Introduction to computational earthquake dynamics: a sample problem

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November 4, 2004

Abstract

The purpose of this note is to introduce readers that are familiar with numerical methods, but not with earthquake seismology, to current issues in the modeling of earthquake dynamics. The physical background of a standard earthquake model is shortly presented. We then formulate a reduced problem that contains the most relevant physical features of the standard model. Its numerical solution, obtained by boundary, finite or spectral element methods, will be discussed with emphasis on issues that need to be tackled with adaptive methods. We will conclude with a perspective on current challenges in computational earthquake dynamics.

1 Motivations and geophysical background

In the prevailing paradigm of earthquake dynamics modeling, an earthquake is regarded as a sudden episode of slip on a pre-existing frictional fault plane, embedded in a linear elastic medium. This section motivates and describes this standard physical model. The presentation is by no means complete: our aim here is to provide a minimal background to the curious reader, to give a rough idea of orders of magnitude of the relevant quantities and to define some vocabulary.

The Earth's crust (the first few tens of km) is broken into tectonic plates that move around (at some cm/year) driven by the thermal convection of the

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deeper mantle and by gravity. At their boundaries, plates do not slide past each other steadily but stick most of the time and slip suddenly. These fast and sporadic slip episodes are earthquakes. They tend to release the potential energy stored through elastic deformation of the plates during the long periods of "stick" (the interseismic phase). For large destructive earthquakes the interseismic period lasts more than a human lifetime and the earthquake itself (the coseismic phase) spans only some seconds. We focus here on the latter stage.

Although earthquake seismologists often use the term "earthquake rupture", the dominant macroscopic process during an earthquake, in contrast to usual problems in engineering fracture dynamics, is not the breaking of an intact crust by freshly new fractures, but reactivation of pre-existing weak interfaces, the *faults*, generally located at plate boundaries. Moreover, earthquakes are not opening cracks but are analogous to shear cracks. *Slip*, defined as the tangential offset across the fault, is the main variable that describes the kinematics of an earthquake. Slip is far from being uniform or simple. Earthquakes and fracture are often thought of as the "turbulence of solids". Earthquakes come in all sizes, from meters to tens of km, the smaller being the most frequent. They generally involve slip only over a limited section of the plate boundary. Earthquakes nucleate, propagate and stop: slip starts on a relatively small region of the fault, usually close to stress concentrations left by previous earthquakes; then the slip region expands at velocities close to the shear wave velocity of the crust (some km/s); finally slip is arrested in regions of lower stress or higher strength. The advancing boundary of the slip region is called the *rupture front*. Earthquake rupture propagation is complex: the spatio-temporal distribution of slip and the history of the rupture front are highly irregular, heterogeneous.

Part of the earthquake energy is released through the radiation of elastic waves. Seismic waves can have destructive effects on structures, hence the direct societal impact of earthquake research. On the other hand waves are routinely recorded by seismograph networks and convey information about the physics of the earthquake source. Seismograms (time series recordings of ground displacement at selected stations on the Earth's surface) are the basic observables that constrain our models of earthquakes physics. A main goal in computational earthquake seismology is to generate physically acceptable histories of coseismic slip, for specific or generic earthquake scenarios. Derived outcomes are parameters of the ground shaking for engineering purposes (usually refered as *strong motion seismology*) and computed seismograms for comparison with observed seismograms [1]. There are two approaches to this problem: kinematic and dynamic modeling. In *kinematic modeling* the space-time distribution of coseismic slip is assumed to be known, and the problem reduces to a simulation of linear wave propagation with specified Dirichlet boundary conditions on the fault. In contrast, *dynamic modeling* involves a non-linear problem where slip is an unknown that must be generated spontaneously from the prescription of an initial state (of fault stress, essentially) and of a physical criterion for the propagation of earthquake rupture.

Due to our poor knowledge of the fine scale heterogeneities of the crust, the interpretation of seismograms is performed in a limited frequency band, tipically below 1 Hz. In earthquake engineering applications (linear structural analysis) frequencies up to 5 Hz are tipically required. Corresponding wavelengths are in the km range. However, the dynamics of earthquake rupture propagation (a non linear process) involve much smaller length scales and higher frequencies. Moreover, the dimensions of the slip area of large earthquakes is of many tens of km and the distance to useful seismograph stations is often of that same order. Although this is rarely assimilated in current approaches to earthquake modeling, earthquake dynamics problems are multi-scale.

The dynamic stresses close to the earthquake source can be very high and expectedly non linear processes, such as damage, plasticity or secondary fractures, may come into play in some volume around the fault. Natural faults are not dry contact between rocks along perfect planes but contain sandwiched granular material, called *gouge*, and fluids and are surrounded by a volume of damaged rock, the *fault zone*. However the standard approach assumes that the fault is a definite surface of tangential displacement discontinuity, embedded in a linear elastic crust: all the eventual off-fault non linearities are lumped into a fault interface condition. This condition is usually formulated as a friction law that relates tractions to slip, slip rate or other fault state variables, and is regarded as a constitutive relation for the fault.

Friction laws for earthquake dynamic modeling range from empirical laws constrained by laboratory experiments to theoretical laws aimed to account for a variety of processes such as contact ageing, shear melting and lubrication

[1] Another branch of computational seismology is devoted to global seismic tomography: seismograms also contain information about the media through which waves travel and are used for imaging the structure of the Earth's interior.

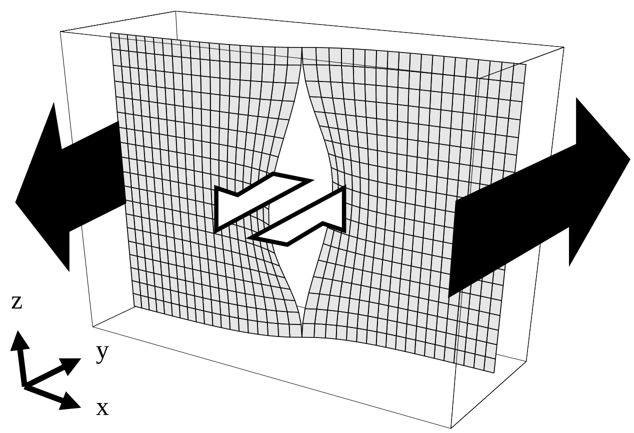

Figure 1: Geometry of the antiplane shear problem.

(slip rates reach some m/s), thermal pressurization, gouge dilatancy, etc. By analogy to the problem of a cohesive shear crack in Fracture Dynamics, an essential ingredient is that some energy must be irreversibly spent in the process of slip. This is the *fracture energy*. Another necessary ingredient is *slip weakening*: for an earthquake to develop as a frictional instability the strength of the fault must decrease with increasing slip.

Certainly the most rudimentary earthquake model is a rigid block sliding on a rigid substrate and pulled by a spring. The contact plane and the pulling force mimic the fault and the driving tectonic forces, respectively. This model is useful to grasp the basics of the stick-slip phenomenon. In particular, a frictional instability occurs if the spring is softer than the slip weakening rate (the restoring force is too "slow" to balance the dynamic reduction of strength). However, the mass inertia of the block is only a poor representation of elastodynamic effects, this model does not generate waves; and the uniform slip of the rigid block is too simple. In the next section we introduce a minimal model where we can generate complex slip histories and wave propagation.

2 Formulation of the model problem

A reduced, yet interesting enough, geometry is the so-called 2D antiplane shear configuration of Figure 2. In Fracture Mechanics this set up is known as a Mode III crack. The fault is the boundary between two linearly elastic half spaces, located at $x = 0$. The dimensionality of the problem is reduced by assuming invariance along the y direction. Further reduction to a scalar problem is obtained by assuming that displacements $\mathbf{d}$ are parallel to the y direction (off the plane), $\mathbf{d} = d(x, z, t)\ \mathbf{y}$. For a vertical fault with horizontal slip, like the San Andreas Fault, this amounts to consider a vertical cross section of the crust, transverse to the fault, and to assume that slip $\mathbf{\Delta d}$ is uniform in the horizontal direction, but not necessarily along depth, $\mathbf{\Delta d} = \Delta d(z, t)\ \mathbf{y} = \mathbf{d}(0^+, z, t) - \mathbf{d}(0^-, z, t)$. By assuming infinite half-spaces the effects of the free surface are disregarded.

The governing equation for the bulk of each half-space is the 2D scalar wave equation, which reads in non dimensional form:

$$\frac{\partial^2 d}{\partial t^2} = \frac{\partial^2 d}{\partial x^2} + \frac{\partial^2 d}{\partial z^2} \tag{1}$$

with homogeneous initial conditions:

$$d(x, z, 0) = 0 \tag{2}$$

$$\frac{\partial d}{\partial t}(x, z, 0) = 0 \tag{3}$$

Here $d(x, z, t)$ is understood as a displacement perturbation with respect to some initial state of static equilibrium. Displacements may be discontinuous across the fault and slip $\Delta d(z, t)$ has been already defined as the displacement offset:

$$\Delta d(z, t) \doteq d(0^+, z, t) - d(0^-, z, t) \tag{4}$$

The perturbations of shear traction, on each side of the fault,

$$\tau^{\pm}(z, t) = \mp \frac{\partial d}{\partial x}\ (0^{\pm}, z, t) \tag{5}$$

are continuous in the following sense:

$$\tau^- = -\tau^+ \tag{6}$$

If $\tau_0(z)$ is the stress on the fault plane in the initial equilibrium state, the total fault stress is written as

$$\tau(z, t) = \tau^-(z, t) + \tau_0(z) \tag{7}$$

We further assume that slip is always positive, which is expected under reasonable initial stress conditions with $\tau_0(z) \geq 0$.

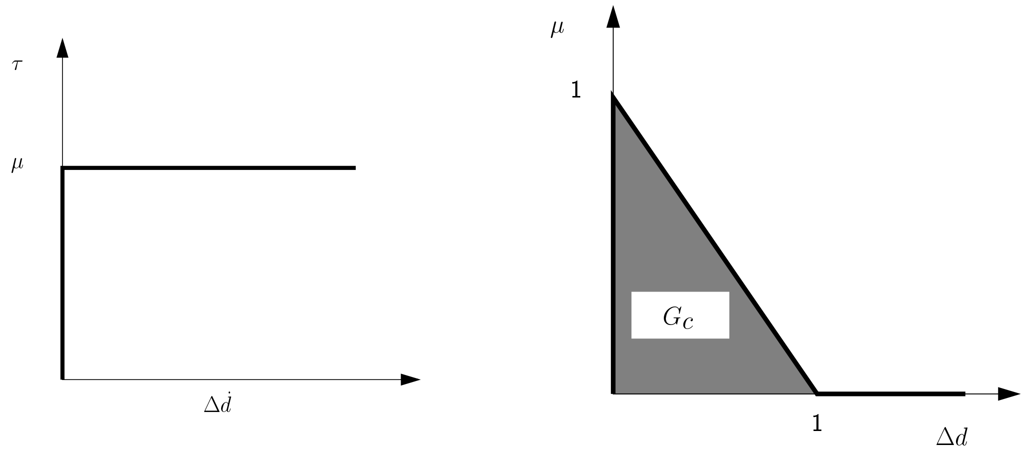

Figure 2: (a) Friction graph relating fault stress τ to slip rate $\Delta \dot{d}$. μ is the current friction coefficient (also called here fault strength). (b) Linear slip weakening friction law prescribing the evolution of μ with ongoing slip Δd. The fracture energy G_c is the area of the shaded surface. All quantities are dimensionless.

The previous equations are closed by boundary conditions on the fault. These are of a mixed nature: traction and slip are related by *friction*. Friction couples both sides of the fault (the boundaries of both half-spaces), so it is an *interface* condition rather than a simple boundary condition. Friction is non regular, it is not a functional relation, and can be represented by a graph as in Figure 2 (a). While at rest (when $\Delta \dot{d} = 0$), the fault stress $\tau(z,t)$ can be anywhere below a *friction coefficient*, $\mu(z)$. Slip can start if τ reaches μ, and during sliding (when $\Delta \dot{d} > 0$) both remain equal. In this context we will also refer to μ as the fault *strength*[2]. Introducing the auxiliary variable $\Phi(z,t) \doteq \tau - \mu$, this is formulated as a set of complementary relations:

$$\Delta \dot{d} \geq 0 \quad (8)$$

$$\Phi \leq 0 \quad (9)$$

$$\Delta \dot{d}\, \Phi = 0 \quad (10)$$

Additionally, the friction coefficient μ is not a constant but evolves with slip in a way that features finite fracture energy and slip weakening. We

[2]More precisely, strength is the product of μ and the normal stress σ. However in this simple problem σ remains constant, and has a normalized value $\sigma = 1$, so the distinction between friction coefficient and strength is not relevant.

adopt the piecewise linear curve of Figure 2 (b), known as the *linear slip weakening* friction law. After the initial strength (= 1) is first overcomed, the fault weakens linearly with increasing slip. Beyond a critical slip value (= 1) the fault remains at a low residual strength (= 0)[3].

$$\mu(\Delta d) = \max(1 - \Delta d, 0) \tag{11}$$

Fracture energy G_c is the total dissipation by the frictional forces, the area of the shaded region between the slip weakening curve and its final strength level (here $G_c = 1/2$). After local arrest we assume instantaneous healing: the strength μ is resumed to its initial value.

The bakground initial stress is uniform and at equilibrium below the initial strength:

$$\tau_0(z) = \tau^\infty < 1 \quad \text{almost everywhere} \tag{12}$$

As we want to focus on the fast propagation stages of earthquake rupture, the physics of the spontaneous earthquake initiation, or nucleation, can be disregarded for the time being. Our model earthquake is triggered by arbitrary stressing a central region of size L_0 above the static strength:

$$\tau_0(z) = 1 + \epsilon_0 \quad \text{over} \quad |z| \leq L_0/2 \tag{13}$$

It is known that if fracture energy is finite, this nucleation region must be larger than some critical size, of order 1.

The earthquake is stopped rather artificially, when it reaches a size L, by an abrupt barrier, a zone of very high strength:

$$\mu(\Delta d) = \mu^b \quad \text{beyond} \quad |z| \geq L/2 \tag{14}$$

We can take for instance $\tau^\infty = 0.7$, $L_0 = 2$, $\epsilon_0 = 1.1$, $\mu^b = 100$ and $L = 10$.

3 Numerical solution and discussion

[This section is incomplete ...]

[3] Actually, residual strength is generally > 0. But, as we are working with perturbative quantities with respect to an initial equilibrium, setting the final strength to $= 0$ amounts to a shift in the stress scale, which is irrelevant for the dynamics.

Finite difference, finite elements and boundary elements in time domain are widespread in the computational earthquake seismology community. More recently, the Spectral Element Method has been succesfully applied to this class of problems. All these methods currently work on fixed discrete meshes. We will discuss now numerical results of our problem obtained by the SEM and by the spectral boundary integral equation method (SBIEM). Codes are available online from the author's homepage[4].

In the FEM and SEM, the implementation of the fault interface conditions require locating the fault along inter-element boundaries and splitting the nodes to define each side of the fault as an internal boundary. As internal boundaries of this kind are not very usual in existing codes, it may be desirable to circumvent their technical implementation. The trick is to exploit the symmetry with respect to the fault plane, $d(-x, z, t) = -d(x, z, t)$, so only one of the two half-spaces ($x \leq 0$) needs to be simulated and the fault becomes a classical external boundary. The friction law can then be writen in terms of $d(0^+, z, t) = \Delta d(z, t)/2$.

Figure XXX shows the space-time distributions of slip, slip rate and fault shear traction ...

Figure XXX shows the evolution of the rupture front, defined as the contour of slip rate at a conventional small value (0.01). The early contours correspond to initiation of slip, whereas other contours are related to slip arrest. Also shown is the contour $\Delta d = 1$, the locus of fault points reaching residual friction. The zone between this and the slip initiation contour defines the *process zone*, the vicinity of the propagating crack tip that is experiencing slip weakening. It is known that, if fracture energy is constant, this process zone shrinks as the earthquake grows.

Figure XXX shows snapshots of particle velocity on one side of the fault. The radiated field is dominated by a few number of concentrated wavefronts. Clearly as the process zone shrinks the characteristic width of these wavefronts decreases and the numerical method with fixed mesh suffers from resolution problems. Convergence with consecutive mesh refinements is too slow to allow for high resolution 3D simulations at reasonable computational cost.

Some straightforward variations of this model are of interest:

- In the inplane shear version (mode II cracks), with both compressional and shear waves, there is an interesting transition from sub-shear rup-

[4] http://geoweb.princeton.edu/people/ampuero/software.html

ture velocity to supershear, intersonic, that generates a shock wave and has serious implications on seismic hazard.

- Earthquakes are not smooth at all, complexity is usually explored by initial heterogeneities of stress or friction parameters along the fault.
- The nucleation and arrest stages are also of particular interest for earthquake radiation and because of the information they convey about fault friction properties.

4 Current challenges in computational earthquake dynamics

[This section is incomplete ...]

We have illustrated through a simple example the multi-scale nature of problems in earthquake dynamics. But there is more ...

- multi-scale physics (off-fault non linear processes: plasticity, damage, secondary faulting and branching)
- micro-macro approaches to the formulation of new constitutive laws, such as dynamic damage accounting for nucleation and coalescence of micro-fractures ahead of the main front, require intensive simulations
- coupling to pore fluid flow, thermal effects, gouge dilatancy
- multiple time-scales of the seismic cycle (slow scales constrained by geodesy)
- real problems are large scale, with complicated fault geometries and complicated media, but poorly constrained at the smaller scales
- the dynamic source inversion problem aims to infer constitutive parameters of the fault (friction properties) from observed seismograms. It is a highly non linear inverse problem that is solved by iterative procedures requiring thousands of forward simulations.
- stochastic analysis (stochastic initial stress or friction properties) also require a large number of simulations. This approach is useful to define families of stochastic kinematic source models with dynamically consistent complexity.