad SV_0^{(3)} = \frac{1}{8} ek b^2 \quad SH_0^{(3)} = 0 \\ P_0^{(4)} &= \frac{1}{8} k^3/a \quad SV_0^{(4)} = \frac{1}{8} ek \quad SH_0^{(4)} = 0 \\ P_0^{(5)} &= -a^3 \quad SV_0^{(5)} = -ek b^2 \quad SH_0^{(5)} = 0 \end{align*} \]

\[ m = 1 \begin{align*} P_1^{(1)} &= \frac{1}{2} ek^3 \quad SV_1^{(1)} = \frac{1}{2} k^3 b \quad SH_1^{(1)} = -\frac{1}{2} e k_0^2 k \\ P_1^{(2)} &= \frac{1}{2} ek^3 \quad SV_1^{(2)} = \frac{1}{2} k^3(2k^2 - k_0^2) \quad SH_1^{(2)} = \frac{1}{2} e k_0^2 k \\ P_1^{(3)} &= -2ek_0^2 \quad SV_1^{(3)} = -b(2k^2 - k_0^2) \quad SH_1^{(3)} = ek_0^2 b^2/lk \\ P_1^{(4)} &= -2ek_0^2 \quad SV_1^{(4)} = -2k^2 b \quad SH_1^{(4)} = 0 \\ P_1^{(5)} &= 0 \quad SV_1^{(5)} = 0 \quad SH_1^{(5)} = -\frac{1}{2} e k_0^2 k \\ P_1^{(6)} &= 0 \quad SV_1^{(6)} = 0 \quad SH_1^{(6)} = \frac{1}{2} e k_0^2 k \end{align*} \]

\[ m = 2 \begin{align*} P_2^{(1)} &= \frac{1}{2} k^2/a \quad SV_2^{(1)} = \frac{1}{2} ek b^2 \quad SH_2^{(1)} = -\frac{1}{2} e k_0^2 b \\ P_2^{(2)} &= 2k^2/a \quad SV_2^{(2)} = ek(2k^2 - k_0^2) \quad SH_2^{(2)} = -k_0^2 b \\ P_2^{(3)} &= \frac{1}{2} k^4/a \quad SV_2^{(3)} = \frac{1}{2} ek^3 \quad SH_2^{(3)} = -\frac{1}{2} e k_0^2 b^2/lb \\ P_2^{(4)} &= \frac{1}{2} k^4/a \quad SV_2^{(4)} = \frac{1}{2} ek^3 \quad SH_2^{(4)} = 0 \\ P_2^{(5)} &= 0 \quad SV_2^{(5)} = 0 \quad SH_2^{(5)} = -\frac{1}{2} e k_0^2 b^2/lb \end{align*} \]

\[ m = 3 \begin{align*} P_3^{(1)} &= \frac{1}{8} k^6/a \quad SV_3^{(1)} = \frac{1}{8} k^3 b \quad SH_3^{(1)} = -\frac{1}{8} e k_0^2 k \\ P_3^{(2)} &= \frac{1}{8} k^6/a \quad SV_3^{(2)} = \frac{1}{8} k^3(2k^2 - k_0^2) \quad SH_3^{(2)} = -\frac{1}{8} e k_0^2 k \\ P_3^{(3)} &= \frac{1}{8} k^6/a \quad SV_3^{(3)} = \frac{1}{8} k^3 \quad SH_3^{(3)} = 0 \end{align*} \]

\[ m = 4 \begin{align*} P_4 &= \frac{1}{8} k^6/a \quad SV_4 = \frac{1}{8} ek^3 \quad SH_4 = -\frac{1}{8} e k_0^2 k^2/lb \end{align*} \]

\[
A_0^{(1)} = (3M_{11} + M_{22})\Delta(\xi_1\xi_1) + (M_{11} + 3M_{22})\Delta(\xi_2\xi_2) + 4M_{13}\Delta(\xi_1\xi_2)
\]
\[
A_0^{(2)} = M_{13}\Delta(\xi_1\xi_3) + M_{23}\Delta(\xi_2\xi_3)
\]
\[
A_0^{(3)} = (M_{11} + M_{22})\Delta(\xi_3\xi_3)
\]
\[
A_0^{(4)} = M_{33}\Delta(\xi_1\xi_1) + \Delta(\xi_2\xi_2)
\]
\[
A_0^{(5)} = M_{33}\Delta(\xi_3\xi_3)
\]
\[
A_1^{(1)} = [(3M_{11} + M_{22})\Delta(\xi_1\xi_1 + 2M_{12}\Delta(\xi_2\xi_3))\cos A_z + [(M_{11} + 3M_{22})\Delta(\xi_2\xi_3) + 2M_{12}\Delta(\xi_1\xi_3)]\sin A_z
\]
\[
A_1^{(2)} = [M_{13}(3\Delta(\xi_1\xi_1) + \Delta(\xi_2\xi_2)) + 2M_{23}\Delta(\xi_1\xi_2)]\cos A_z + [M_{23}(\Delta(\xi_1\xi_1) + 3\Delta(\xi_2\xi_2)) + 2M_{13}\Delta(\xi_1\xi_2)]\sin A_z
\]
\[ A^{(3)}_1 = (M_{13} \cos A_x + M_{23} \sin A_x) \Delta(\xi_3 \xi_5) \]
\[ A^{(4)}_1 = M_{33}[\Delta(\xi_3 \xi_5) \cos A_x + \Delta(\xi_5 \xi_3) \sin A_x] \]
\[ A^{(5)}_1 = -2(M_{11} + M_{22})[\Delta(\xi_1 \xi_5) \cos A_x + \Delta(\xi_5 \xi_1) \sin A_x] \]
\[ A^{(6)}_1 = -4[\Delta(\xi_1 \xi_5) + \Delta(\xi_5 \xi_1)] (M_{13} \cos A_x + M_{23} \sin A_x) \]
\[ A^{(7)}_1 = [(M_{22} - M_{11}) \cos 2A_x - 2M_{12} \sin 2A_x] \Delta(\xi_3 \xi_5) \]
\[ A^{(2)}_2 = [M_{23} \Delta(\xi_3 \xi_5) - M_{13} \Delta(\xi_1 \xi_5)] \cos 2A_x - [M_{13} \Delta(\xi_3 \xi_5) + M_{23} \Delta(\xi_1 \xi_5)] \sin 2A_x \]
\[ A^{(3)}_2 = [M_{11} \Delta(\xi_3 \xi_5) - M_{22} \Delta(\xi_3 \xi_5)] \cos 2A_x + [(M_{11} + M_{22}) \Delta(\xi_1 \xi_5) + M_{12} \Delta(\xi_1 \xi_5) + \Delta(\xi_2 \xi_5)] \sin 2A_x \]
\[ A^{(4)}_2 = M_{33}[(\Delta(\xi_3 \xi_5) - \Delta(\xi_1 \xi_5)) \cos 2A_x - 2\Delta(\xi_3 \xi_5) \sin 2A_x] \]
\[ A^{(5)}_2 = [M_{11} \Delta(\xi_3 \xi_5) - M_{22} \Delta(\xi_3 \xi_5)] \cos 2A_x - [(M_{11} + M_{22}) \Delta(\xi_1 \xi_5) + M_{12} \Delta(\xi_1 \xi_5) + \Delta(\xi_2 \xi_5)] \sin 2A_x \]
\[ A^{(6)}_2 = [(M_{22} - M_{11}) \Delta(\xi_3 \xi_5) + 2M_{12} \Delta(\xi_3 \xi_5)] \cos 2A_x + [(M_{22} - M_{11}) \Delta(\xi_1 \xi_5) + \Delta(\xi_2 \xi_5)] \sin 2A_x \]
\[ A^{(7)}_2 = [M_{13} \Delta(\xi_3 \xi_5) - M_{23} \Delta(\xi_1 \xi_5)] \cos 2A_x + [M_{12} \Delta(\xi_3 \xi_5) + 2M_{23} \Delta(\xi_1 \xi_5)] \sin 2A_x \]
\[ B^{(1)}_1 = -[M_{11} (3\Delta(\xi_1 \xi_5) + 2\Delta(\xi_2 \xi_5))] \sin A_x + [(M_{11} + M_{22}) \Delta(\xi_1 \xi_5) - 2M_{12} \Delta(\xi_1 \xi_5)] \cos A_x \]
\[ B^{(2)}_1 = -[M_{13} (3\Delta(\xi_1 \xi_5) - 3\Delta(\xi_2 \xi_5))] \sin A_x + [M_{13} + M_{22} \Delta(\xi_1 \xi_5) + 2M_{12} \Delta(\xi_1 \xi_5)] \cos A_x \]
\[ B^{(3)}_1 = -(M_{11} \sin A_x + M_{23} \cos A_x) \Delta(\xi_3 \xi_5) \]
\[ B^{(4)}_1 = M_{33}[-(\Delta(\xi_1 \xi_5) \sin A_x + \Delta(\xi_2 \xi_5) \cos A_x) \]
\[ B^{(5)}_1 = 2(M_{11} + M_{22})[\Delta(\xi_1 \xi_5) \sin A_x - \Delta(\xi_2 \xi_5) \cos A_x] \]
\[ B^{(6)}_1 = 4[\Delta(\xi_1 \xi_5) + \Delta(\xi_2 \xi_5)] (M_{13} \cos A_x + M_{23} \sin A_x) \]
\[ B^{(7)}_1 = -[(M_{22} - M_{11}) \sin 2A_x + 2M_{12} \cos 2A_x] \Delta(\xi_3 \xi_5) \]
\[ B^{(2)}_2 = -[M_{23} \Delta(\xi_3 \xi_5) - M_{23} \Delta(\xi_3 \xi_5)] \sin 2A_x + [M_{11} + M_{22}) \Delta(\xi_1 \xi_5) + M_{12} \Delta(\xi_1 \xi_5) + \Delta(\xi_2 \xi_5)] \cos 2A_x \]
\[ B^{(3)}_2 = -[(M_{22} - M_{11}) \sin 2A_x + 2M_{12} \cos 2A_x] \Delta(\xi_3 \xi_5) \]
\[ B^{(4)}_2 = -[M_{23} \Delta(\xi_3 \xi_5) - M_{23} \Delta(\xi_3 \xi_5)] \sin 2A_x - [M_{11} + M_{22}) \Delta(\xi_1 \xi_5) + M_{12} \Delta(\xi_1 \xi_5) + \Delta(\xi_2 \xi_5)] \cos 2A_x \]
\[ B^{(5)}_2 = -33[(\Delta(\xi_3 \xi_5) - \Delta(\xi_1 \xi_5)) \sin 2A_x + 2\Delta(\xi_3 \xi_5) \cos 2A_x] \]
\[ B^{(6)}_2 = -[(M_{22} - M_{11}) \Delta(\xi_3 \xi_5) + 2M_{12} \Delta(\xi_3 \xi_5)] \sin 3A_x + [(M_{22} - M_{11}) \Delta(\xi_1 \xi_5) + \Delta(\xi_2 \xi_5)] \cos 3A_x \]
\[ B^{(7)}_2 = -[(M_{22} - M_{11}) \Delta(\xi_3 \xi_5) - M_{23} \Delta(\xi_3 \xi_5)] \sin 3A_x + [(M_{22} - M_{11}) \Delta(\xi_1 \xi_5) - M_{12} \Delta(\xi_1 \xi_5) + \Delta(\xi_2 \xi_5)] \cos 3A_x \]
\[ A_4 = [M_{23} \Delta(\xi_2 \xi_5) - \Delta(\xi_1 \xi_5) + 2M_{23} \Delta(\xi_1 \xi_5) - \Delta(\xi_2 \xi_5) + 4M_{12} \Delta(\xi_1 \xi_5)] \cos 4A_x + [2(M_{22} - M_{11}) \Delta(\xi_1 \xi_5) + 2M_{12} \Delta(\xi_1 \xi_5) - \Delta(\xi_1 \xi_5)] \sin 4A_x \]
\[ B_4^{(1)} = (M_{22} - M_{11}) \Delta(\xi_1 \xi_5) + M_{12} \Delta(\xi_1 \xi_5) - \Delta(\xi_2 \xi_5) \]
\[ B_4^{(2)} = M_{13} \Delta(\xi_1 \xi_5) - M_{23} \Delta(\xi_1 \xi_5) \]
\[ B_4^{(3)} = B_4^{(4)} = B_4^{(5)} = 0 \]
\[ B_4^{(6)} = -[(3M_{11} + M_{22}) \Delta(\xi_1 \xi_5) + 2M_{12} \Delta(\xi_2 \xi_5)] \sin A_x + [(M_{11} + 3M_{22}) \Delta(\xi_2 \xi_5) + 2M_{12} \Delta(\xi_1 \xi_5)] \cos A_x \]

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