Estimating the Effect of Earth Elasticity and Variable Water Density on Tsunami Speeds

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The speed of tsunami waves is typically calculated using the shallow-water approximation over a rigid-body Earth. Recent comparisons of tsunami arrival times from the March 11, 2011 tsunami suggest, however, that the standard formulation has errors around the one percent level, and it has been suggested that the elasticity of the Earth can explain the discrepancy. While previous work has indeed shown that such elastic deformation can modify tsunami speeds, the effect has been neglected partly due to the difficulty in understanding how large this elastic effect is. Here, we remedy this by providing a new derivation and expression for how to incorporate the first-order effect that solid Earth elasticity and ocean water compressibility has on tsunami speeds. This result is shown to agree approximately with previous theory, and helps to explain observed timing discrepancies from the March 11, 2011 tsunami. The dispersive elastic correction and the non-dispersive compressibility correction together may account for the majority of the observed discrepancy.

1. Introduction

The theory for understanding tsunami wave propagation has been well known since at least 1845 [*Airy*, 1845; *Thomson*, 1887; *Takahasi*, 1943, 1949; *Murty*, 1977]. The classical theory treats the problem of gravity waves in a uniform ocean of arbitrary depth, from which the tsunami solution can be derived as the shallow water wave approximation to the full solution. Although a few authors have pointed to a few potential deficiencies in the standard theory [e.g., *DeDontney and Rice*, 2011], the vast majority of the tsunami modeling community relies on this standard (nonlinear) shallow water formulation, in which the local tsunami phase speed, c, is non-dispersive and given by $\sqrt{gH}$ where g is the acceleration of gravity and H is the local ocean depth [*Ward*, 1980; *Titov and Synolakis*, 1998; *Titov et al.*, 2005; *Simons et al.*, 2011].

However, recently, it has been noticed that far-field observations of tsunamis, including those from the 2010 Maule Earthquake and the 2011 Tohoku-Oki Earthquake, have main arrivals that are later than expected from the standard theory [*Simons et al.*, 2011; *Watada et al.*, 2011; *Kusumoto et al.*, 2011; *Yamazaki et al.*, 2012; *Grilli et al.*, 2012; *Kusumoto et al.*, 2012]. Yet even though there exist theoretical frameworks [*Sells*, 1965; *Ward*, 1980; *Okal*, 1982; *Comer*, 1984; *Lynett and Liu*, 2004] which suggest various potential improvements to the standard theory, the relative complexity of these formulations makes it challenging to understand what effect is most important and what this effect is most sensitive to. To address this issue, here we present a new derivation for the effect of solid Earth elasticity and ocean water compressibility on tsunami wave speeds. The elastic correction to tsunami phase velocity is shown to be proportional to wavelength, inversely proportional to an average shear modulus, and has no dependence on ocean depth. Inclusion of this correction seems to explain a significant part of the difference between observed and modeled tsunami waveforms from the two recent earthquakes mentioned previously, especially at relatively long wavelengths. The correction is also shown to be consistent, to first order, with the results of *Comer* [1984], but the present result has the advantage that it highlights the physical mechanism responsible for the correction in a straightforward way. Agreement with observations is found to be significantly improved after also accounting for the correction from ocean water density increasing with depth due to compressibility.

2. Theory

2.1. Basic Theory

In any conservative mechanical system, the total energy, E, remains constant, and can be written as $E = U + T$ where U is potential energy and T is kinetic energy. Energy oscillates between U and T, with the sum not changing. In the simplest case, a single parameter, ξ, can be used to define the state of the system, where ξ often signifies a displacement, and ξ undergoes simple harmonic motion. In such a case, $U = 1/2 \cdot K \cdot \xi^2$ and $T = 1/2 \cdot M \cdot (d\xi/dt)^2$, where K is an effective stiffness, M is an effective mass and t is time. For example, for a simple spring-block system, ξ is displacement of the block, K is the spring's elastic constant, and M is the mass of the block. The frequency of oscillation, ω, in such a system can be expressed as $\omega^2 = K/M$.

2.2. Theory Applied to the Standard Tsunami Problem with Rigid Bottom

As has been shown previously [e.g., *Lighthill*, 1978; *Stevenson*, 2005], the basic theory described in Section 2.1 can be used to derive the standard (linear shallow water) tsunami dispersion relation and speed. We summarize this simple derivation here. The ocean is assumed to be of fixed water depth H at rest, and $h(x,t) = \xi(t)\cos(kx)$ is taken to be the vertical displacement of the sea surface. As shown in Fig. 1, ξ is the amplitude of the sinusoidal wave, k is the wavenumber, and x is position in the direction of wave propagation. (In this analysis, although the wave is assumed to be a standing wave, a propagating wave can be constructed as a superposition of 2 standing waves.) From this picture of shallow water waves ($kH \ll 1$), we first note that horizontal velocities, $\dot{u} \equiv \partial u/\partial t$, are approximately uniform with depth and can be determined by mass conservation. In an incompressible ocean, this leads to

$$H\dot{u}(x) = -\int_0^x \dot{h}(s)ds = -\frac{\sin(kx)}{k}\frac{d\xi}{dt}. \tag{1}$$

Since vertical velocities, $\dot{w}$, are of order $d\xi/dt$ and since $kH \ll 1$, Eq. (1) shows that $\dot{u} \gg \dot{w}$ (by a factor of $1/(kH)$) so that vertical velocities can be safely ignored in computations of T. With this approximation, both U and T can be calculated by direct integration. Per unit area, averaged over a wavelength, we have

$$U = U_0 \equiv \frac{k}{2\pi}\int_0^{2\pi/k} \rho g(H+h)\cdot\frac{h}{2}dx = \frac{\rho g \xi^2}{4}, \tag{2}$$

and

$$T \approx \frac{k}{2\pi}\int_0^{2\pi/k} \frac{\rho H \dot{u}^2}{2}dx = \frac{\rho H}{4(kH)^2}\cdot\left(\frac{d\xi}{dt}\right)^2, \tag{3}$$

where ρ is water density, g is the acceleration of gravity, and U is referenced to the center of mass of the undisturbed ocean. We can therefore identify $K = K_0 \equiv \rho g/2$, $M = \rho H/(2k^2H^2)$, so that $\omega^2 = gHk^2$. From this dispersion relation, one can then calculate phase velocity as

$$c \equiv \omega/k = \sqrt{gH} \equiv c_0, \tag{4}$$

and we recover the standard expression for tsunami wave propagation speed. (Similarly, group velocity $c_g = \partial\omega/\partial k = \sqrt{gH}$.)

2.3. Tsunami Problem with an Elastic Substrate

It is straightforward to extend the analysis of Section 2.2 to account for elasticity of the substrate rather than having a rigid ocean bottom (see Fig. 1). For this new problem, all that must be done is to calculate a new expression for U and T that accounts for elastic deformation of the substrate. In the following paragraphs, we will find that to a first approximation, T is unchanged whereas $U = U_0 + U_e + \Delta U_0$ where U_0 is the standard tsunami energy of Eq. (2), U_e is an elastic energy stored in the elastic medium due to the added pressure from the water wave, and ΔU_0 is a correction term that accounts for the extra energy of the ocean due to displacement of the ocean floor. Once a new U and T are calculated, we perform the same identification of K and M to calculate ω and c as in Section 2.2. Note that hereafter h denotes the perturbation of the total water column depth, ξ its sinusoidal amplitude, and w_s the vertical displacement of the seafloor.

To determine U_e, we solve the static elasticity problem of periodic (excess) pressure loading of

$$p(x) = \rho g \xi \cos(kx). \tag{5}$$

For the simplest case of an elastic halfspace, the solution for surface displacement is well known and given by *Jeffreys* [1976] as

$$w_s(x) = -\frac{(1-\nu)\rho g \xi \cos(kx)}{\mu k}, \tag{6}$$

where μ is the shear modulus and ν is the Poisson's ratio of the elastic medium. Eq. (6) is valid for the problem of interest so long as $w_s \ll h$, which is indeed the case over the entire range of relevant parameter space (see values listed later). U_e (per unit area, as before) can then be calculated as

$$U_e = \frac{k}{2\pi}\int_0^{2\pi/k} \frac{-p(x)w_s(x)}{2}dx = \frac{(1-\nu)\rho^2 g^2 \xi^2}{4\mu k}. \tag{7}$$

With $w_s(x)$ given by Eq. (6), ΔU_0 can be calculated as

$$\Delta U_0 = \frac{k}{2\pi}\int_0^{2\pi/k} \rho g[H + h(x)]w_s(x)dx = -2U_e, \tag{8}$$

and so $U = U_0 - U_e$. We therefore find

$$K = \frac{\rho g}{2}\left[1 - \frac{(1-\nu)\rho g}{\mu k}\right]. \tag{9}$$

For the opposite extreme end-member, where the elastic zone is of thickness $H_e \ll 1/k$ (and is rigid underneath), the same pressure loading of Eq. (5) results in effectively one-dimensional loading (i.e. all strains are zero except for vertical strains), in which case

$$w_s(x) = w_{H_e}(x) \equiv -\frac{(1-2\nu)\rho g H_e \xi \cos(kx)}{2(1-\nu)\mu}, \tag{10}$$

and

$$K = K_{H_e} \equiv \frac{\rho g}{2}\left[1 - \frac{(1-2\nu)\rho g H_e}{2(1-\nu)\mu}\right]. \tag{11}$$

For a layered elastic halfspace, the solution is more complicated and, in general, must be numerically calculated. However, we note that a fair approximation may be obtained by using Eq. (9) where $(1-\nu)/\mu$ is taken as a weighted average over depth, z, with weighting e^{-kz} to account for the exponential decay of strain with depth. For the extreme case of Eq. (10), this estimate would be identical except for a dependence on Poisson's ratio as $1-\nu$ instead of $(1-2\nu)/(2-2\nu)$. For reasonable values of ν, the estimate of the correction term for K is therefore off by a factor of about 2 in this extreme case.

For T, we note that the elastic kinetic energy term scales as w_s^2/k whereas the standard kinetic energy term scales as $h^2/(Hk^2)$ so that $w_s \ll h$ and $kH \ll 1$ imply that the elastic kinetic energy term is negligible compared to the T calculated in Eq. (3).

With the new estimate of K in Eq. (9) and the original estimate of T from Eq. (3), we calculate

$$\omega^2 = gHk^2 \left[1 - \frac{(1-\nu)\rho g}{\mu k}\right], \tag{12}$$

and therefore

$$c = \sqrt{gH} \cdot \sqrt{1 - \frac{(1-\nu)\rho g}{\mu k}} \approx \sqrt{gH} \left[1 - \frac{(1-\nu)\rho g}{2\mu k}\right], \tag{13}$$

where the approximation is valid since $w_s \ll h$. From Eq. (13) we can immediately estimate the decrease in tsunami speed expected from incorporating the elasticity of the solid Earth. To make a preliminary estimate of the magnitude of this correction, we take $k = 2\pi/1000$ km, $\mu = 5 \cdot 10^{10}$ Pa, $\nu = 0.3$, $\rho = 1000$ kg/m^3, $g = 9.8$ m/s^2, which yields an estimate of a -1.1% phase velocity correction due to elasticity. We therefore see that this correction is significant and, for example, is much larger than the correction of using the non-shallow-water equations ($c = \sqrt{g \tanh(kH)/k}$ rather than $c = \sqrt{gH}$) for 1000 km wavelengths and typical ocean depths (≈ 4 km). The conclusion that this correction is significant differs from that of *Okal* [1982] potentially because his work does not fully account for perturbations in the gravitational potential of the solid Earth.

We note that our result can be shown to agree to first order with the expression of *Comer* [1984] derived with a more classical approach (including non-shallow-water waves). In his case, he ends up with a relation that can be simplified and approximated as

$$\begin{aligned} c^2 &= \frac{g}{k} \cdot \frac{\tanh(kH) - (3\rho c^2/4\mu)}{1 - (3\rho c^2/4\mu)\tanh(kH)} \\ &\approx \frac{g}{k} \cdot [kH - (3\rho g H/4\mu)] = gH\left(1 - \frac{3\rho g}{4\mu k}\right), \end{aligned} \tag{14}$$

when assuming $\nu = 1/4$ and, as before, that $w_s \ll h$ and $kH \ll 1$. This then suggests the non-shallow analog of Eq. (13) to be

$$\begin{aligned} c &= \sqrt{\frac{g\tanh(kH)}{k}} \cdot \sqrt{\frac{1 - \frac{(1-\nu)\rho g}{\mu k}}{1 - \frac{(1-\nu)\rho g}{\mu k}\tanh^2(kH)}} \\ &\approx \sqrt{\frac{g\tanh(kH)}{k}} \cdot \sqrt{1 - \frac{(1-\nu)\rho g}{\mu k}}, \end{aligned} \tag{15}$$

where we still assume $w_s \ll h$, and the approximation is valid to first order in the small parameter kH.

2.4. Tsunami Problem with Variable Water Density

Accounting for the increase of ocean water density with depth (due to ocean water compressibility, which dominates over the effects of temperature and salinity on density) also yields a systematic correction. In the standard formulation (e.g., Eq. (4)), it is observed that dependence on the density of ocean water disappears since both K and M are proportional to ρ. However, if density increases with depth, the derivations of Eqs. (1)-(3) must be revised. We begin our rederivation by assuming density increases approximately linearly with depth from the disturbed surface, i.e. $\rho(z') = \rho_0[1 + \rho_0 g(H + h - z')/\mathcal{K}]$ where $\mathcal{K}$ is the bulk modulus, ρ_0 is the surface density, and z' is the height above the ocean floor. Assuming that the pressure is still given by the static density field, the linearized momentum equation, $\rho(z')\ddot{\mathbf{u}} = -\nabla p$, can then be simplified as

$$\rho(z')\ddot{u} = -\rho(z')g\frac{\partial h}{\partial x}, \tag{16}$$

and we find that $\dot{u}$ is still constant with depth. The general continuity equation, $\dot{\rho}+\nabla\cdot(\rho\dot{\mathbf{u}})=0$ can be simplified as

$$\frac{\rho_0 g}{\mathcal{K}}\dot{h}+\frac{\partial}{\partial x}[\rho(z')\dot{u}]+\frac{\partial}{\partial z}[\rho(z')\dot{w}]=0. \quad (17)$$

Integrating z' from the bottom to the surface, and ignoring higher order terms then yields

$$\rho_H\dot{h}+\rho_{avg}H\frac{\partial\dot{u}}{\partial x}\approx 0, \quad (18)$$

where $\rho_H\approx\rho_0[1+\rho_0 gH/\mathcal{K}]$ is the density of water at the ocean bottom and $\rho_{avg}\approx\rho_0[1+\rho_0 g/(2\mathcal{K})]$ is the average density of water from the ocean bottom to the surface. Eq. (18) can then be rewritten in the same form as Eq. (1) as

$$H\dot{u}\approx-\frac{\rho_H}{\rho_{avg}}\int_0^x\dot{h}(s)ds. \quad (19)$$

The integrated continuity equation (mass conservation) also dictates that the average surface height no longer remains constant and we must now distinguish between the reference water depth H_0 and the average water depth $H(t)=H_0(1+\alpha(t))$. Mass (averaged over one wavelength) is given by

$$\begin{aligned}\mathcal{M} &= \frac{k}{2\pi}\int_0^{2\pi/k}\int_0^{H+h}\rho(z)dzdx \\ &\approx \mathcal{M}_0+\rho_0 H_0\left[\alpha+\frac{\rho_0 g\xi^2}{4\mathcal{K}H_0}\right], \quad (20)\end{aligned}$$

where $\mathcal{M}_0$ is the mass in the reference state when $\xi=0$. Mass conservation then implies $\alpha=-\rho_0 g\xi^2/(4\mathcal{K}H_0)$. U can now be calculated as the sum of two contributions, $U=U_g+U_{pV}$ where U_g is the gravitational potential energy and U_{pV} is the pressure-volume energy stored. U_g

is given by

$$\begin{aligned} U_g &= \frac{k}{2\pi}\int_0^{2\pi/k}\int_0^{H+h}\rho(z')gz'dz'dx \\ &= \frac{1}{2}\rho_0 gH^2[1+\frac{\rho_0 gH}{3\mathcal{K}}]+\frac{1}{4}\rho_0 g\xi^2[1+\frac{\rho_0 gH}{\mathcal{K}}] \\ &\approx \frac{1}{2}\rho_0 gH_0^2(1+\frac{\rho_0 gH_0}{3\mathcal{K}})+\frac{1}{4}\rho_0 g\xi^2 \\ &= U_{g0}+\frac{1}{4}\rho_0 g\xi^2, \end{aligned} \tag{21}$$

where U_{g0} is U_g in the reference state. Similarly, U_{pV} can be calculated as

$$\begin{aligned} U_{pV} &= \frac{k}{2\pi}\int_0^{2\pi/k}\int_0^{H+h}\int_0^h \frac{\rho_0 g\cdot p(h',H,z')}{\mathcal{K}}dh'dz'dx \\ &= \frac{k}{2\pi}\int_0^{2\pi/k}\int_0^{H+h}\rho_0 g(H+\frac{h}{2}-z')\frac{\rho_0 gh}{\mathcal{K}}dz'dx \\ &= \frac{\rho_0 g}{4}\frac{\rho_0 gH}{\mathcal{K}}\xi^2. \end{aligned} \tag{22}$$

Adding the two contributions therefore gives

$$U = U_g + U_{pV} - U_0 \approx \frac{\rho_H g\xi^2}{4}. \tag{23}$$

With Eq. (19) replacing Eq. (1) and Eq. (23) replacing Eq. (2), we can now identify new expressions for K and M as $K \approx \rho_H g/2$ and $M = \rho_H^2 H_0/(2\rho_{avg})$ so that

$$c \approx \sqrt{\frac{\rho_{avg}}{\rho_H}}\sqrt{gH} \approx \left(1-\frac{\Delta\rho}{4\rho_{avg}}\right)\sqrt{gH}, \tag{24}$$

where $\Delta\rho = \rho_0^2 gH_0/\mathcal{K} \approx \rho_H - \rho_0$. We note that this correction of Eq. (24) is asymptotically similar to other density corrections proposed previously but is different in the numerical factor. For example, where our Eq. (24) has $1/4$, *Okal* [1982] obtains a factor of $1/6$ from solving a normal mode problem, whereas the classical compressible tsunami solution has a factor of $1/2$ [D. Wang, personal communication]. In both of these alternative derivations, it seems that density is assumed constant with depth but compressible, whereas our derivation assumes a more realistic density profile increasing linearly with depth (in accordance with the compressibility). We attribute the difference between our prediction and those others as due to this difference in assumption, but a full analysis of why these discrepancies exist is beyond the scope of this paper. We also note that our correction can also be obtained simply by substituting Eq. (16) into Eq. (18) and solving the resulting wave equation. Therefore, the energy approach is seen to be a completely compatible alternative to the classical approach (under the same assumptions).

For average ocean water (surface temperature of 17°C, salinity of 3.5%, depth of $H = 3800$ m), surface density is $\rho_0 = 1025.5$ kg/m^3, and standard high pressure corrections [*Millero et al.*, 1980] predict close to the assumed linear increase of density with depth, with $\rho_H = 1044.7$ kg/m^3 and $\rho_{avg} = 1036.1$ kg/m^3. We therefore expect c to be roughly 0.5% slower than the standard shallow-water theory prediction due to water density increasing with depth.

Finally, we note that a similar correction could be made to Eq. (15). While we do not attempt a full derivation of this, examination of the various terms involved in the previous derivation suggests that the non-shallow water analogue to Eq. (24) is approximately correct when written as

$$c = \sqrt{\frac{\rho_{avg}}{\rho_H}} \cdot \sqrt{\frac{g \tanh(kH)}{k}} \cdot \sqrt{\frac{1 - \frac{(1-\nu)\rho g}{\mu k}}{1 - \frac{(1-\nu)\rho g}{\mu k} \tanh^2(kH)}}, \quad (25)$$

where ρ_{avg} and ρ_H now pertain to the portion of the water column that has significant motion. For example, in the deep-water limit, both densities may be expected to be equal to the surface density ρ_0 and therefore not have an effect on c.

3. Application to the Earth

To more accurately compute the elastic correction of Eq. (13), we perform the layered elastic halfspace approximation suggested in Section 2.3. For our layered elastic model, we take PREM

[*Dziewonski and Anderson*, 1981] from the ocean bottom (at 3 km depth) to the core-mantle boundary (at 2891 km depth). As shown in Fig. 2 (dashed blue curve), the elastic correction predicts a discrepancy of about 0.3% from standard shallow-water predictions at a wavelength of 200 km, and a discrepancy of close to 1.0% at 1000 km. Once standard non-shallow water effects are included (with standard dispersion $\omega^2 = gk \tanh kH$, the solid blue curve in Fig. 2), the simple correction is similar to the preliminary results computed by *Watada et al.* [2011].

Including the additional (non-dispersive) ocean water density correction of Eq. (25) (thick solid green curve in Fig. 2), we find that our predicted correction is now significantly larger. In particular, the correction at shorter wavelengths (200 km) is about a factor of 2 larger (going from 0.5% to 1.0%), and the correction at longer wavelengths (1000 km) goes from about 1.0% to 1.5%. This predicted discrepancy reproduces reasonably well both the dominant-period discrepancies observed by a number of authors [e.g., *Simons et al.*, 2011; *Yamazaki et al.*, 2012; *Grilli et al.*, 2012] as well as the preliminary measurements of *Watada et al.* [2011], *Kusumoto et al.* [2011] and *Kusumoto et al.* [2012].

4. Discussion and Conclusions

In this work, we find that an energy argument can be used to derive the shallow water tsunami wavespeed with the inclusion of elastic Earth deformation as well as non-uniform water density. Both effects are shown to result in significant deviations from the standard shallow water prediction. In particular, the density effect slows the tsunami by about 0.5% and the elastic effect slows the tsunami by 0.2-1.0% as wavelength increases from 200 to 1000 km. Together, the two effects are able to explain much of the discrepancy for the March 11, 2011 Tohoku earthquake tsunami observed across the Pacific Ocean [*Simons et al.*, 2011; *Watada et al.*, 2011; *Yamazaki* *et al.*, 2012; *Grilli et al.*, 2012]. Since the corrections in Eq. (25) that we suggest are straightforward to calculate (for example, much easier than performing the lengthy computations that *Watada et al.* [2011] undertakes), we suggest that the correction can eventually be included in standard tsunami models.

We would like to stress, however, that we have only attempted to understand two corrections to the standard shallow water theory that we have calculated to be important. There remain at least three major issues. First, there are other potentially important corrections. For example, we have not attempted to include the effect of radiation of elastic energy (and hence dissipation of tsunami energy), classical dispersion as in *DeDontney and Rice* [2011], interactions with a thin viscous muddy layer as in *Gade* [1958] or other types of bottom friction, or Coriolis effects, and we do not attempt to address the potential bias resulting from using coarsely sampled bathymetry. Some of these effects, like that of elastic radiation, may be relatively straightforward to include within the framework we have constructed, but we make no attempt here to do so, and some of these effects may be important. Secondly, we have only provided a physical explanation of the corrections discussed and have not attempted to put the equations derived into a form that is commonly used by tsunami modelers. Additional work will be necessary to provide this information. A third issue is that comparisons of the various 'data' [e.g., *Simons et al.*, 2011; *Watada et al.*, 2011; *Kusumoto et al.*, 2011; *Yamazaki et al.*, 2012; *Grilli et al.*, 2012; *Kusumoto et al.*, 2012] yield comparable but not completely identical results, and we do not claim that any of them are entirely accurate. Nonetheless, we believe our theoretical results to be applicable and useful, independent of the precise observations. Our suggested corrections represent an improved physical model and, moreover, predict corrections that seem to account for a majority of the current discrepancies between actual tsunami arrival times and the standard tsunami model predictions.

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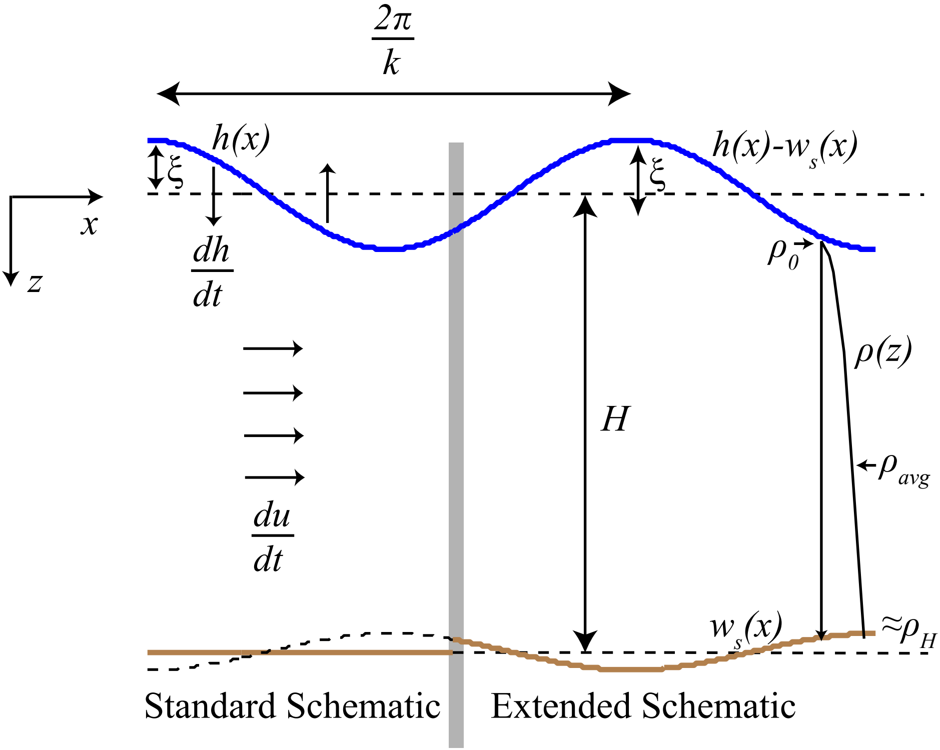
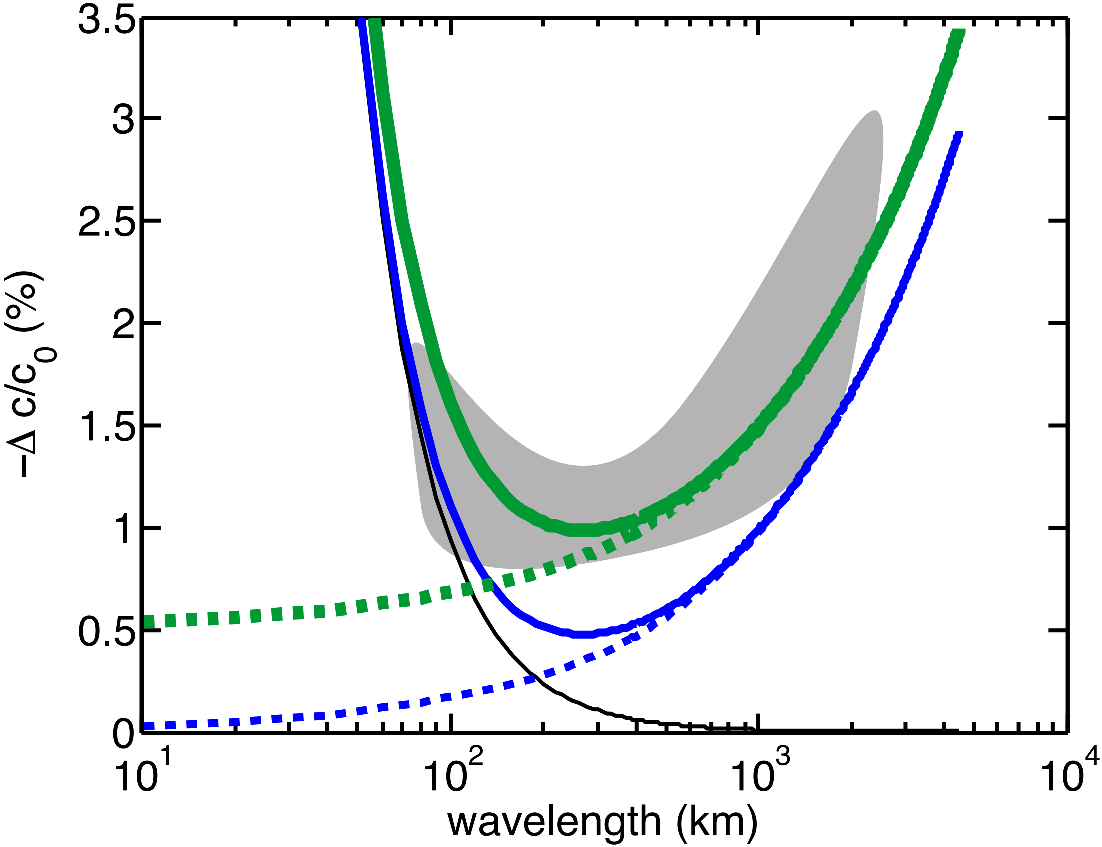
Figure 1. Schematic depicting tsunami model parameters. The blue curve denotes the sea surface and the brown curve denotes the ocean bottom. The left side is for the standard rigid bottom case of Section 2.2 whereas the right side is for the extensions as discussed in Sections 2.3 and 2.4. In both cases, $h(x) = \xi \cos(kx)$. The density profile $\rho(z)$ is assumed to be primarily a function of depth from the sea surface, as drawn.

Figure 2. Deviation from the standard shallow water speed. Solid thin black curve is the standard non-shallow water correction ($\omega^2 = gk \tanh kH$). Dashed blue curve is the prediction based only on including the elastic deformation correction of Eq. (13) and the solid blue curve is using Eq. (15) (i.e. Eq. (13) corrected by the black curve). Solid thick green curve is the prediction using both the ocean density correction and the elastic correction of Eq. (25), and the dashed thick green curve is using the shallow-water approximation to Eq. (25). Although the observed discrepancies are not very well documented, the shaded gray region denotes the approximate range of measured tsunami travel-time discrepancies from *Simons et al.* [2011], *Kusumoto et al.* [2011], *Watada et al.* [2011], *Yamazaki et al.* [2012], *Grilli et al.* [2012], and *Kusumoto et al.* [2012].



2π/k
h(x)
ξ
h(x)-w_s(x)
ξ
x
z
dh/dt
ρ_0
ρ(z)
H
du/dt
ρ_avg
w_s(x)
≈ρ_H
Standard Schematic
Extended Schematic


3.5
3
2.5
2
1.5
1
0.5
0
$-\Delta c/c_0$ (%)
10^1
10^2
10^3
10^4
wavelength (km)