EFFECTS OF NON-LINEAR WEAKENING ON EARTHQUAKE SOURCE SCALINGS

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ABSTRACT
An earthquake is usually modeled as an extension of the Dugdale approach: as in LEFM, fracture energy G_c is completely spent in a relatively small region close to the crack tip, the process zone; and the fault strength is described by a slip weakening law that reaches a residual level at a characteristic slip D_c. Beyond its regularizing properties, slip weakening is a phenomenological fact of experimental rock mechanics. Recent laboratory observations show that slip weakening can be a persistent process over large amounts of slip, in contradiction to our usual view of a finite D_c. New seismological constraints from seismic nucleation phases and from the scaling of radiated energy with magnitude, seem to point to a similar interpretation. Persistent weakening can also be viewed as a lumped representation of off-fault non linear processes occurring in the wake of the process zone. On this basis, power law weakening laws have been proposed that feature a very steep weakening rate in the short slip range followed by a long tailed, non linear, weakening process with no characteristic slip. On the other hand there is growing interest in the apparent scaling of fault properties, such as D_c and G_c, with earthquake size. This is a key issue in understanding how much can be learned about large destructive earthquakes from the observation of smaller but more frequent ones and from laboratory experiments. I explore here the possible effects of non linear strength drop on macroscopic earthquake source parameters, such as magnitude, rupture size and radiated energy, and their inter-relations mainly via numerical simulation of earthquake dynamics under a general family of empirical non-linear slip weakening laws.

1 INTRODUCTION

In the simulation of earthquake rupture dynamics the linear slip weakening model is the simplest rupture criterion that accounts for a finite fracture energy G_c and regularizes the crack tip stress fields [1]. This allows for a straightforward implementation in numerical codes, which is an argument for his widespread use in earthquake seismology. An additional ingredient, already present in the Dugdale model, is that during an earthquake the strength of the fault drops completely to a residual value beyond a characteristic slip D_c that has been often regarded as a material property. In particular, in the linear slip weakening model the strength drop is linear with slip. In this contribution I will address the effects of relaxing the assumption of complete strength drop by testing an empirical family of non-linear, long-tailed, slip dependent weakening laws. This abstract reviews the experimental, seismological and theoretical motivations for such modeling. The results of the simulations and their analysis will be thoroughly discussed in the oral presentation.

2 EXPERIMENTAL CONSTRAINTS

Slip weakening is generally observed in experimental rock mechanics [2]. In the early 80's a modern description of second order effects in rock friction was developed, leading to the rate-and-state formalism [3, 4]. Rate-and-state contains the slip-weakening ingredient in some limit and has been successfully applied to slow earthquake initiation, post-seismic relaxation, earthquake clustering and triggering and other long term processes. However, experimental rock mechanics still has to reach some of the important conditions proper for real earthquakes, such as fast slip rates (≈ 1 m/s) and large displacements (> 1 m). For this reason, earthquake dynamics simulations are often performed under hypothetical strength weakening laws that enclose a limited set of physical ingredients into a tractable mathematical form. The most fundamental common ingredients are the fracture energy G_c and the characteristic slip D_c.

Recently, the experimental frontier of large displacements was explored by shearing samples of granular material in a large rotary apparatus [5]. The granular material simulates the gouge believed to be present in the core of deep active fault zones. As deformation spontaneously localizes at the inner boundary of the sample, the experimental behavior is consistent with our usual view of fault slip. A typical rate-and-state behavior with short D_c was consistently observed but was found to be a relatively minor effect. The dominant behavior was a persistent slip weakening process: the gouge approached residual strength only after slipping several meters. This is in sharp contrast with the usually small values of $D_c \approx 0.1$ mm observed in small scale experiments and even contradicts our usual view of a finite D_c. On this basis, a robust slip weakening law was proposed [5] in the form of an inverse power law:

$$\Delta strength \propto (\epsilon + slip)^{-p} \tag{1}$$

with exponent $p \approx 0.4$ and a less well resolved "regularization" slip $\epsilon < 1$ mm. The distinctive features of this new family of laws are: a very steep weakening rate in the short slip range followed by a long tailed, non linear, weakening process with no characteristic slip. Whereas caution is still required when extrapolating these low velocity experiments to natural earthquake conditions, we will see next that some seismological observations seem to converge to similar interpretations.

3 SEISMOLOGICAL CONSTRAINTS

One of the major goals of earthquake seismology is to use recorded seismograms to constrain dynamic fault properties. For large and well instrumented earthquakes, near-field data can now be inverted for the distribution of strength excess, strength drop and D_c [7]. Resolution is problematic due to the limited frequency band of the data. However a more fundamental issue is the strong trade-off between the inverted parameters [8]. This is a signature of the more fundamental role of G_c, which is a compound parameter: the integral of the strength weakening curve up to D_c. Theory indicates that only few aspects of dynamic rupture depend on higher order properties of the weakening law: some seismic nucleation phases (the initial shape of seismograms) depend on the weakening rate [9] and the transition to supershear rupture velocity depends strongly on strength excess [10]. Unfortunately both phenomena are rarely observed, although in the first case optimal conditions should be found in the new generation of deep borehole instrumentation through active faults. So far it seems that we should conform to a minimal description of fault strength in terms of fracture energy G_c.

Moreover most earthquakes are not favorable to the detailed studies of near-field strong-motion seismology, either because of their small magnitude or due to the lack of dense instrumentation. Frequently only macroscopic constraints are available. Among the usual macroscopic quantities two are fundamental and their inter-relations are the subject of ongoing debates: seismic moment M_0 and radiated energy E_r. Whereas the former is routinely reported, estimation of the latter has proved more problematic [11], although much progress has been done recently [12]. Inversion schemes for higher order properties of the seismic moment [13] should soon give further consistent constraints from the far-field on earthquake size, rupture velocity, duration and directivity.

Instead of focusing on very large earthquakes the key is in trying to retrieve clues from as large as possible a magnitude range. An important goal is to bridge the gap between the well observed large earthquakes and the well constrained laboratory scales: how much can be learned about destructive earthquakes from the observation of smaller but more frequent ones and from laboratory experi-

ments? This requires the development of specialized methods to analyze microseismicity, such as the forthcoming data from deep borehole experiments; and an improved synergy between earthquake source seismology and mining and induced hydro-fracture seismology.
In the meantime, the E_r/M_0 scaling debate has been enriched with interpretations based on the slip weakening framework. It was recently shown [14] that a broad magnitude range of E_r/M_0 observations was consistent with power law slip weakening. This supports the previously discussed laboratory derived weakening laws, eqn (1).
On the other hand, the hypothesis of linear slip weakening has been tested through new seismological constraints from seismic nucleation phases [9]. In non-LEFM, large scale yielding, models of nucleation under linear weakening, the initial shape of seismograms is predicted to depend on the weakening rate W, through a balance between slip weakening and fault impedance (radiation damping). On a broad magnitude range, the estimated weakening rates were observed to be magnitude dependent, $W \propto M_0^{-1}$. A possible interpretation is non-linear strength weakening $W \propto slip^{-1}$, which points again to the family of laws of eqn (1).

4 THEORETICAL CONSTRAINTS

The steep weakening rate in the short slip range featured by eqn (1) is essential to understand the coexistence of large and small earthquakes. If G_c is a material property LEFM predicts the existence of a minimal stable crack size which, considering the seismological estimates of G_c for large earthquakes, would be in contradiction with the observation of microearthquakes. Scale-dependency of G_c is one way out, but an alternative view comes from a non-LEFM, large scale yielding, analysis of earthquake nucleation. It has been shown [15] that under linear slip weakening and general loading conditions the minimal earthquake nucleation size is related to the weakening rate W. It is reasonably expected that non linear weakening will allow for a broad range of nucleation sizes.
The search for complexity in simulated seismic cycles under deterministic continuum mechanics was elusive until recently [16]. One of the ingredients that were found to be required to generate complex synthetic seismicity (Gutenberg-Richter distribution of earthquake sizes) is the presence of two decoupled slip-scales in the weakening law. It is then reasonable to expect that the slip weakening family eqn (1), with practically no characteristic slip or alternatively with a continuum of slip-scales, will be favorable to the generation of realistic earthquake cycle complexity.
The seismological observations presented in the previous section can alternatively be interpreted as a signature of scale-dependent fault properties. Some have pursued this idea within the framework of fault friction [17], with controversial propositions. It is perhaps more natural to approach the issue from the standpoint of off-fault non linear processes. At the outset of modern dynamic earthquake modeling [1] it was recognized that non linearities of the material surrounding the fault might play an essential role in fault dynamics. A simple yielding bulk rheology was shown to account for usual experimental and seismological facts: low limiting rupture velocities, a moderate fraction of shear wave velocity; and very large apparent fracture energies, the estimates of G_c for large earthquakes being many orders larger than the laboratory estimates. Surprisingly enough, simulation of earthquake dynamics has been since conducted under the more restricted fault friction paradigm. Among the arguments in favor of the friction framework are: the advent of the modern experimentally derived friction laws, some evidence for intense coseismic strain localization and its numerical and analytical tractability. Despite isolated efforts to study the feedback of secondary dynamic faulting on main fault rupture propagation [18], the development of a complete description of non linear off-fault processes is still a rather unexplored frontier of earthquake seismology. In the meantime it should be born in mind that the surface friction model is a lumped representation of fully tri-dimensional dynamic processes. This reconciles the apparent scale-dependency of fault

friction parameters: they should not be understood as material properties but as effective properties, just as toughness is understood in fracture mechanics in the presence of R-curve behavior [19]. The apparent scale dependency is the result of the dynamic evolution of the tri-dimensional process region, which is expected to grow with rupture size [1].

The development of a dynamic bulk damage or elasto-plastic model is out of the scope of this contribution. I will instead focus on the non linear friction laws of the form of eqn (1). However, non linear weakening can be regarded as a preliminary step to that ultimate goal: it can be seen as a lumped model for non linear processes occurring within the wake of the off-fault process zone.

5 NUMERICAL SIMULATIONS

Spontaneous dynamic rupture is simulated in 2D mode III configuration on a planar fault embedded in a linear elastic unbounded medium. The problem is numerically solved with a spectral boundary integral equation method, including time-truncated kernels to allow for long runs containing slow nucleation stages [20]. The first rupture is nucleated by a slow uniform ”tectonic” load on top of a peaked non uniform initial stress that crudely mimics the state of stress left by previous ruptures [15]. Subsequent events are spontaneously triggered by the ”tectonic” load. Instantaneous or time-dependent healing can be included.

I will focus on static and dynamic macroscopic properties of the simulated ruptures such as magnitude/size scalings and magnitude/radiated energy scalings. In the usual seismological interpretation of macroscopic quantities it is implicitly assumed that local quantities, such as stress drop and fracture energy consumption, have weak fluctuations along the fault [12]. This is not a very robust assumption when weakening is strongly non-linear and persistent. I will illustrate the possible bias with the help of numerical modeling. This will also shed some light on how to infer weakening properties from far-field observations. The results of the numerical simulations will be reported and discussed during the meeting.

6 CONCLUSION

Experimental and seismological observations are converging towards new paradigms of earthquake dynamics modeling. I suggest that some longstanding open questions of source seismology, specially those related to source scaling relations, may find a way out with the advent of models accounting for off-fault dynamic non linearities and their feedback onto main fault rupture propagation. As a preliminary step I explore the possible effects of an empirical non linear slip weakening law on macroscopic earthquake source parameters. The main motivations have been exposed here. The results of numerical simulations will be reported and discussed during the meeting.

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