DYNAMIC RUPTURE WITH OFF-FAULT CONTINUUM DAMAGE

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INTRODUCTION

High stress concentration close to a dynamic rupture front can produce rock damage (reduction of elastic moduli) in the material surrounding the fault. Off-fault yielding and energy absorption in the damage process should reduce the amplitude of the ground motion. However, the reduced elastic moduli in the damaged zone can amplify locally the motion and create a waveguide that may allow the motion to propagate with little geometric attenuation. In addition, the asymmetric damage generated in the in-plane rupture mode may produce bimaterial interfaces that can reduce the frictional dissipation and increase the radiation efficiency. Off-fault damage is of special importance for ruptures running along faults that separate rocks of different elastic properties, but they can generate asymmetric patterns of material degradation that might be observable in the field.

Previous studies incorporated plastic yielding in simulations of dynamic rupture (Andrews, 1975, 2005; Ben-Zion and Shi, 2005; Templeton et al., 2007) while keeping the elastic moduli unchanged. Here we examine dynamic earthquake ruptures on a frictional fault considering a continuum damage rheology for the evolution of elastic moduli above a yielding level (e.g., Lyakhovsky and Ben-Zion, 2008). Our numerical simulations are based on the spectral element method and involve slip-weakening and velocity-weakening friction. We are interested in how the parameters of the friction law, damage rheology and background stress control the rate of growth of the off-fault damage zone, the terminal rupture speed, the energy balance, and the maximum slip rate and ground motion.

CONTINUUM DAMAGE MODEL

We adopt the continuum damage formulation by Lyakhovsky et al. (1997) including damage-related plasticity as introduced by Hamiel et al. (2004) and modified for 2D plane strain.

The first and second invariants of the 2D elastic strain tensor are defined as $I_1 = \epsilon^e_{kk}$ and $I_2 = \epsilon^e_{ij}\epsilon^e_{ij}$, respectively. A strain invariant ratio is defined as $\xi = I_1/\sqrt{I_2}$. The following non-linear stress-strain relation is assumed:

$$\sigma_{ij} = (\lambda - \gamma/\xi)\, I_1 \delta_{ij} + (2\mu - \gamma\xi)\, \epsilon^e_{ij} \tag{1}$$

where γ is an additional elastic modulus. The elastic moduli depend on a scalar damage variable, α, that varies between $\alpha = 0$ (intact) and $\alpha = 1$

$$\begin{aligned} \lambda &= \lambda_0 && (2)\\ \mu &= \mu_0 + \gamma_r \xi_0\, \alpha && (3)\\ \gamma &= \gamma_r\, \alpha && (4) \end{aligned}$$

where λ_0 and μ_0 are Lamé's parameters for the intact material. The parameter ξ_0 is the threshold value of the strain invariant ratio ξ at the onset of damage. It is related to the internal friction angle ϕ in a cohensionless Mohr-Coulomb yield criterion by

$$\xi_0 = \frac{-\sqrt{2}}{\sqrt{1 + (\lambda_0/\mu_0 + 1)^2\, \sin^2\phi}} \tag{5}$$

The scaling factor γ_r is chosen such that convexity is lost at $\alpha = 1$ when $\xi = \xi_0$. The evolution equation for the damage variable is

$$\dot{\alpha} = \begin{cases} C_d I_2(\xi - \xi_0) & \text{if } \xi > \xi_0 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

The total strain is the sum of an elastic and a plastic contribution, $\epsilon = \epsilon^e + \epsilon^p$. The evolution of the plastic strain is driven by the damage variable α:

$$\dot{\epsilon}^p_{ij} = \begin{cases} \tau_{ij} C_v \dot{\alpha} & \text{if } \dot{\alpha} \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where $\tau_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\,\delta_{ij}$ is the deviatoric part of the stress tensor. The parameter C_v is of order $1/\mu_0$ and is related to the seismic coupling coefficient $0 < \chi < 1$ by

$$C_v = \frac{1-\chi}{\chi}\,\frac{1}{\mu_0} \tag{8}$$

Loss of convexity of the elastic strain potential, associated to macroscopic failure, occurs at a critical value of damage α that depends on ξ. Upon loss of convexity the constitutive model should include additional dissipation mechanisms. However, this is not implemented yet. The simulation presented here are limited to moderate damage levels.

NUMERICAL METHOD

We implemented the continuum damage constitutive law in SEM2DPACK (Ampuero, 2008), a 2D spectral element code for seismic wave propagation and earthquake dynamics. The spectral element method is a high order spatial discretization technique that inherits the geometrical flexibility of the finite element method and the accuracy of spectral methods. The practical implementation is analogous to that of any explicit mass-lumped finite element method.

References

Ampuero, J.-P. (2008). *SEM2DPACK, A Spectral Element Method tools for 2D wave propagation and earthquake source dynamics, User's Guide, version 2.3.0*. Available online at `http://www.sourceforge.net/projects/sem2d/`.

Hamiel, Y., Y. F. Liu, V. Lyakhovsky, Y. Ben-Zion, and D. Lockner (2004). A viscoelastic damage model with applications to stable and unstable fracturing. *Geophys. J. Int. 159*(3), 1155–1165.

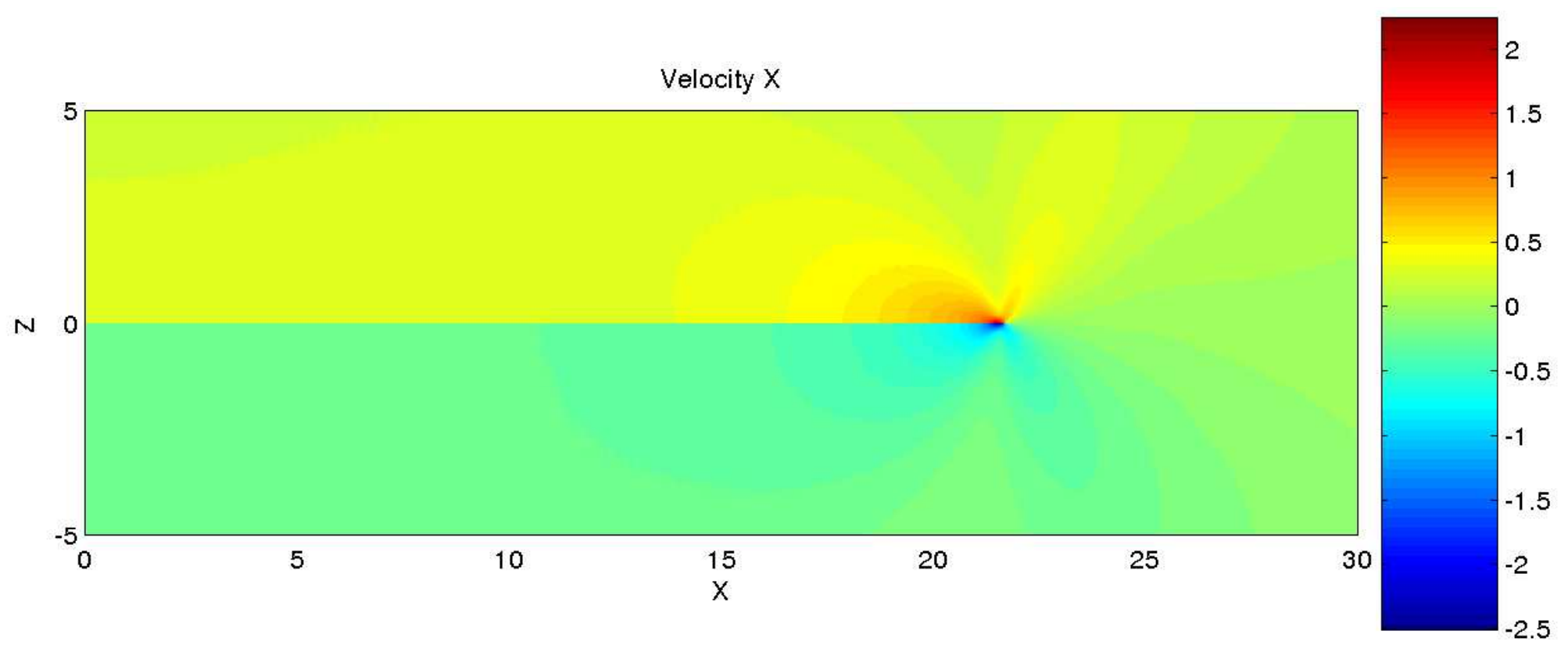
Lyakhovsky, V., Y. Ben-Zion, and A. Agnon (1997). Distributed damage, faulting, and friction. *J. Geophys. Res. 102*(B12), 27635–27649.

SLIP WEAKENING DYNAMIC CRACKS

We consider a planar fault governed by linear slip weakening friction, $\mu(D) = \max\left[\mu_d, \mu_s - (\mu_s - \mu_d)\, D/D_c\right]$, with $\mu_s = 0.6$ and $\mu_d = 0.1$. The initial conditions are defined by the angle Ψ between the maximum principal stress and the fault plane, and by the strength excess parameter $S \doteq (\sigma\mu_s - \tau_0)/(\tau_0 - \sigma\mu_d)$. We show two cases $\Psi = 14^o$ and $\Psi = 56^o$, both with $S = 1$. Initial rupture growth at moderate speed ($0.5c_S$) is enforced by time weakening. Beyond a critical propagation distance rupture accelerates and continues to grow spontaneously.

The inverse timescale of damage evolution in the examples is set as $C_d = 0.25\mu c_S/(\sigma(\mu_s - \mu_d)D_c)$ and the plasticity parameter is $C_v = \mu_0$ ($\chi = 0.5$). Increasing C_d increases the amount and extent of damage, and requires increasing C_v to avoid loss of convexity.

Slip is normalized by D_c, distances by $L_c = \mu D_c/\sigma(\mu_s - \mu_d)$, stresses by $\sigma(\mu_s\mu_d)$, time by L_c/c_S, velocities by $c_S\sigma(\mu_s - \mu_d)/\mu$.



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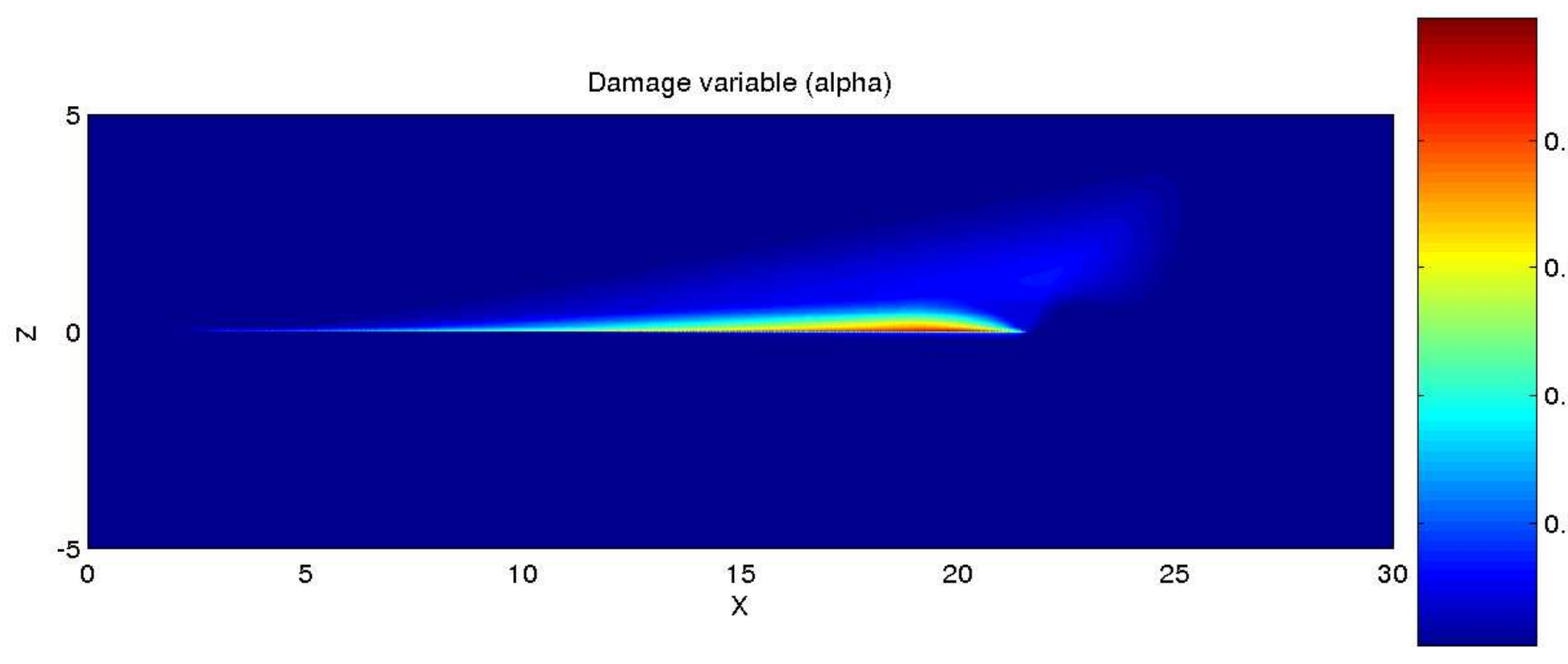

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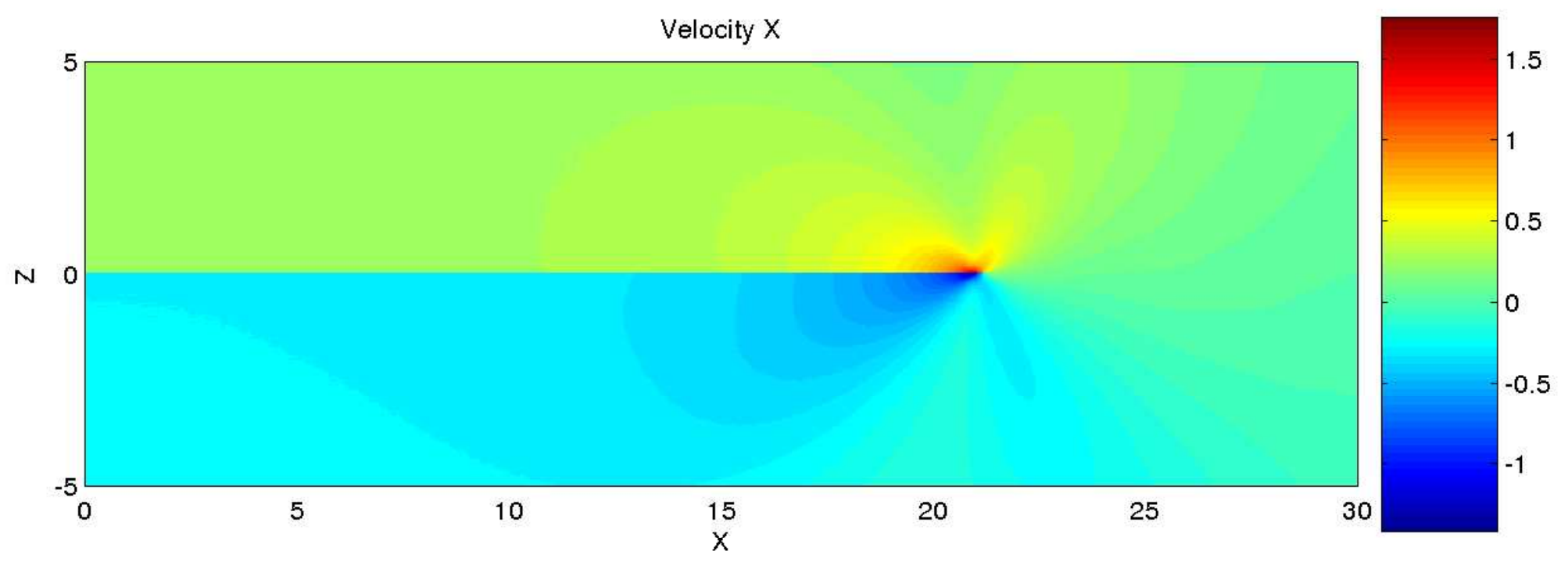

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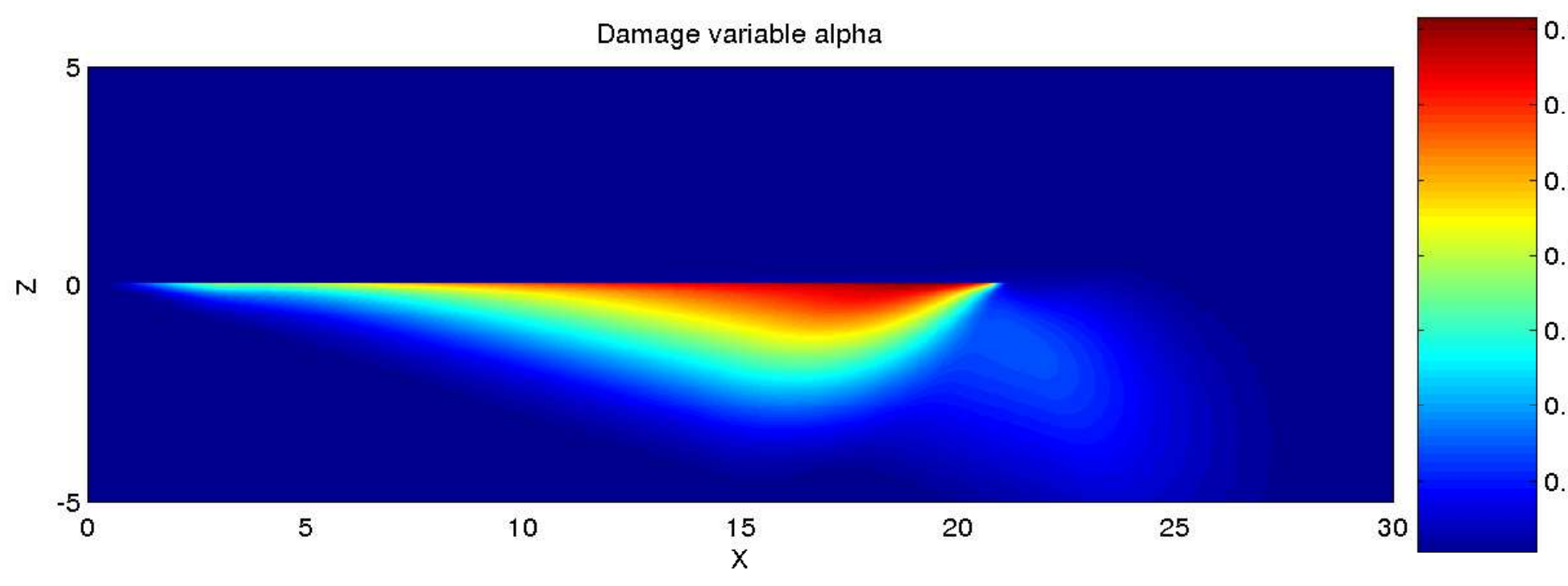

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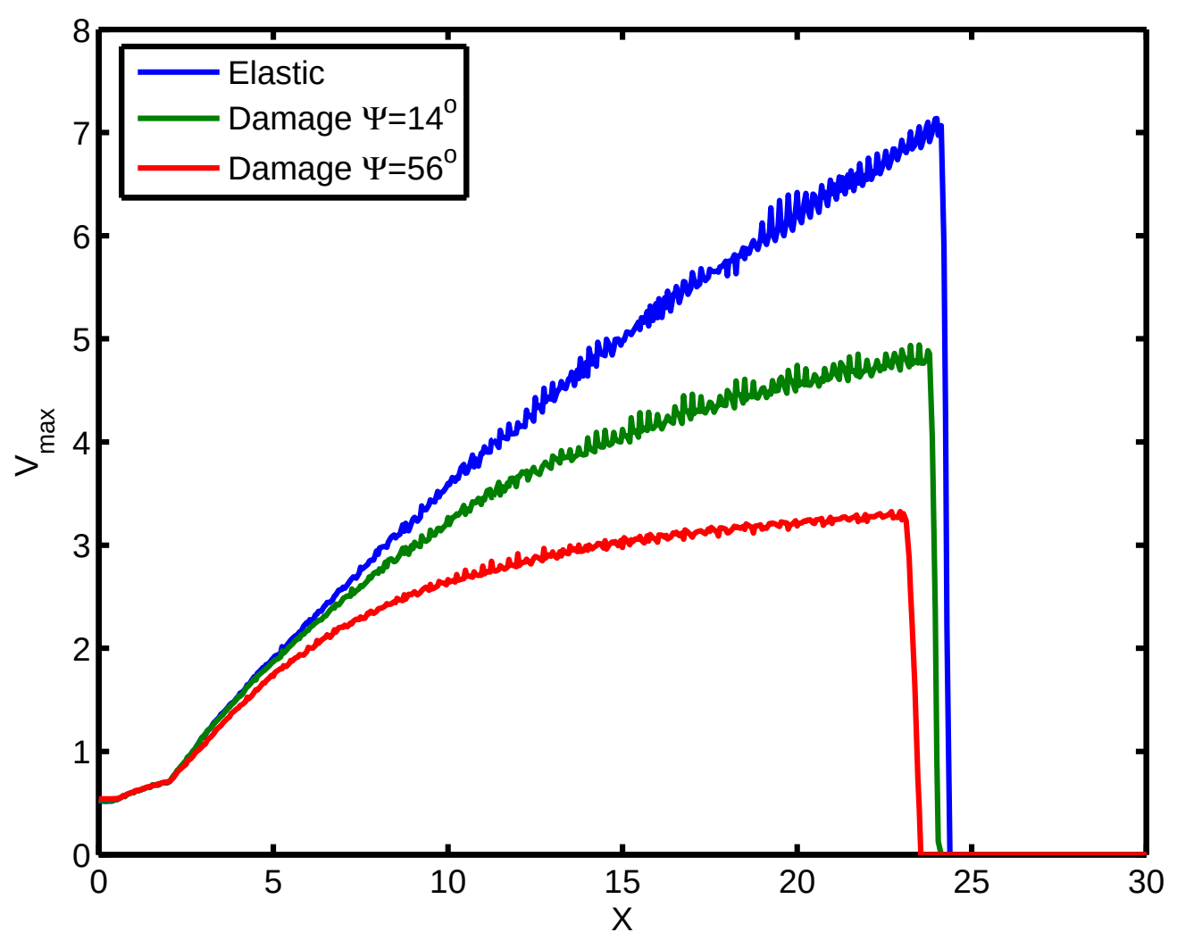
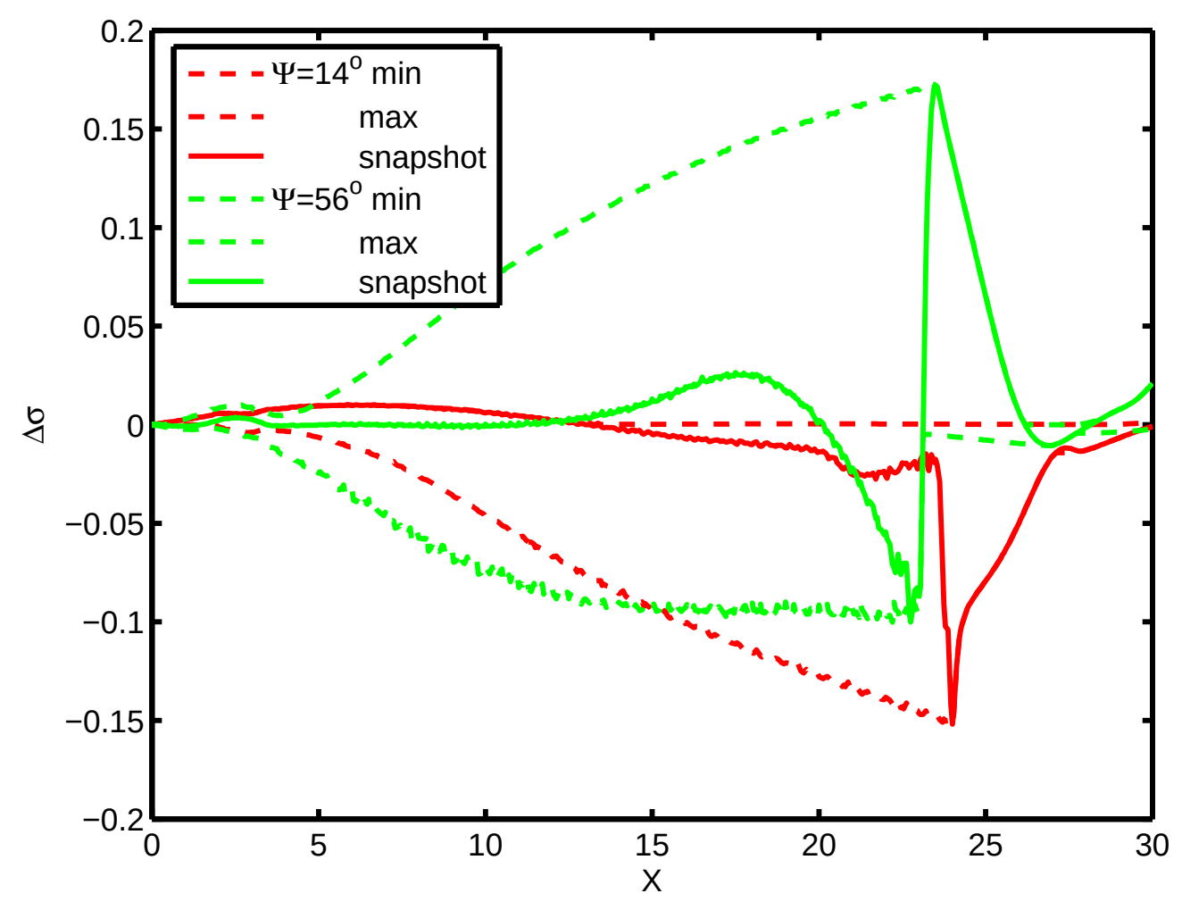


2)

[1.a] Horizontal velocity snapshot, $\Psi = 14^o$.
[1.b] Damage variable (α) for $\Psi = 14^o$.
[1.c] Horizontal velocity for $\Psi = 56^o$.
[1.d] Damage variable for $\Psi = 56^o$.
The off-fault damage zone grows proportionally to rupture distance. The dependence on Ψ is consistent with theoretical expectations (Poliakov et al, 2002).

[2] Peak slip velocity as a function of distance. Damage reduces the peak slip velocity and the rupture speed. The effect is larger for larger C_d and C_v.

[3] Minimum, maximum and last snapshot of normal fault stress changes (positive is tensile).



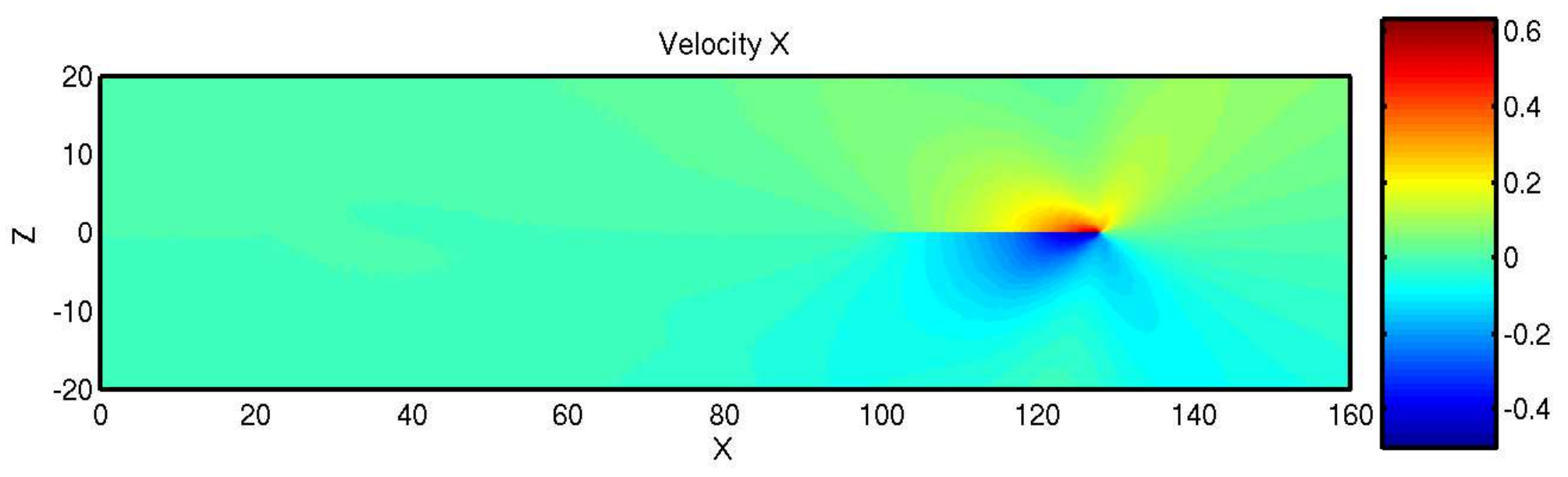
3)

VELOCITY WEAKENING DYNAMIC PULSES

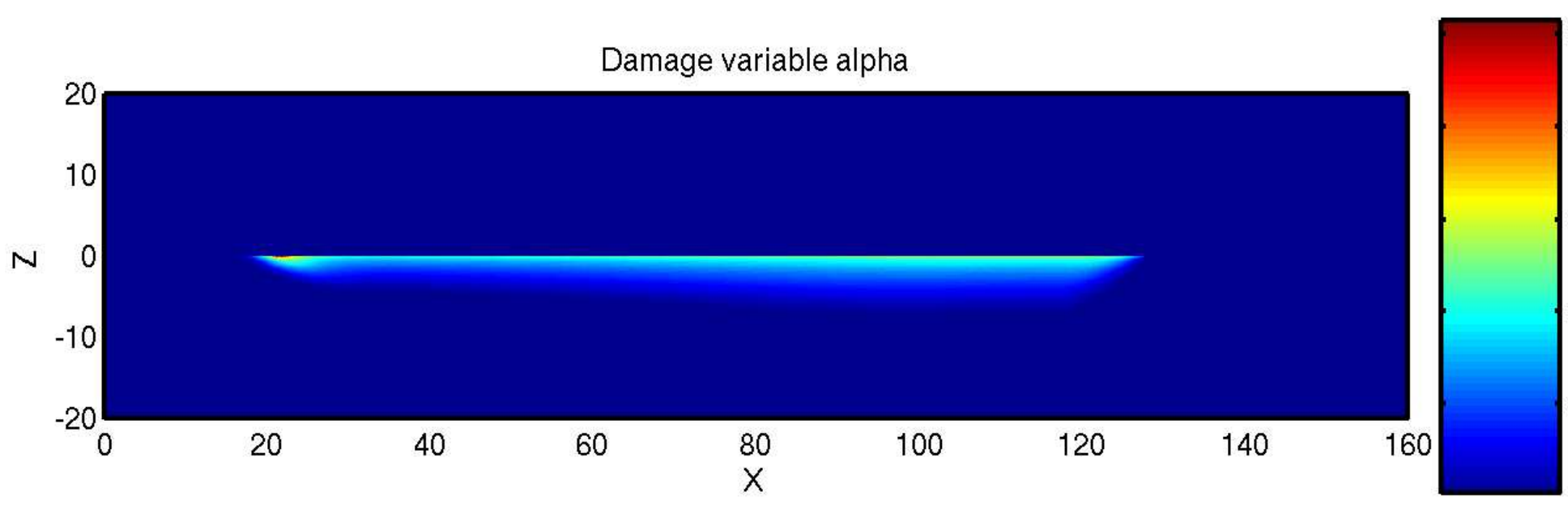
We consider now a fault governed by a rate and state dependent friction law, with strong velocity weakening at high slip rates ($\propto 1/V$), and regularization by a direct effect [Ampuero and Ben-Zion, 2008]:

$$\mu_f = \mu_d - \frac{\alpha}{1 + V/V_c} + \frac{\beta}{1 + \theta/V_c} \qquad \text{and} \qquad \dot{\theta} = \frac{V - \theta}{D_c/V_c}$$

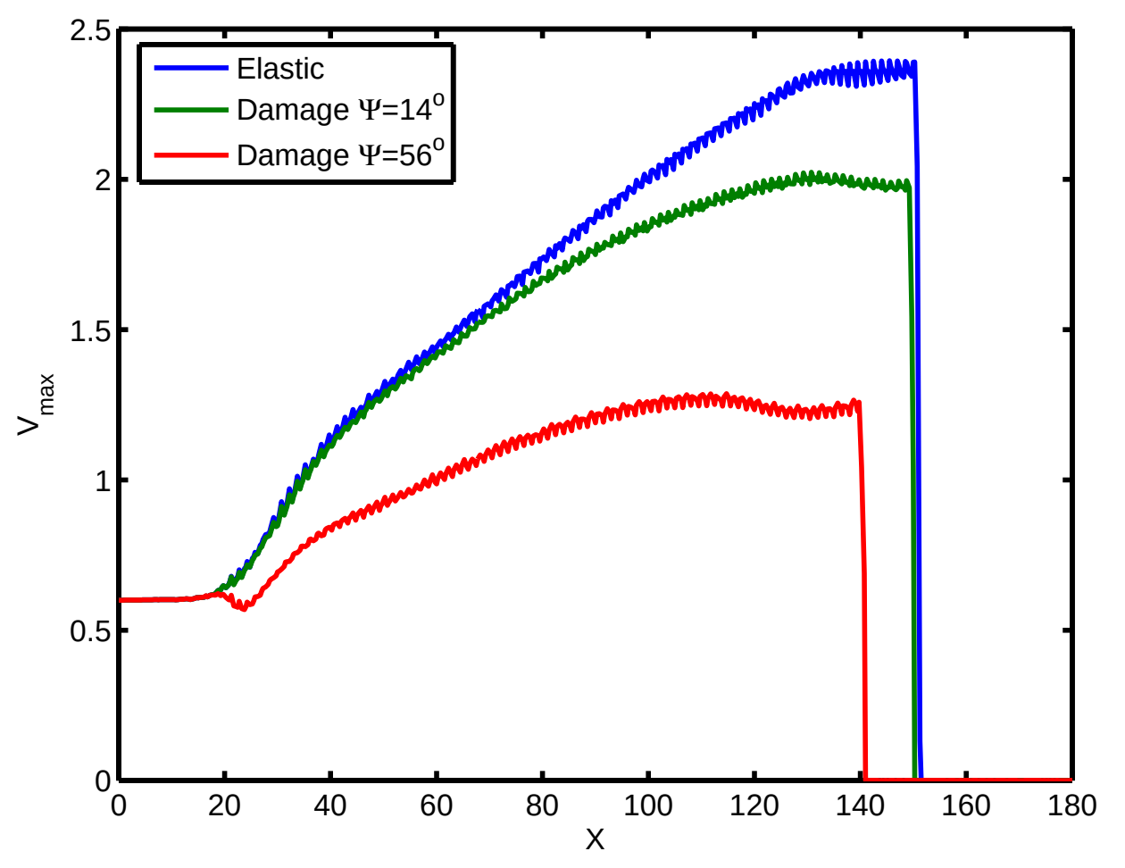
Slip-weakening and velocity-weakening are limit cases (high and low V_c, respectively). The example shown below has strong velocity-weakening, and results in sustained pulse-like rupture. Rupture is triggered by a slightly overstressed patch.



4.a)



4.b)

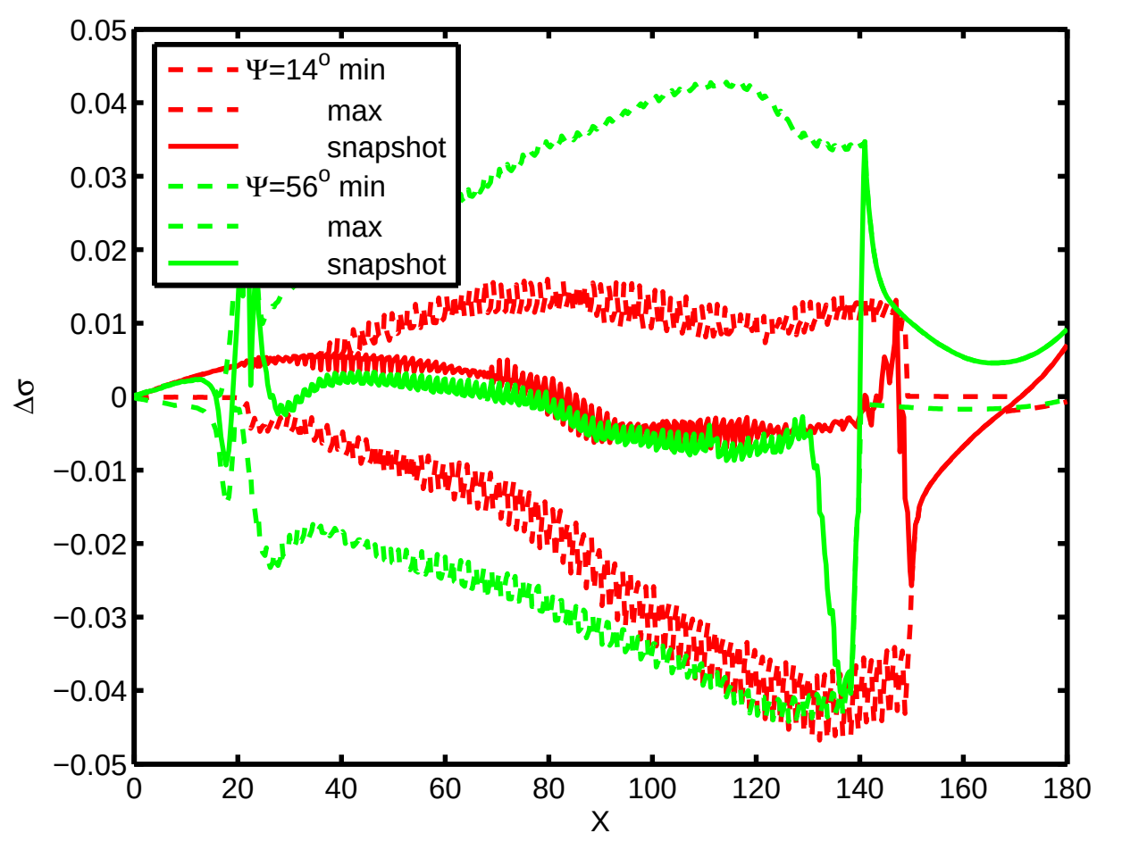


5)

[4.a] Horizontal velocity snapshot, $\Psi = 56^o$.
[4.b] Damage variable (α) for $\Psi = 56^o$.
The growth of the off-fault damage zone is more limited than in the crack case.

[5] Peak slip velocity as a function of distance. Dynamic damage can modify the stability of the pulse, from self-sustained to decaying.

[6] Minimum, maximum and last snapshot of normal fault stress changes (positive is tensile).



6)

SUMMARY AND PERSPECTIVES

- We have succesfully implemented a continuum damage rheology in a 2D spectral element code, to explore the role of off-fault rock damage in earthquake dynamics.
- Slip weakening crack ruptures show self-similar growth of the off-fault damage zone and reduction of peak slip velocity and rupture speed, as also observed in simulations with Coulomb plasticity. The damaged fault zones have wave speed reduction up to 30%. The damage variable α (a proxy for microcrack density) decays logarithmically as a function of distance to the fault.
- In velocity-weakening pulses, the extent of off-fault damage is more limited if the pulse is steady, but grows if the pulse is a self-similar one. Due to their higher susceptibility to small scale perturbations, pulses are more clearly affected by the feedback with off-fault damage.
- The scope was here limited to moderate damage (before loss of convexity). Further extension through an additional plastic mechanism will allow to unravel stronger effects. Interestingly, in simulations that reached loss of convexity, failure was preceded by detachment of a daughter pulse from the main rupture front, reminiscent of a wrinkle-like pulse detachment mechanism operating in bimaterial faults. Actually, dynamic damage generates locally a bimaterial interface.