Effects of low-rigidity layers on earthquake-cycle dynamics in long-term fault slip simulations

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Abstract. We develop a spectral element method for the simulation of long-term histories of spontaneous seismic and aseismic slip on faults subjected to slow tectonic loading and governed by rate-and-state friction. Our approach reproduces all stages of earthquake cycles: accelerating slip before dynamic instability, rapid dynamic propagation of earthquake rupture, post-seismic slip and interseismic creep. We apply the developed methodology to study the effects of low-rigidity layers on the dynamics of the earthquake cycle in 2-D. We consider two cases: small earthquakes on a planar fault surrounded by a damaged fault zone and large earthquakes on a vertical strike-slip fault that cuts through a shallow low rigidity layer. Our results indicate how the source properties of small repeating earthquakes are affected by the presence of a damaged fault zone: compared to faults in homogeneous media we find (i) reduction in the earthquake nucleation size, (ii) amplification of slip rates during dynamic rupture propagation, (iii) increase in the recurrence interval, and (iv) smaller amount of aseismic slip. Based on a linear stability analysis, we derive a theoretical estimate of the nucleation size as a function of the width and rigidity reduction of the fault zone layer, which is in good agreement with simulated nucleation sizes. We further examine the effects of vertically-stratified bulk layers (e.g., sedimentary basins) on the nature of shallow coseismic slip deficit. For the set of parameters we consider, low-rigidity shallow bulk materials alone do not lead to coseismic slip deficit. While the low-rigidity materials lead to lower interseismic stress accumula-
tion, they also cause dynamic amplification of coseismic slip rates, with the net effect on slip being nearly zero.
1. Introduction

Earthquake cycle simulations are important for understanding earthquake mechanics and physics-based hazard analysis. Modeling long-term slip histories of faults remains, however, quite challenging due to a wide range of spatial and temporal scales involved. To simulate a spontaneous earthquake sequence on a planar fault, a model needs to incorporate and resolve slow tectonic loading during the interseismic periods, nucleation and propagation of rupture during earthquakes that involve rapid changes in stress and slip rate at the propagating dynamic rupture tips, and the subsequent postseismic deformation and aseismic afterslip. In addition, destructive large earthquakes occur on faults that extend tens to hundreds of kilometers while variations in stress changes and slip rate at the rupture tip occur over distances of the order of meters.

Several approaches to modeling long-term histories of fault slip have been proposed [e.g., Shibazaki and Matsu’ura, 1992; Rice, 1993; Cochard and Madariaga, 1996; Tullis, 1996; Kato, 2004; Duan and Oglesby, 2005; Liu and Rice, 2005; Hillers et al., 2006] but all of them used simplified treatments of either aseismic slip processes (e.g., nucleation, fault creep, and afterslip) or inertial effects during dynamic rupture. Lapusta et al. [2000] and Lapusta and Liu [2009] developed 2-D and 3-D boundary integral methods (BIMs) capable of capturing both seismic and aseismic slip and the gradual process of earthquake nucleation. Those studies are restricted to planar faults embedded into a uniform elastic space or a half-space. At the same time, observations indicate complex crustal structures with variable bulk properties, fault damage zones, and non-planar fault geometries. Hence it is important to include those factors into long-term earthquake cycle models, combining them with laboratory-derived constitutive fault relations.
In this work, we develop a spectral element method (SEM) that can enable us to simulate long-term history of spontaneous seismic and aseismic slip on a fault subjected to laboratory-derived rate and state friction and slow tectonic loading. In particular, we present a quasi-static SEM with a time updating scheme that can be used to model long-term deformation histories and that is suitable for a fault boundary governed by a rate and state friction formulation. Our model merges the explicit scheme presented in Kaneko et al. [2008] for simulating dynamic rupture propagation with the quasi-static SEM developed in this work for modeling slow tectonic loading and the associated crustal deformation and fault slip. The SEM with an explicit time scheme was built upon prior studies by Komatitsch and Vilotte [1998], Komatitsch and Tromp [1999], and Ampuero [2002]. The combined algorithm is able to resolve all stages of an earthquake cycle, including gradual nucleation processes, dynamic rupture propagation, postseismic slip, and aseismic processes throughout the loading period.

Our methodology is described in sections 2 and 3. Section 3 also illustrates its potential by presenting simulations of long-term slip on a fault segment with relatively simple distributions of fault friction properties. To validate the developed SEM approach, we compare SEM and BIM simulation results in a 2-D model of small repeating earthquakes. We then consider two application examples to explore the potential effects of variable bulk properties on repeating earthquakes (sections 4 and 5).

In section 4, we consider small repeating earthquakes on a fault that bisects a meter-scale fault-parallel low-rigidity zone embedded in undamaged country rock. Such a model configuration is motivated by damaged zones surrounding fault cores often found on exhumed faults [e.g., Chester et al., 1993] and meter-thick zones of lower P and S wave speeds surrounding localized fault surfaces at about 2.6-2.8 km depths observed in the Earth-Scope’s San Andreas Fault Observatory at Depth (SAFOD) project [Zoback et al.,
Simulations of single earthquake rupture suggest that such a mechanical configuration leads to perturbed rupture speeds and slip velocity of propagating rupture, resulting in high-frequency oscillations in the slip function near the rupture front [Harris and Day, 1997]. Here we examine additional effects of a fault-parallel low-rigidity zone on earthquake source properties of simulated repeating earthquakes. We compare the model response in a layered bulk with that in a homogeneous bulk, and investigate how earthquake source properties, such as stress drop, recurrence intervals, and nucleation sizes, depend on the width of a low-rigidity layer. Given the recent successes of rate and state models in explaining several properties of repeating earthquake sequences [Chen and Lapusta, 2009; Chen et al., 2010], it is important to consider these additional effects for proper interpretation of observations.

In section 5, we consider the effects of stratified bulk properties (e.g., a sedimentary basin) on simple repeating earthquakes that rupture the entire seismogenic depth. Fialko et al. [2005] pointed out that, for several large (~M7) strike-slip earthquakes, coseismic slip in the uppermost crust is systematically lower than that at seismogenic depth. A reduction of coseismic slip at shallow depths (< 3-4 km), referred to as ‘shallow slip deficit’, has been inferred for several large strike-slip crustal earthquakes [e.g., Simons et al., 2002; Fialko et al., 2005; Bilham, 2010], including the 1992 M7.3 Landers earthquake, the 1999 M7.1 Hector Mine earthquake, the 2005 M6.5 Bam earthquake, and the 2010 M7.0 Haiti earthquake. Several mechanisms have been proposed to explain the shallow slip deficit, including the presence of velocity-strengthening friction at shallow depths that releases accumulated strain by fault creep [e.g., Marone et al., 1991; Marone, 1998; Kaneko et al., 2008] and low pre-stress in low-rigidity shallow bulk materials resulting from uniform tectonic strain [e.g., Rybicki, 1992; Rybicki and Yamashita, 1998]. Using the developed SEM, we investigate whether a vertical strike-slip fault embedded in a vertically stratified
bulk structure can cause shallow coseismic slip deficit without the presence of a shallow velocity-strengthening region.

2. A quasi-static SEM algorithm for simulations of long-term deformation histories

2.1. Discretized elastodynamic relations

The SEM dynamic model presented in Kaneko et al. [2008] relies on an explicit time updating scheme, the approach commonly used in SEMs for wave propagation [e.g., Komatitsch and Vilotte, 1998; Komatitsch and Tromp, 1999]. However, the explicit time scheme limits the maximum length of each time step $\Delta t$ by the Courant stability condition. For dynamic rupture simulations, the Courant condition can be rewritten in terms of the cohesive zone size $\Lambda$ divided by the wave speeds of the medium:

$$
\Delta t \leq \frac{CA}{\sqrt{DNV_p or s}},
$$

where $D$ is the dimension of the problem, $C$ is a stability parameter that depends only on the time scheme and is of order one, and $N$ is the number of fault-plane node points within the cohesive zone. (In SEM the critical time step is actually smaller due to the non-uniform distribution of the Gauss-Lobatto-Legendre nodes inside each spectral element.) $N$ should be at least 3-5 for well-resolved simulations of dynamic rupture in the cases with slip-weakening or weakly rate-dependent friction laws [Day et al., 2005; Kaneko et al., 2008; Kaneko and Lapusta, 2010]. For dynamic rupture simulations with cohesive zone sizes of 1-100 meters, $\Delta t \sim 10^{-4} - 10^{-2}$ s, and hence simulating tens to thousands of years of deformation histories is not computationally feasible. To take a longer time step, one needs to use an implicit time updating scheme. SEMs with implicit schemes have been used to solve elastic and acoustic wave equations [e.g., Ampuero, 2002; Zampieri and Pavarino, 2006; Dupros et al., 2010]. Here we develop a quasi-static SEM with an
adaptive time stepping and merge it with the fully dynamic SEM to simulate long-term
slip histories on a rate-and-state fault.

As in Kaneko et al. [2008], we start from the discretized weak form of the equation of
motion in its matrix form:

\[ M\ddot{u} = -Ku + B\tau, \] (2)

where \( M \) and \( K \) are the mass and stiffness matrix respectively, \( B \) is the fault-boundary
matrix, \( \tau = T - \tau_0 \) is the relative traction vector on the fault, \( T \) is the total traction and
\( \tau_0 \) is the traction in the reference static-equilibrium state. Vectors \( u, \dot{u}, \) and \( \ddot{u} \) collect
the values of displacements, particle velocities and accelerations, respectively, of all the
computational nodes of the bulk mesh.

In the case of quasi-static static problems, equation (2) becomes

\[ Ku = B\tau, \] (3)

Let us decompose the displacement vector \( u \) into the values on fault nodes, denoted by
\( u^f \), and the values on nodes within the medium, denoted by \( u^m \). Then

\[ K_{11}u^f + K_{12}u^m = B\tau, \] (4)

\[ K_{21}u^f + K_{22}u^m = 0, \] (5)

where \( K_{11} \) and \( K_{12} \) are the parts of the stiffness matrix corresponding to \( u^f \), and \( K_{21} \) and
\( K_{22} \) are the parts corresponding to \( u^m \). From equation (5), we have

\[ K_{22}u^m = -K_{21}u^f. \] (6)

Given the displacement on the fault, \( u^f \), this equation yields the corresponding displace-
ment field in the medium, \( u^m \).

In equation (4), let us introduce \( A \equiv K_{11}u^f + K_{12}u^m \). We now write equation (4) for
the fault nodes with the ± signs indicating the values of field variables on the two sides
of the fault (Figure 1):

$$B_{\pm} \tau_{\pm} = A_{\pm}.$$ \hspace{1cm} (7)

Subtracting the minus side from the plus side, and using the sign convention $\tau = -\tau_+ = \tau_-$, where $\tau_{\pm}$ are defined with respect to the outward normal from the fault boundary $\Gamma_\pm$ (Figure 1), we obtain

$$\tau = - (B_+ + B_-)^{-1} (A_+ - A_-).$$ \hspace{1cm} (8)

Equation (6) allows to eliminate the off-fault degrees of freedom, $u^m$, to obtain a formulation (8) involving only the degrees of freedom on the fault ($\tau$ and $u^f$). This procedure is known as static condensation or sub-structuring in computational mechanics. Because the $B_{\pm}$ matrices are diagonal, the expression (8) is a local relation which can be computed node by node on the fault once the $A_{\pm}$ terms are known or predictor values are assumed.

It is convenient to rewrite equation (8) in terms of total traction, $T = \tau_0 + \tau$:

$$T = \tau_0 + \tau = - (B_+ + B_-)^{-1} (A_+ - A_-).$$ \hspace{1cm} (9)

Note that for the cases we consider in this study, the fault-normal component of traction $\tau$ remains unchanged, and hence the fault-normal components of $\tau$ is zero.

### 2.2. Fault constitutive response: Rate and state friction laws

The fault resistance to sliding is described by laboratory-derived rate and state friction laws, which were developed to incorporate observations of rock friction experiments at low slip rate \cite{Dieterich, 1978, 1979; Ruina, 1983; Blanpied et al., 1995, 1998; Marone, 1998}. For time-independent effective normal stress $\bar{\sigma}$, the shear strength $T$ on the fault is expressed as

$$T = \psi(\dot{\delta}, \theta) = \bar{\sigma} \left[ f_0 + a \ln \left( \frac{\dot{\delta}}{\dot{\delta}_0} \right) + b \ln \left( \frac{\dot{\delta}_0 \theta}{L} \right) \right],$$ \hspace{1cm} (10)
where \( a > 0 \) and \( b > 0 \) are rate and state constitutive parameters with magnitudes of the order of 0.01, \( \dot{\delta} \) is the magnitude of slip velocity, \( f_0 \) is a reference friction coefficient corresponding to a reference slip velocity \( \dot{\delta}_0 \), \( \theta \) is a state variable which is typically interpreted as the average age of the population of contacts between two surfaces, and \( L \) is the characteristic slip for state evolution [Dieterich, 1978, 1979; Rice and Ruina, 1983; Ruina, 1983; Dieterich and Kilgore, 1994]. Two types of state-variable evolution laws are commonly used in modeling:

\[
\frac{d\theta}{dt} = 1 - \frac{\dot{\delta} \theta}{L} \quad \text{(aging law),} \tag{11}
\]

\[
\frac{d\theta}{dt} = -\frac{\dot{\delta} \theta}{L} \ln \left( \frac{\dot{\delta} \theta}{L} \right) \quad \text{(slip law).} \tag{12}
\]

The parameter combination \( a - b < 0 \) corresponds to steady-state velocity-weakening friction and can lead to unstable slip, whereas \( a - b > 0 \) corresponds to steady-state velocity-strengthening and leads to stable sliding [Rice and Ruina, 1983; Ruina, 1983]. Throughout this article, we omit the words “steady-state” and simply refer to velocity weakening/strengthening.

In expression (10), shear frictional strength \( T \) is undefined for slip velocities \( \dot{\delta} = 0 \), which is unphysical. To regularize (10) near \( \dot{\delta} = 0 \), we follow the approach of Rice and Ben-Zion [1996], Ben-Zion and Rice [1997], and Lapusta et al. [2000] in using a thermally activated creep model of the direct effect term \( a \ln \left( \frac{\dot{\delta}}{\dot{\delta}_0} \right) \) to obtain

\[
T = \psi(\dot{\delta}, \theta)
= a \sigma \arcsinh \left( \frac{\dot{\delta}}{2 \dot{\delta}_0} \exp \left( \frac{f_0 + b \ln(\delta_0 \theta / L)}{a} \right) \right). \tag{13}
\]

This regularization is used in our simulations. It produces a negligible change from equation (10) in the range of slip velocities explored by laboratory experiments; the difference in \( \dot{\delta} \) at \( \dot{\delta} \sim \dot{\delta}_0 \) is of the order of \( \exp(-2f_0/a) \) or less, and the typical value of \( f_0/a \) in this study is 40.
Under slow tectonic loading, frictional instability (i.e., an earthquake) is able to develop only if the velocity-weakening region of the fault exceeds the nucleation size $h^*$ [Rice and Ruina, 1983; Rice, 1993; Rubin and Ampuero, 2005]. Two theoretical estimates of the earthquake nucleation size for 2-D problems are given by

$$h_{RR}^* = \frac{\pi \mu^* L}{4 \sigma (b - a)},$$

$$h_{RA}^* = \frac{2 \pi \mu^* L b}{\pi (b - a)^2},$$

where $\mu^* = \mu$ for mode III and $\mu^* = \mu / (1 - \nu)$ for mode II. The estimate $h_{RR}^*$ was derived from the linear stability analysis of steady sliding by Rice and Ruina [1983], while $h_{RA}^*$ was obtained for the parameter regime $a/b > 0.5$ by Rubin and Ampuero [2005] on the basis of energy balance for a quasi-statically expanding crack. Note that Rubin and Ampuero [2005] gave formulae for half of the nucleation size but we use full nucleation sizes here.

2.3. Updating scheme: Advancing one evolution time step

We have developed an updating scheme appropriate for the rate and state fault boundary condition. Here, we discuss how values of field variables are updated over one evolution time step. We adopt a multi-stage predictor-corrector strategy to solve the statically condensed problem. Suppose that the discretized values of displacement $u$ and particle velocity $\dot{u}$ are known at the $n$th time step. To find the values of the field variables at the $(n + 1)$th time step, we perform the following steps.

1. Predict the values of displacements on the fault $u^f$, based on the known values at the $n$th time step:

$$u_{n+1}^{sf} = u_n^f + \Delta t \dot{u}_n^f.$$

2. Solve for the displacement field in the medium $u_{n+1}^{sm}$ using equation (6):

$$K_{22} u_{n+1}^{sm} = -K_{21} u_{n+1}^{sf}.$$
This is solved by a preconditioned conjugate gradient method, an iterative method for solving symmetric-positive-definite systems of equations. The algorithm we use is based on Hestenes and Stiefel [1952] and is summarized in Trefethen and Bau [1997]. Because the stiffness matrix $K_{22}$ is large ($\sim 10^4$ by $10^4$), a direct method such as Gaussian elimination cannot be used. Fortunately, the matrix $K_{22}$ is sparse, and the product $Ku$ is always computed at a local elemental level as in the case of the dynamic SEM [Kaneko et al., 2008]. This is why we use an iterative method.

3. Compute $A^* = K_{11}u_{n+1}^e + K_{12}u_{n+1}^m$ and $T_{n+1}^*$ in equation (9):

$$T_{n+1}^* = \tau_0 - (B_+ + B_-)^{-1}(A_{+, n+1}^* - A_{-, n+1}^*).$$  \hfill (18)

4. Determine the first prediction of the state variable, $\theta_{n+1}^*$. By integrating the evolution law (11) or (12) with the constant magnitude $\dot{\delta}_n$ of slip velocity $\dot{\delta}_n = u_{n}^+ - u_{n}^-$ during the time step, we obtain

$$\theta_{n+1}^* = \theta_n \exp \left( -\frac{\dot{\delta}_n \Delta t}{L} \right) + \frac{L}{\dot{\delta}_n} \left( 1 - \exp \left( -\frac{\dot{\delta}_n \Delta t}{L} \right) \right)$$  \hfill (19)

for the aging law, and

$$\theta_{n+1}^* = \frac{L}{\dot{\delta}_n} \left( \frac{\dot{\delta}_n \theta_n}{L} \right) \exp \left( -\frac{\dot{\delta}_n \Delta t}{L} \right)$$  \hfill (20)

for the slip law.

5. Find the first prediction of slip velocity $\dot{\delta}_{n+1}^*$ by equating the magnitude of shear stress in equation (18) and strength in equation (13). The directions of shear traction vector $T_{n+1}^*$ and slip velocity vector $\dot{\delta}_{n+1}$ have to coincide. From equation (18), the traction $T_{n+1}^*$ and $A_{n+1}^*$ have the same direction. By projecting (18) onto that direction and using an inverted form of equation (13), we obtain

$$\dot{\delta}_{n+1}^* = \psi(T_{n+1}^*, \theta_{n+1}^*).$$  \hfill (21)
Using the directional cosines constructed from the components of \( T_{n+1}^* \), we obtain the components of \( \delta_{n+1}^* \).

6. Calculate the final prediction of displacement and slip on the fault, \( u_{n+1}^{\text{sf}} = \frac{1}{2} \delta_{n+1}^{\text{sf}} \), at the \((n+1)\)th time step by

\[
\mathbf{u}_{n+1}^{\text{sf}} = \mathbf{u}_{n}^{f} + \frac{\Delta t}{2} \left( \dot{\mathbf{u}}_{n}^{f} + \dot{\mathbf{u}}_{n+1}^{f} \right). \tag{22}
\]

7. Make the corresponding prediction \( u_{n+1}^{\text{sf}} \) of the displacement in the medium using the \( u_{n+1}^{\text{sf}} \) as in step 2.

8. Make the corresponding prediction \( T_{n+1}^{**} \) and \( \theta_{n+1}^{**} \) by repeating steps 3 and 4 and by replacing \( \delta_{n} \) in equation (19) or (20) with \( (\delta_{n} + \delta_{n+1}^{*})/2 \).

9. Find the final prediction \( \dot{\delta}_{n+1}^{**} \) and the components of \( \delta_{n+1}^{**} \) by repeating step 5 with \( T_{n+1}^{**} \) and \( \theta_{n+1}^{**} \) instead of \( T_{n+1}^{*} \) and \( \theta_{n+1}^{*} \).

10. Declare the values of \( \dot{\delta}_{n+1}, \theta_{n+1}, \) and \( T_{n+1} \) on the fault, and the values of displacement of the entire medium \( \mathbf{u}_{n+1} \), to be equal to the predictions with the superscript double asterisks.

There is a convergence criterion in the updating scheme described above. In step 2, we terminate the conjugate gradient iteration when \( \| \text{LHS}_2 - \text{RHS}_2 \| / \| \text{RHS}_2 \| < \varepsilon_{\text{CG}} = 10^{-5} \), where \( \text{LHS}_2 \) and \( \text{RHS}_2 \) are the left-hand side and right-hand side of equation (17), respectively. The resulting algorithm is similar to Heun’s method, which can be seen as an extension of the Euler method into a two-stage second-order Runge-Kutta method. While Heun’s method is usually qualified as an explicit method, we still need to solve a large linear system, equation (6), during static condensation. Hence, we prefer to qualify our algorithm as implicit.

3. Implementation example

3.1. Formulation of a 2-D model
The response of faults to tectonic loading is characterized by long periods of quasi-static deformation combined with short periods of fast dynamic slip. To simulate such response, we adopt the variable time stepping procedure of Lapusta et al. [2000], in which the time step is set to be inversely proportional to slip velocity on the fault as described in appendix A. As a result, relatively large time steps, a fraction of a year, are used in the interseismic periods, while small time steps, a fraction of a second or smaller, are used to simulate fast seismic slip. Note that the stability of the stepping procedure relies on the presence of the positive direct effect in the rate and state formulation, a feature that has ample laboratory confirmation.

The updating scheme introduced in the previous section can be merged with the explicit time stepping scheme of the fully dynamic SEM problem. The main challenge is to find proper criteria for switching from the quasi-static implicit scheme to the dynamic explicit scheme and vice versa. At the onset of earthquakes (or at the end of nucleation processes), slip velocities abruptly increase from values much smaller than typical plate loading rates ($\sim 10^{-10} - 10^{-9}$ m/s) to coseismic values ($\sim 1 - 10$ m/s), and the time step progressively becomes smaller. Hence we switch from one scheme to the other based on the values of the maximum slip velocity. For the problems discussed below, we switch from the quasi-static to dynamic scheme at $\dot{\delta}_{QD}^{\text{max}} = 0.5$ mm/s and from the dynamic to quasi-static scheme at $\dot{\delta}_{DQ}^{\text{max}} = 0.2$ mm/s. Ideally, one could formulate a switching criterion based on the relative importance of the inertial term in the governing equation (2). However, we find that such quantity is numerically more unstable than the criteria based on the values of the maximum slip velocity (appendix B). We confirm that the criteria we adopt here ensures that, at the times of the switch, the inertial term in the governing equation is much smaller ($\sim 10^{-4}$%) relative to the other terms (appendix B) and that the results compare well with BIM methods as discussed below.
To demonstrate how the ideas outlined so far are combined to produce long-term deformation histories, let us consider the response of a 2-D fault model of a vertical strike-slip fault embedded in an elastic medium (Figure 2). On the fault, a potentially seismogenic patch borders regions steadily moving with the prescribed slip rate $V_{pl} = 2 \text{ mm/yr}$, as illustrated in Figure 2. That steady motion provides loading. The fault motion is in the along-strike direction $y$, but only variations with depth $z$ are considered, so that the fault behavior is described by strike-parallel slip $\delta(z,t)$, slip velocity (or slip rate) $\dot{\delta}(z,t) = \partial\delta(z,t)/\partial t$, and the relevant component of shear stress $T(z,t)$. The symmetries of the problem allow us to restrict the computational domain to the medium on one side of the fault ($x \geq 0$).

It is convenient to express the formulae in terms of variables \((u(x,z,t) - V_{pl}t/2)\) and \((\dot{u}(x,z,t) - V_{pl}/2)\), in which case $\tau_0(x,z,t)$ becomes independent of time and equal to the initial stress $\tau_0(x,z)$. This approach was used for the BIM model of Lapusta et al. [2000]. For the 2-D problems we consider here, the medium across the fault boundary has equal and opposite motion by symmetry consideration. Then the relation (9) on the fault becomes

\[
T(z,t) = \tau_0(z) - \frac{1}{2B(z)} \left( K_{11}(y,z,t) \left[ u_f(z,t) - \frac{V_{pl}t}{2} \right] \right) + K_{12}(y,z,t) \left[ u_m(z,t) - \frac{V_{pl}t}{2} \right].
\] (23)

Note that our mesh is conformal and hence $B = B_+ = B_-.$

The SEM model consists of a 90 m by 60 m rectangular domain (Figure 2). To allow comparison to the BIM, the domain is replicated using periodic boundary conditions on both sides of the domain (Figure 2). The fault boundary obeys rate and state friction with the aging law (11). The model contains variations in steady-state friction properties that create rheological transitions (Figure 3). The parameters used in the simulations are...
listed in Table 1. The effective normal stress \( \bar{\sigma} \) and characteristic slip \( L \) are uniform along the fault.

We use the criteria for spatial discretizations developed in the work by Perfettini et al. [2009] and Lapusta and Liu [2009], which showed that resolving a cohesive zone size is a more stringent requirement than resolving the nucleation size, for the aging formulation of rate and state friction and typical rate and state parameters. In our simulations, we use an average node spacing \( \Delta x = 0.25 \) m, which results in \( \Lambda_0 / \Delta x \approx \mu L / (b \bar{\sigma} \Delta x) \approx 5 \) where \( \Lambda_0 \) is the cohesive zone size at the rupture speed \( V_r \to 0^+ \). Such resolution has shown to be adequate in the work of Day et al. [2005] and Lapusta and Liu [2009], and it leads to stable results in our simulations that do not change due to finer discretizations.

The selected spatial discretization corresponds to \( h_{RA}^* / \Delta x \approx 50 \) (Figure 3b), where \( h_{RA}^* \) is the estimate of the nucleation size obtained by Rubin and Ampuero [2005] for \( a/b \gtrsim 0.5 \), see equation (15).

### 3.2. Comparison of simulation results obtained with 2-D SEM and 2-D BIM

To assess the accuracy of numerical results, we conduct comparison of simulation results obtained using the developed SEM model with those of the BIM spectral formulation of Lapusta et al. [2000], which resolves all stages of each earthquake episode under a single computational scheme. Figure 2 illustrates the geometry of the antiplane SEM and BIM models. In BIM, wave propagation is analytically accounted for by boundary integral expressions. The method assumes the fault is repeated periodically, which we also enforce in the SEM model.

Earthquake sequences simulated in SEM and BIM models are shown and compared in Figure 4. The solid lines are plotted every 0.5 years and show the continuous slow sliding (creep) of the steady-state velocity-strengthening regions. That slow slip creates stress
concentration at its tip and penetrates into the velocity-weakening region. In due time, an earthquake rupture nucleates and propagates bilaterally; its progression is shown by dashed lines. After an earthquake arrests, the velocity-strengthening region experiences accelerated sliding, or afterslip, due to the transferred stress. The interseismic period between two successive events is about 6 years. The overall agreement of spatial slip distributions between two models during coseismic as well as interseismic periods validates our developed SEM approach (Figure 4a). The histories of shear stress and slip velocity at the center of the fault in these models are virtually identical, and the timing of the onset ($\dot{\delta}_{\text{max}} > 1 \text{ cm/s}$) of the 4th seismic event in these models differs by 0.005% (Figure 4b,c).

The agreement is very good given that, in the SEM simulation, there are a total of $\sim 40,000$ adaptive time steps, each of which includes 1 to 500 conjugate gradient iterations. To make sure that the solution is accurate, we have checked that the result of a BIM simulation with the twice higher resolution shows identical slip patterns and timings of seismic events, confirming that our results are grid independent.

Figure 5 shows the evolution of the displacement and velocity fields during the interseismic and coseismic periods in the SEM model. In Figure 5a, warm colors indicate a larger amount of displacement relative to the displacement given by the plate loading, whereas cold colors correspond to a smaller amount of displacement. About one minute before the onset of the seismic event, the displacement field localizes near the eventual nucleation site. The change in strain field associated with this localization can be detected if a strain-meter were placed at an off-fault distance comparable to the size of the nucleation region. This is why observations of premonitory slip prior to the eventual mainshock are difficult, consistent with the study by Tullis [1996]. After the seismic event (Figure 5b) that occurred between the 4th and 5th panels in Figure 5a, the red-colored region gradually expands due to afterslip on the velocity-strengthening segments of the fault. The
corresponding change in the displacement field for the afterslip is much larger than that for the nucleation process. Therefore, if nucleation of small and large events is governed by the same physical processes, one should look for signals associated with afterslip of small repeating earthquakes before searching for smaller signals associated with the earthquake nucleation process. Of course, the possibility remains that the nucleation of at least some large events proceeds differently from small events.

4. Effects of a fault-parallel low-rigidity layer on seismic and aseismic slip

We use the SEM model developed in section 2 and validated in section 3 to investigate the effects of variable bulk properties on repeating earthquakes. The model set up is similar to the one shown in Figure 2 except that a fault parallel low-rigidity layer of width $H$ and rigidity $\mu_D$ is added in the vicinity of the fault, to mimic a damaged fault zone. The friction-related parameters and the distribution of effective normal stress are the same as in section 3. We examine how earthquake source properties, such as stress drop, recurrence intervals, and nucleation sizes, depend on the width of the low-rigidity layer.

Figure 6 shows simulated earthquake sequences for two scenarios: a case with a low-rigidity layer of width $H = 1.5$ m and the other with a homogeneous bulk with rigidity $\mu_D = 20.5$ GPa. Note that the rigidity ratio is $\mu_D/\mu = 0.64$ and the corresponding ratio of the $S$-wave speeds is $V_{sD}/V_s = 0.8$. As Figure 4a and Figure 6 show, the earthquake source properties are affected by the width of the low-rigidity layer. We further perform several simulations with different values of $H$ and quantify the dependence of earthquake source properties on $H$ (Figure 7). In the following, we summarize several key findings.
4.1. Reduction of nucleation sizes in low-rigidity fault zones

The theoretical estimates of a nucleation size given in equations (14) and (15) predict that the nucleation size on a planar fault embedded in a homogeneous medium is linearly proportional to rigidity $\mu$ of the medium. This is consistent with our simulation results for the scenarios with homogeneous bulk (Figure 7a). To compute nucleation sizes in our simulations we use a criterion based on rupture speed: we define the onset of instability as the time when a tip of the actively slipping zone moves with the speed that exceeds a fraction (10%) of the shear wave speed of the surrounding elastic medium [Kaneko and Lapusta, 2008]. The tips of the actively slipping zone are found as the locations of peak shear stress.

The dependency of nucleation size on fault zone width $H$ has two extreme regimes. The nucleation size approaches the length $h^*$ estimated with the rigidity $\mu$ of the undamaged host rock when $H$ is large compared to that length. As $H$ decreases, the influence of the low-rigidity layer on the nucleation size becomes greater. The nucleation size approaches the length $h^*$ estimated with the rigidity $\mu_D$ of the fault zone damaged rock when $H$ is a small fraction of that length.

In the transition between these two extreme regimes, linear stability analysis (appendix C) provides a theoretical estimate of the nucleation size $h_{lay}^*$ on rate and state faults embedded in a simple layered medium, given as the solution of the following equation (C3):

$$h_{lay}^* \tanh \left[ H \frac{\pi}{2h_{lay}^*} + \text{arctanh} \left( \frac{\mu_D}{\mu} \right) \right] = h_{\text{hom}}^{*\mu_D}, \quad (24)$$

where $h_{\text{hom}}^{*\mu_D}$ is the estimate of a nucleation size in a homogeneous medium with rigidity $\mu_D$. The theoretical prediction of $h_{lay}^*$ in equation (24) obtained by setting $h_{\text{hom}}^{*\mu_D} = h_{\text{RA}}^{*\mu_D}$.
where \( h_{RA}^* \) is given by equation (15), agrees fairly well with the simulated nucleation sizes for a range of \( H \) (Figure 7a).

Using friction parameters found in laboratory experiments of Blanpied et al. [1995], \( a - b = 0.004, a = 0.01 \) and \( L = 1-10 \) microns, and assuming that \( \bar{\sigma} = 100 \) MPa at seismogenic depth, the nucleation size is of the order of 0.1-1 m based on equations (14) and (15). This suggests that a nucleation size for natural earthquakes that occur on faults embedded in meter-scale damaged but still cohesive rock are controlled by the properties of the damaged rock and can be smaller than the value estimated using the rigidity of undamaged rocks.

4.2. Dynamic amplification of slip rates in low-rigidity fault zones

As the width of a low-rigidity layer \( H \) becomes larger, the peak slip rate of the propagating dynamic rupture amplifies (Figure 7b). Between the end member cases with homogeneous bulk \( \mu \) and \( \mu_D \), the amplification of the peak slip rate in Figure 7b is about 3.0, larger than the ratio of the rigidity contrast \( \mu/\mu_D = 1.56 \). To understand this behavior, we consider an analytical solution for the maximum slip rate due to a propagating shear crack at a constant speed in a homogeneous medium. In this case, the peak slip rate \( \dot{\delta}_{\text{max}} \) and the rupture speed \( V_r \) are related by [Ida, 1973]:

\[
\dot{\delta}_{\text{max}} \propto \frac{V_r \Delta \tau^{p-s}}{\mu \Lambda_{\text{III}}} ,
\]

where \( V_r \) is a rupture speed of the propagating shear crack, \( \Delta \tau^{p-s} \) is the change from the peak stress to the dynamic sliding stress (i.e., strength drop), and \( \Lambda_{\text{III}}(V_r/V_s) = (1 - V_r^2/V_s^2)^{1/2} \) is a monotonically decreasing universal function of \( V_r \). We find that, in our simulations, the strength drop \( \Delta \tau^{p-s} \) does not depend much on the rigidity \( \mu \) of the medium. The ratio of the peak slip rate between damaged and undamaged fault-zone
scenarios then becomes

\[
\frac{\dot{\delta}_{\text{max}}^D}{\dot{\delta}_{\text{max}}^\text{hom}} = \frac{V_r^D}{V_r^\text{hom}} \frac{\Lambda_{\text{III}}(V_r^\text{hom}/V_s)}{\Lambda_{\text{III}}(V_r^D/V_s)} \frac{\mu}{\mu_D},
\]

(26)

where the quantities with the superscript ‘hom’ refer to the case for the undamaged fault zone (a homogeneous bulk).

Figure 8b shows a comparison between the theoretical prediction (26) and the ratio of the simulated peak slip rate in the damaged fault-zone cases to that in the homogeneous case with \(\mu = 32\) GPa (Figure 7b). We set rupture speed \(V_r^D\) in expression (26) to be the maximum value of \(V_r^D\) during a seismic event in each case (Figure 8a) because the slip rate and rupture speed are generally correlated. The good agreement between the simulations and the theoretical prediction in Figure 8b suggests that the higher peak slip rates are caused by a combination of the rigidity contrast and the difference in effective rupture dimension, i.e., the ratio of the rupture length to the nucleation size. In our models, both the peak slip rate and rupture speed increase as the rupture propagates a longer distance (Figure 8a). Since the effective rupture dimension is larger in the damaged fault zone due to the reduction of the nucleation size (Figure 7a), the resulting rupture speeds are higher. This effect further amplifies the peak slip rate in the damaged fault-zone cases in addition to the rigidity contrast. This result suggests that the peak slip rates of small repeating earthquakes in severely damaged and undamaged rocks may be different by an order of magnitude.

4.3. Larger recurrence interval in low-rigidity fault zones

Interestingly, the recurrence interval of the simulated earthquakes increases as the width of a lower-rigidity layer increases (Figure 7c). One may think that a smaller nucleation size in the case with the low-rigidity bulk would reduce the recurrence interval, but the opposite happens. To understand this, we consider a simpler model of an earthquake
sequence with constant stress drop $\Delta \tau$ and constant stressing rate $\dot{\tau}$. The recurrence interval $T_r$ in this scenario is given by $T_r = \Delta \tau / \dot{\tau}$. Figure 7d shows that the stress drop slightly increases with increasing width of a low-rigidity layer due to the dynamic amplification of slip rates. However, this effect alone cannot fully explain the greater increase of the recurrence interval. We find that the difference in interseismic stressing rates caused by different rigidity of the adjacent rock contributes to the increase of the recurrence interval. Since our model is loaded by the back-slip motion $V_{pl}$ outside of the velocity-strengthening fault segments and the corresponding stressing rate on the fault strongly depends on the rigidity of the medium adjacent to the fault plane, $\dot{\tau}$ is smaller for the lower-rigidity bulk medium. This is why the recurrence interval is larger for the cases with the lower-rigidity medium. Since tectonic loading is applied quasi-statically, the dependence of the recurrence interval on the width of a low-rigidity layer is similar to that of the nucleation size.

4.4. Smaller amount of aseismic slip in low-rigidity fault zone

To quantify the amount of the aseismic slip compared to the seismic one during one earthquake cycle, we compute seismic and total potency in the 2-D model. Potency can not be usefully discussed in the 2-D model where slip extends infinitely, and simultaneously, along strike at any given depth $z$. We therefore use our 2-D modeling to provide input to a 3-D source by mapping the 2-D slip distribution into a 3-D one. Following the approach used in Lapusta and Rice [2003], we compute potency as follows:

$$P(t) = \int \int \delta(y, z, t) dy \, dz = 2\pi \int_{0}^{r_{\text{max}}} \delta(r, t) r \, dr,$$

(27)

where $r = (y^2 + z^2)^{1/2}$, with $y$ and $z$ measured from the rupture origin, and the slip $\delta(y, z, t) = \delta(r, t)$ for a 3-D source with the radial symmetric property. We select the center of the nucleation zone as the effective origin $z_o$ and reinterpret the above integral.
in terms of the computed slip $\delta(z,t)$ from our 2-D modeling as

$$P(t) = \pi \int_{z_{\text{bottom}}}^{z_{\text{top}}} \delta(z,t)|z-z_0|dz.$$  \hspace{1cm} (28)

Note that this procedure is exact (except for a numerical prefactor) if applied to known solutions for cracks of constant stress drop which grow at a constant rupture speed. However, in our case, neither stress drop nor rupture speeds are constant.

The ratio of the seismic potency to the total potency $P_{\text{seis}}/P_{\text{total}}$ computed using equation (28) increases as the width of a low-rigidity layer $H$ increases (Figure 7e). This means that more seismic slip is promoted in the bulk with the lower-rigidity layer. When $H$ is comparable to the source dimension (i.e., the ruptured length), the ratio $P_{\text{seis}}/P_{\text{total}}$ approaches the value for the case with a homogeneous bulk with $\mu_D$. We find that the increase of $P_{\text{seis}}/P_{\text{total}}$ for larger $H$ can be explained by the increase in the effective rupture dimension, i.e., the ratio of the rupture length to the nucleation size (Figure 7f). A smaller nucleation size and the resulting larger rupture dimension for larger $H$ leads to a smaller amount of aseismic slip over the earthquake cycle.

5. Can vertically stratified bulk structure cause shallow coseismic slip deficit?

In this section, we further extend our analysis to the effects of near-surface low-rigidity bulk layers and study how these layers affect the depth dependence of slip in large earthquakes. We model earthquake sequences on a planar vertical strike-slip fault embedded into an elastic half-space (Figure 9a). The setup is similar to the depth-variable model of Lapusta et al. [2000], where friction acts in the top 24 km of the fault and the deeper extension moves with a prescribed plate rate of 35 mm/year. The physical parameters of the simulations presented in this work are shown in Figure 9a,b. The value of $L$ used is 8 mm, in which case the model results in sequences of model-spanning earthquakes as shown in Lapusta et al. [2000]. The variation of friction parameters $a$ and $b$ with depth shown in
Figure 9b is similar to the one in Rice [1993] and Lapusta et al. [2000]; it is derived from laboratory experiments [Blanpied et al., 1995]. The region between 2.0 km and 14.3 km has steady-state velocity-weakening properties. The transition from steady-state velocity weakening to steady-state velocity strengthening at 14.3-km depth is due to temperature increase with depth. The effective normal stress $\bar{\sigma}$ becomes constant and equal to 50 MPa at depths larger than 2.6 km (Figure 9b), due to the assumption that fluid over-pressure prevents further increase of $\bar{\sigma}$ with depth [Rice, 1993; Ben-Zion and Rice, 1997].

We consider four different scenarios of earthquake sequences in: (i) homogeneous bulk structure without the shallow velocity-strengthening fault patch, (ii) layered bulk structure without the shallow velocity-strengthening fault patch, (iii) homogeneous bulk structure with the shallow velocity-strengthening fault patch, and (iv) layered bulk structure with the shallow velocity-strengthening fault patch (Figure 9). The layered bulk model approximately corresponds to the 1D Parkfield velocity structure down to the depth of $\sim 15$ km used in the study by Custódio et al. [2005]. The scenarios with the velocity-strengthening patch at depths less than 2.0 km (Figure 9b) are motivated by laboratory experiments and field observations as discussed in Kaneko et al. [2008].

Figure 10a,b show earthquake sequences simulated in the 2-D SEM model for scenarios (i) and (ii). The solid lines are plotted every 5 years and show the continuous slow sliding (creep) of the steady-state velocity-strengthening region at depth. That slow slip creates stress concentration at its tip and penetrates into the velocity-weakening region. In due time, an earthquake nucleates close to the transition. We show the progression of earthquakes with dashed lines plotted every second.

The scenarios with homogeneous and layered bulk do not lead to shallow coseismic slip deficit in these particular examples (Figure 10a,b,e). From the evolution of shear stress in these cases (black and black dashed lines in Figure 10f), the stress accumulation rates
in the lower-rigidity materials during the interseismic periods are smaller than those in
the materials with higher rigidity. As a result, the prestress on the fault within the low-
rigidity materials becomes smaller. However, as explained in section 4, the coseismic slip
rates get amplified in the low-rigidity materials, resulting in the net effect on slip being
nearly zero.

The small reduction of the coseismic slip near the free surface in scenario (i) (the
black curve in Figure 10e) is caused by minor aseismic creep. Due to this minor creep,
interseismic shear stress near the free surface in scenario (i) decreases for some time (the
green curve in Figure 10f). The occurrence of this minor creep is related to the decrease
in effective normal stress $\bar{\sigma}$ towards the free surface. From the theoretical estimates of
a nucleation size given in equations (14) and (15), $h^*$ is proportional to $\mu/\bar{\sigma}$. Since $\bar{\sigma}$
increases with depth, $h^*$ is larger near the free surface than that at a depth provided that
other parameters are uniform over depth. Theoretical models predict that aseismic slip
would only occur if $h^*$ is larger than the velocity-weakening region of the fault [Rice and
Ruina, 1983]. Hence larger $h^*$ near the free surface in scenario (i) results in minor aseismic
slip there. This effect indeed explains why there is no aseismic creep in scenario (ii). Since
rigidity $\mu$ is also smaller near the free surface in scenario (ii), $h^*$ remains relatively small,
and hence there is no aseismic slip.

Figure 10c,d shows earthquake sequences simulated in the 2-D SEM model for sce-
narios (iii) and (iv). The presence of the shallow velocity-strengthening patch leads to
shallow coseismic slip deficit regardless of the properties of the bulk (red and red dashed
lines in Figure 10e). Due to the interseismic creep and afterslip in the shallow velocity-
strengthening region, the stress accumulation rates during the interseismic periods are
very small in both scenarios (red and red dashed lines in Figure 10f). Hence the coseismic
slip at the shallow parts is driven by the dynamic stress supplied by the incoming ruptures propagating up-dip.

The results here suggest that coseismic slip deficit can be caused by the presence of a shallow velocity-strengthening region, but not by that of low-rigidity shallow bulk materials. In a model embedded in elastic media, the accumulated slip is equal to the sum of co-, inter- and postseismic slip. On velocity-weakening faults accommodating little inter- and postseismic slip, the coseismic slip at a given point has to catch up with that on the rest of the fault plane. Hence the cancellation of the net effect of dynamic amplification and low interseismic stress accumulation in Figure 10 is reasonable. The studies by Rybicki [1992] and Rybicki and Yamashita [1998], which proposed that the reduction of coseismic slip at shallow depths can be caused by the presence of low-rigidity materials, did not consider dynamic amplification of coseismic slip, even though they considered a wider range of conditions. While exploring the wider range of parameters may be important, our conclusions should still be valid, unless certain conditions lead to significant interseismic creep or afterslip on faults with velocity-weakening friction.

6. Conclusions

We have developed a 2-D SEM algorithm for simulating long-term histories of seismic and aseismic fault slip on a vertical strike-slip fault embedded in heterogeneous bulk media subjected to slow tectonic loading. Our approach reproduces all stages of earthquake cycles from accelerating slip before dynamic instability, to rapid dynamic propagation of earthquake rupture, to postseismic slip, and to interseismic creep. We have set up an antiplane benchmark problem and have validated the developed SEM approach by comparing SEM and BIM simulation results in a 2-D model of small repeating earthquakes.
Using the developed formulation, we have investigated the effects of variable fault-zone bulk properties on source properties of small repeating earthquakes. Our results suggest that source properties of small repeating earthquakes depend on the width of a lower-rigidity bulk (or a damaged zone) and its rigidity value. We find that a fault bisecting a lower-rigidity layer, compared to the one in undamaged country rock, leads to the following changes in the properties of earthquakes and their cycles: (i) reduction in the earthquake nucleation size, (ii) amplification of slip rates during dynamic rupture propagation, (iii) increase in the recurrence interval, and (iv) smaller amount of aseismic slip. Note that changes (ii-iv) are due to a combined effect of the presence of the lower-rigidity layer and of the change (i).

We have further examined the effects of vertically-stratified bulk layers on the nature of shallow coseismic slip deficit. For the set of parameters we have considered, low-rigidity shallow bulk materials alone do not lead to coseismic slip deficit. While the low-rigidity materials do cause lower interseismic stress accumulation, they also cause dynamic amplification of coseismic slip rates, with the net effect on slip being nearly zero. At the same time, the addition of velocity-strengthening friction to shallow parts of the fault leads to coseismic slip deficit in all cases we have considered.

The developed SEM methodology can be used to study a number of fault slip phenomena that bridge the broad spectrum of earthquake behavior, from rupture dynamics to long-term crustal deformation. The SEM model allows for more flexibility in fault geometry and heterogeneous and non-elastic bulk properties in long-term simulations of fault slip. Furthermore, while the methodology is presented using the 2-D antiplane problem, it can be readily extended to the 2-D in-plane and 3-D formulations. The developed models would be useful for interpreting seismic and geological data collected and sampled close to structurally-complex fault zones at SAFOD and other drilling sites.
Appendix A: Variable evolution time step in 2-D antiplane problems

Simulations of long-term deformation histories with periods of rapid dynamic slip (earthquakes) require time steps that change by orders of magnitude. For the SEM with the implicit scheme, we adopt the time-stepping scheme developed for BIM by Lapusta et al. [2000] for a 2-D antiplane problem. This scheme also works well for our SEM model. Note that the maximum time step is limited by the Courant condition and constant in the SEM with the explicit scheme, but not in the implicit scheme (unconditionally stable). The variable time step $\Delta t$ is chosen as:

$$\Delta t = \max\{\Delta t_{\text{min}}, \Delta t_{\text{ev}}\}, \quad (A1)$$

where $\Delta t_{\text{min}}$ is the minimum time step, and $\Delta t_{\text{ev}}$ depends on slip velocity at each time step. The minimum time step is set by the Courant condition and given by

$$\Delta t_{\text{min}} = C \Delta x_{\text{min}}/V_s, \quad (A2)$$

where $C = 0.6$ is used in our 2-D antiplane problem. The same condition is used for modeling single dynamic ruptures in the 2-D antiplane test problem in [Kaneko et al., 2008]. The time step $\Delta t_{\text{ev}}$ is set to be inversely proportional to slip velocity:

$$\Delta t_{\text{ev}} = \min[\xi_i L_i/\dot{\delta}_i], \quad (A3)$$

where $L_i$, $\dot{\delta}_i$, and $\xi_i$ are the characteristic slip, the current slip velocity and a prescribed parameter for the $i$th fault node of the discretized domain, respectively. $\xi_i$ is a function of friction properties from linear stability analysis [Lapusta et al., 2000], and it is constrained to satisfy $\xi_i \leq \xi_c$, where $\xi_c$ is a constant, to ensure that slip at each time step does not exceed $\xi_c L_i$. As in Lapusta et al. [2000], we use $\xi_c = 1/2$ in our 2-D SEM and BIM models.

Appendix B: Switching criteria between quasi-static and fully-dynamic SEMs
The quasi-static SEM formulated in section 2 can be merged with the fully dynamic SEM [Kaneko et al., 2008]. We switch from the quasi-static to the dynamic SEM and vice versa based on the values of the maximum slip velocity on the fault described in section 3.1. To make sure that at the times of the switch the inertial term is negligible relative to the other terms in the governing equation, we define and compute the percentage \( P_{Ma} \) of inertial term on the fault at each time step:

\[
P_{Ma} = \max \left( \max \left( \frac{|Ma_{eff}|}{|Ku|} \right), \max \left( \frac{|Ma_{eff}|}{|B\tau|} \right) \right) \times 100 \quad (B1)
\]
during the quasi-static regime, and

\[
P_{Ma} = \max \left( \max \left( \frac{|Ma|}{|Ku|} \right), \max \left( \frac{|Ma|}{|B\tau|} \right) \right) \times 100 \quad (B2)
\]
during the dynamic regime. The effective acceleration computed during the quasi-static regime is computed as \( a_{eff} \equiv (v_{n+1} - v_n)/\Delta t \). Figure 11 shows \( P_{Ma} \) and values of the maximum slip velocity on the fault as a function of time before, during and after one of the simulated earthquake events shown in Figure 4. At the times of the switch, the inertial term \( Ma \) is negligible relative to the other terms in the governing equation, validating the use of our switching criteria based on the values of the maximum slip velocity.

**Appendix C: Nucleation size for a layered elastic medium**

We derive the theoretical estimate of a nucleation size on rate and state faults embedded in a simple layered medium shown in Figure 2. The static stiffness \( k_{hom} \) for sliding on a patch of characteristic size \( h \) in a homogeneous elastic solid is given by [e.g., Dieterich, 1992]:

\[
k_{hom} = \frac{\gamma \mu}{h}, \quad (C1)
\]
where \( \gamma \) is a parameter of order one that depends on the geometry of the slip patch and assumptions relating to slip or stress conditions on the patch, and \( \mu \) is the rigidity.
Considering sinusoidal perturbations of slip of wavelength $h$, for which $\gamma = \pi$, Rice and Ruina [1983] and Ruina [1983] showed that frictional sliding with the rate and state friction is always stable for the perturbations of the shortest wavelengths, but, if the sliding surfaces have steady-state velocity-weakening properties, then a critical wavelength exists, associated to a critical stiffness $k_c$, such that larger wavelengths are unstable. This means that only the long wavelengths of a perturbation, with $k_{\text{hom}} < k_c$, diverge exponentially and trigger an instability. For rate and state friction with the aging law (11), the critical stiffness is $k_c = \bar{\sigma}(b - a)/L$ [Ruina, 1983]. Setting $k_c = k_{\text{hom}}$ and solving for the critical size $h$ leads to equation (14), which we denote $h_{\text{hom}}^*$ in this appendix.

Ampuero et al. [2002] derived an analytical expression of the static stiffness $k_{\text{lay}}$ for sinusoidal slip perturbations of wavelength $h$ in a simple layered medium. We slightly modify their expression by introducing a parameter $\gamma$ that accommodates more general slip modes, inspired by equation (C1):

$$k_{\text{lay}} = \frac{\gamma \mu D}{h} \cotanh \left[ 2H \frac{\gamma}{h} + \arctanh \left( \frac{\mu D}{\mu} \right) \right],$$

where $\mu D$ and $\mu$ are rigidity of the low-rigidity layer adjacent to the fault plane and that of the undamaged host rock, respectively, and $H$ is the thickness of the low-rigidity layer (Figure 2). Since $k_c$ depends only on the friction law and effective normal stress $\bar{\sigma}$, $k_c$ is the same for both the homogeneous and layered cases. By setting $k_{\text{hom}} = k_{\text{lay}} = k_c$, we obtain

$$h_{\text{lay}}^* \tanh \left[ 2H \frac{\gamma}{h_{\text{lay}}^*} + \arctanh \left( \frac{\mu D}{\mu} \right) \right] = h_{\text{hom}}^*.$$  

Given $h_{\text{hom}}^*$, one can find $h_{\text{lay}}^*$ by numerically solving equation (C3). The parameter $\gamma$ can be determined empirically for a given model geometry. For the antiplane problems we consider here, we find that setting $\gamma = \pi/4$ provides a satisfactory representation of our simulation results.
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References


Table 1. Parameters used in the 2-D SEM and 2-D BIM models of small repeating earthquakes.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear modulus</td>
<td>( \mu )</td>
<td>32.0 GPa</td>
</tr>
<tr>
<td>Shear wave speed</td>
<td>( V_s )</td>
<td>3.464 km/s</td>
</tr>
<tr>
<td>Reference slip rate</td>
<td>( \dot{\delta}_0 )</td>
<td>10^{-6} m/s</td>
</tr>
<tr>
<td>Reference friction coefficient</td>
<td>( f_0 )</td>
<td>0.6</td>
</tr>
<tr>
<td>Characteristic slip distance</td>
<td>( L )</td>
<td>84.0 micron</td>
</tr>
<tr>
<td>Effective normal stress</td>
<td>( \bar{\sigma} )</td>
<td>120 MPa</td>
</tr>
<tr>
<td>Rate and state parameter ( a )</td>
<td>( a )</td>
<td>0.0144</td>
</tr>
<tr>
<td>Rate and state parameter ( b )</td>
<td>( b )</td>
<td>0.0191 (^a)</td>
</tr>
</tbody>
</table>

\(^a\) The indicated value of \( b \) is valid for the steady-state velocity-weakening region in Figure 2.

\[
\Gamma_+ \quad n^+\bigtriangleup \tau^+ \quad \tau^- \bigtriangleup n^- \quad \Gamma_-
\]

Figure 1. The fault divided into two non-overlapping surfaces \( \Gamma_{\pm} \).
Figure 2. 2-D fault models of a vertical strike-slip fault. Small repeating earthquakes at seismogenic depths in a region indicated by a black rectangle are modeled using these models. By symmetry consideration, the medium across the fault boundary has equal and opposite motion. Unless otherwise noted, the medium is assumed to be homogeneous. In section 4, we investigate the effect of a fault-parallel low-rigidity layer of a width $H$ and rigidity $\mu_D$ on seismic and aseismic slip.

Figure 3. (a) Depth-variable distribution of friction parameters $a$ and $(a-b)$. (b) Distribution of the ratio $h^*/\Delta x$. Two theoretical estimates $h^*$ of the nucleation size by Rubin-Ampuero (RA) and Rice-Ruina (RR) are shown.
Figure 4. Comparison of earthquake sequences simulated in BIM and the developed SEM. (a) Solid lines show slip accumulation every 0.5 years for BIM (blue) and SEM (red). Dashed lines are intended to capture dynamic events and are plotted every 1 millisecond (ms) during the simulated earthquakes (or $\dot{\delta}_{\text{max}} > 1 \text{ cm/s}$). Evolution of aseismic slip and seismic slip of the 2nd event are shown. Spatial distributions of slip contours in these models agree very well. (b) Shear-stress and (c) slip-velocity histories at the center of the fault. The 2nd, 3rd, and 4th earthquake events are shown. The timings of earthquake events in these models are nearly identical as quantified in the text, validating our SEM implementation.
Figure 5. (a) Snapshots of the displacement field (relative to the plate motion) in the 2-D SEM model. The line \( x = 0 \) corresponds to the fault. (b) Snapshots of the SH particle velocity field every 3 milliseconds during the seismic event.
Figure 6. Simulated earthquake sequences with (a) the layered bulk structure with the width of a low-rigidity layer $H = 1.5$ m and (b) homogeneous bulk structure with $\mu_D = 20.5$ GPa corresponding to $H = \infty$. Solid and dashed lines have the same meaning as in Figure 4a. The model geometry is shown in Figure 2a.
Figure 7. Effects of a fault parallel low-rigidity layer of width $H$ on small repeating earthquakes. (a) The relation between simulated nucleation sizes and $H$. The nucleation sizes and the widths are non-dimensionalized by the theoretical estimate $h_{RA}^{\mu_D}$, with $h_{RA}^{\mu_D} = 7.9$ m for the parameters used. The solid curve is the theoretical prediction (24), which fits the simulated nucleation sizes fairly well. (b) The peak slip rate during a typical earthquake as a function of $H$. (c) Recurrence interval between successive earthquakes as a function of $H$. (d) Stress drops averaged over the region of positive stress drop as a function of $H$. (e) Ratio of seismic potency $P_{\text{seis}}$ to total potency $P_{\text{total}}$ released on the velocity-weakening patch over one earthquake cycle. (f) $P_{\text{seis}}/P_{\text{total}}$ as a function of the effective rupture dimension (i.e., the ratio of the rupture length to the simulated nucleation size). The solid line is the least-square fit to the data points. The increase of the effective rupture dimension due to increasing $H$ leads to the increase of $P_{\text{seis}}/P_{\text{total}}$ and hence more seismic slip. The case with $H/h_{RA}^{\mu_D} = 2.5$ in panels (a-e) corresponds to the case for a homogeneous bulk structure with $\mu_D = 20.5$ GPa shown in Figure 6b.
**Figure 8.** (a) Rupture speeds $V_r$ normalized by the $S$-wave speed $V_s$ of the medium adjacent to the fault during a seismic event for the cases with a homogeneous bulk $\mu$ and $\mu_D$ and for a layered case with $H = 1.5$ m. Rupture speed is determined by computing average rupture speed over each spectral element (or 5 computational nodes) and plotting the obtained value with respect to the center of the element. The normalized rupture speeds become larger for a larger width $H$ of a low-rigidity layer due to the increase in the effective rupture dimension. (b) The peak slip rate $\dot{\delta}_{\text{max}}$ shown in Figure 7b divided by that in the case for a homogeneous bulk with $\mu = 32$ GPa ($\dot{\delta}_{\text{hom}}^{\text{max}}$). The ratio of the peak slip rates $\dot{\delta}_{\text{max}}/\dot{\delta}_{\text{hom}}^{\text{max}}$ predicted by equation (26) is in good agreement with the ratio of the simulated peak slip rates.
Figure 9. (a) 2-D SEM model of a vertical strike-slip fault. Layered bulk properties used in some scenarios we consider are indicated. (b) Depth-variable distribution of effective normal stress, the rate-and-state constitutive parameters \(a\) and \((a - b)\) over the fault segment where friction acts.
Figure 10. Simulated earthquake sequences and event characteristics of models with: (a) homogeneous bulk structure without the shallow velocity-strengthening fault patch, (b) layered bulk structure without the shallow velocity-strengthening fault patch, (c) homogeneous bulk structure with the shallow velocity-strengthening fault patch, and (d) layered bulk structure with the shallow velocity-strengthening fault patch. Solid lines show slip accumulation every 5 yrs. Dashed lines are intended to capture dynamic events and are plotted every 1 s during the earthquakes. (e) Seismic slip of a representative event in each case. Low-rigidity shallow bulk materials alone do not lead to coseismic slip deficit. (f) Shear stress evolution at the depth of $z = -2$ km for all four cases and at the depth of $z = -1$ km for the case (i) (panel a). Time is normalized by the recurrence interval for each case, and $t = 0$ corresponds to the time just after the prior seismic event.
Figure 11. Percentage \( P_{Ma} \) of the relative importance of the inertial term in the governing equation and the maximum slip velocity on the fault as a function of time before, during and after the simulated earthquake shown in Figure 4. \( P_{Ma} \) is defined in equations (B1) and (B2). At the times of the switch from quasi-static to dynamic and vice versa, the inertial term \( Ma \) is much smaller (~10\(^{-4}\%\)) than the other terms in the governing equation (2).