Seismic radiation from regions sustaining material damage

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Summary

We discuss analytical results for seismic radiation during rapid episodes of inelastic brittle deformation that include, in addition to the standard moment term, a damage-related term stemming from changes of elastic moduli in the source region. The radiation from the damage-related term is associated with products of the changes of elastic moduli and the total elastic strain components in the source region. Order of magnitude estimates suggest that the damage-related contribution to the motion in the surrounding elastic solid, which is neglected in standard calculations, can have appreciable amplitude that may in some cases be comparable to or larger than the moment contribution. A decomposition analysis shows that the damage-related source term has an isotropic component that can be larger than its *DC* component.

Keywords: Earthquake dynamics, Earthquake ground motions, Theoretical seismology, Dynamics and mechanics of faulting, Earthquake source observations.

1. Introduction

Seismic representation theorems establish fundamental quantitative connections between rapid actions at source regions that may sustain permanent inelastic deformation and generated motion at observational points in the surrounding elastic material (e.g., Aki and Richards, 2002; Ben-Zion, 2003, Ampuero and Dahlen, 2005). In addition to being important on theoretical grounds, the representation theorems provide the foundation for multitudinous applications ranging from estimating source properties of earthquakes and explosions from observed seismograms to simulating ground motions from hypothetical sources for engineering purposes. During brittle inelastic deformation, the cracking and associated changes of the internal surface area and void space modify the elastic moduli (i.e., produce material damage) in regions where the elastic limit is exceeded. The commonly-used representation with moment sources for motions produced by earthquakes and other episodes of brittle deformation ignores the changes of the effective elastic properties in the source regions. While this is a useful approximation, it is logically inconsistent and may lead as will be shown below to appreciable errors in some cases.

Mal and Knopoff (1967) provided a representation that incorporates changes of elastic properties and mass density across internal boundaries in a solid. Knopoff and Randall (1970) used that representation to analyze the radiation from regions that sustain changes of the density, the bulk modulus, the Lamé parameter λ and the shear modulus μ. They showed that changes of the first three parameters lead to isotropic radiation, while changes of the shear modulus produce motion that can be represented by a superposition of a double-couple, an isotropic source and a compensated linear vector dipole (CLVD). The representation of Mal and Knopoff (1967) involves products of the changes of material parameters and the existing elastic displacements (or strains) in the source region. Knopoff and Randall (1970) inconsistently employed the jumps of the displacements (or strains) in the source region from the state before to that after the episode leading to the parameter changes. The definition of the relevant strain fields does not affect the main results of Knopoff and Randall (1970) on the forms of radiation patterns, but is important for understanding the relative contributions of slip and damage to the radiation.

Motivated by recent interest in the maximum ground motion that can be generated by seismic sources, we re-examine the representation of radiation by brittle deformation phases that include changes of elastic properties in the source region. In the next section we obtain expressions for such a representation that are consistent with the equations of Mal and Knopoff (1967). The results indicate that the seismic motion generated by a brittle failure process that includes rock damage has, in addition to the classical moment term associated with displacement discontinuities, a contribution associated with the product of the tensor of the changes of elastic moduli and the tensor of elastic strain in the source region. Decreasing elastic moduli in the source region, as produced generally by brittle deformation of low-porosity rocks and explosions, increase the seismic radiation to the bulk. In contrast, increasing moduli in the source region, which may be produced during the formation of compaction bands in porous rocks, decrease the radiation. The radiation from the damage-related source term may have significant isotropic and *CLVD* components. Basic estimates indicate that the contribution to motion from the changes of elastic moduli in the source region can be a substantial fraction of, and in some cases larger than, the contribution from the classical moment term.

2. Representation of seismic sources with material damage

Given the lack of clarity on the fields involved in the seismic representation theorem that includes changes of elastic moduli, we provide a detailed derivation of results. Consider a solid with an internal source region that sustains at a given time interval inelastic brittle deformation. The total strain tensor at (***x***, *t*) is written as a sum of elastic (ε_{ij}) and inelastic (p_{ij}) contributions $\varepsilon_{ij}^t = \varepsilon_{ij} + p_{ij}$. The elastic stress-strain relation in the initial state is given by

$$\sigma_{ij} = c_{ijkl}^i \varepsilon_{kl}\,, \tag{1a}$$

where σ_{ij} is the elastic stress tensor, c_{ijkl}^i is the initial tensor of elastic moduli, and the summation convention for repeating subscripts is used. The elastic stress-strain relation in the final state following the brittle deformation episode is given by

$$\sigma_{ij} = c_{ijkl}^f (\varepsilon_{kl}^t - p_{kl})\,, \tag{1b}$$

where c_{ijkl}^f is the final tensor of elastic moduli that was modified by the brittle deformation process. The terms characterizing the change of state in (1b) may be written as

$$p_{kl} = \begin{cases} \varepsilon_{kl}^T & \text{inside the source region} \\ 0 & \text{outside} \end{cases}\,, \tag{2a}$$

and

$$c_{ijkl}^{f} = \begin{cases} c_{ijkl}^{i} + \Delta c_{ijkl} & \text{inside the source region} \\ c_{ijkl}^{i} & \text{outside} \end{cases} , \tag{2b}$$

where ε_{kl}^{T} is the transformational strain tensor that resets (Eshelby, 1957) the reference levels of the elastic stress and strain tensors after the event. We now write the elastic stress-strain relation in the final state as

$$\begin{aligned} \sigma_{ij} &= (c_{ijkl}^{i} + \Delta c_{ijkl})(\varepsilon_{kl}^{t} - \varepsilon_{kl}^{T}) \\ &= c_{ijkl}^{i}\varepsilon_{kl}^{t} - c_{ijkl}^{i}\varepsilon_{kl}^{T} + \Delta c_{ijkl}\varepsilon_{kl} \end{aligned} , \tag{3}$$

with the second and third terms being non-zero only in the source region and $\varepsilon_{ij}^{t} = \varepsilon_{ij}$ outside the source region.

The Cauchy equation of motion for a continuum is

$$\sigma_{ij,j} + f_i = \rho \ddot{u}_i , \tag{4a}$$

where f_i is the *i* component of body force per unit volume, u_i is the *i* component of the displacement vector $\boldsymbol{u}(\boldsymbol{x}, t)$, the comma after a subscript implies a spatial derivative, and the over dots indicate time derivatives. The equation of motion at the initial state, before the source terms produce dynamic motions, is

$$\sigma_{ij,j} + f_i = 0 , \tag{4b}$$

Subtracting (4b) from (4a) and using equations (1)-(3), we get for the field generated by the source terms of (2)

$$\frac{\partial}{\partial x_j}[c_{ijkl}^{i}(\varepsilon_{kl}^{t} - \varepsilon_{kl}^{i})] - \frac{\partial}{\partial x_j}(c_{ijkl}^{i}\varepsilon_{kl}^{T}) + \frac{\partial}{\partial x_j}(\Delta c_{ijkl}\varepsilon_{kl}) = \rho \ddot{u}_i , \tag{5a}$$

or

$$\frac{\partial}{\partial x_j}(c_{ijkl}^{i}\Delta\varepsilon_{kl}) + f_i^{eff} = \rho \ddot{u}_i , \tag{5b}$$

where $\Delta\varepsilon_{kl} = \varepsilon_{kl}^{t} - \varepsilon_{kl}^{i}$ is the incremental strain field generated by a distribution of effective body forces $f_i^{eff} = -m_{ij,j} + d_{ij,j}$. The term $m_{ij} = c_{ijkl}^{i}\varepsilon_{kl}^{T}$ is the standard stress glut or seismic moment density tensor, while $d_{ij} = \Delta c_{ijkl}\varepsilon_{kl}$ is an additional source term stemming from the existence of material damage in the failure region. We note that d_{ij} is associated with the *total elastic* (rather than the incremental elastic or the transformational) strain in the source region. This is consistent with Mal and Knopoff (1967). We also note that both source terms are symmetric ($m_{ij} = m_{ji}$, $d_{ij} = d_{ji}$).

Outside the source region the incremental strain field is purely elastic. The solution for the displacement field associated with the dynamic changes of the elastic strain outside the source region can be written with the aid of the Green's function of the intact medium $G_{ij}(\boldsymbol{x},t;\boldsymbol{x'},t')$ as

$$
\begin{aligned}
u_i(\boldsymbol{x},t) &= \int_{-\infty}^{t} dt' \int_V G_{ij}(\boldsymbol{x},t;\boldsymbol{x'},t') f_j(\boldsymbol{x'},t') dV' \\
&+ \int_{-\infty}^{t} dt' \int_S G_{ij}(\boldsymbol{x},t;\boldsymbol{x'},t') T_j(\boldsymbol{x'},t') dS'
\end{aligned}
\quad . \tag{6}
$$

Assuming that the tractions T_j vanish on the outer surface S, and using the notations introduced in (5b), we have

$$
u_i(\boldsymbol{x},t) = \int_{-\infty}^{t} dt' \int_V G_{ij}(\boldsymbol{x},t;\boldsymbol{x'},t') \left[-\frac{\partial m_{jk}(\boldsymbol{x'},t')}{\partial x'_k} + \frac{\partial d_{jk}(\boldsymbol{x'},t')}{\partial x'_k} \right] dV' . \tag{7a}
$$

We write each of the multiplications inside the integrand of (7a) as

$$
G_{ij} \frac{\partial Y_{jk}}{\partial x'_k} = \frac{\partial (G_{ij} Y_{jk})}{\partial x'_k} - \frac{\partial G_{ij}}{\partial x'_k} Y_{jk} , \tag{7b}
$$

where Y_{jk} represents either m_{jk} or d_{jk}. Using Gauss' theorem to convert the volume integrals associated with the first term on the right side of (7b) to surface integrals, and recognizing that those vanish if m_{jk} and d_{jk} are zero at the surface, we get

$$
u_i(\boldsymbol{x},t) = \int_{-\infty}^{t} dt' \int_V \frac{\partial G_{ij}(\boldsymbol{x},t;\boldsymbol{x'},t')}{\partial x'_k} \left[m_{jk}(\boldsymbol{x'},t') - d_{jk}(\boldsymbol{x'},t') \right] dV' , \tag{8a}
$$

or

$$
u_i(\boldsymbol{x},t) = \int_{-\infty}^{t} dt' \int_V \frac{\partial G_{ij}}{\partial x'_k} \left[c^i_{jklm} \varepsilon^T_{lm} \right] dV' - \int_{-\infty}^{t} dt' \int_V \frac{\partial G_{ij}}{\partial x'_k} \left[\Delta c_{jklm} \varepsilon_{lm} \right] dV' . \tag{8b}
$$

Equation (8b) can be written concisely as

$$
u_i(\boldsymbol{x},t) = u_i(\boldsymbol{x},t)^{trad} - u_i(\boldsymbol{x},t)^{damage} , \tag{8c}
$$

where $u_i(\boldsymbol{x},t)^{trad}$ is the traditional seismic representation theorem and $u_i(\boldsymbol{x},t)^{damage}$ is the additional damage-related contribution to the motion. As seen, the latter is produced by the product of the tensor characterizing the change of elastic moduli multiplied by the tensor of elastic strain in the source volume. A more general derivation would add to (8) small additional contributions associated with nonzero m_{jk} and d_{jk} at the surface. The results of (8) indicate that accounting for material damage in the source region can increase or decrease the seismic motion in the bulk, depending on the signs and amplitudes of the various Δc_{jklm} components. The second term in (8), associated with material damage, is expected to have generally a volumetric term (trace ≠ 0) and a non-double-couple deviatoric term (determinant ≠ 0). This is discussed in more detail in section 4.

Laboratory studies indicate that brittle deformation leading to shear faulting is accompanied by increasing seismic anisotropy in the source region (e.g., Lockner et al., 1977; Stanchits et al., 2006). Nevertheless, it is useful to assume for simplicity that the elastic moduli in the source region are isotropic both before and after the brittle deformation episode. In this case, the results of (8) reduce to

$$u_i(\boldsymbol{x},t) = u_i(\boldsymbol{x},t)^{trad} - \int_{-\infty}^{t} dt' \int_{V} \frac{\partial G_{ij}}{\partial x'_k} \left[\Delta\lambda \delta_{jk} \varepsilon_{pp} + 2\Delta\mu \varepsilon_{jk} \right] dV' , \tag{9}$$

where δ_{jk} is the Kronecker delta function. The integrand in the right term of (9) is similar to equation (5) of Knopoff and Randall (1970). However, a notable difference that is important in the context of the present work is that the fields in (9) multiplying the changes of moduli are the total elastic strains in the source region, whereas in Knopoff and Randall (1970) they are assumed to be associated with the jumps of the displacement vectors in the source region that accompany the brittle deformation episode.

3. Order of magnitude estimates

To have basic estimates of the relative contributions of the traditional and damage-related source terms to the generated motion, we assume in this section a uniform process with a single modulus *c*, and denote the moment and damage-related contributions as *m* and *d*, respectively. The effective volumes associated with the classical moment representation and the damage process are denoted as V_1 and V_2, respectively. From (8b), the moment and damage-related contributions in this case are to first order given by

$$m \approx \left| c\varepsilon^T \right| \cdot V_1 = \Delta\tau \cdot V_1 , \tag{10a}$$

and

$$d \approx \left| \Delta c \varepsilon \right| \cdot V_2 = \left| \Delta c / c \right| \cdot \tau \cdot V_2 , \tag{10b}$$

where $\Delta\tau$ and τ are the stress drop and absolute stress, respectively. The ratio of the two terms is

$$m / d \approx \frac{c}{\left| \Delta c \right|} \frac{\Delta\tau}{\tau} V_1 / V_2 . \tag{11}$$

Derived earthquake stress drops are typically a small fraction (e.g., 0.1) of the absolute shear stress at seismogenic depth (e.g., Abercrombie, 1995; Shearer et al., 2006). In laboratory fracturing experiments with low-porosity rocks, the modulus reduction on the approach to brittle failure may be several tens of percent (e.g., Gupta, 1973; Lockner et al., 1977; 1992). During the brittle instability itself the modulus reduction may be 50% or more, and the modulus reduction in the source volume of explosions can approach 100%. This suggests that, unless $V_1 >> V_2$, the damage-related source term may have similar or larger contribution to the seismic motion than the classical moment. To have more specific estimates we have to evaluate the volumes associated with the classical moment and the damage terms. This is done below in the context of dynamic fracture mechanics.

The total scalar seismic moment for a classical crack sustaining a uniform stress drop over a rupture area A is given (e.g., Madariaga, 1979; Ben-Zion, 2003) by

$$m = \gamma A R \Delta\tau \,, \tag{12}$$

where γ is a nondimensional constant that depends on the failure geometry and elastic properties (e.g., $\gamma = 16/(7\pi)$ for a circular crack in a Poissonian solid) and R is the shortest characteristic length of the rupture surface. Dynamic crack-like ruptures with off-fault dissipation produce self-similar damage zones that grow linearly with the rupture propagation distance (e.g. Andrews, 2005; see also equation (14c) below). For steady-state pulse-like ruptures, the thickness of the yielding zone is approximately constant and proportional to the slip velocity on the fault (Ben-Zion and Shi, 2005).

The volume associated with the damage process of a circular crack can be written as $V_2 = (2/3)\pi R^2 W$ with W being the width of the damage zone near the final crack edge. We assume that W is of the same order as the yielding zone generated by a singular rupture tip in a solid. The stress field near the tip of the crack (e.g., Freund, 1990; Broberg, 1999) has the form

$$\tau \approx \tau_0 + K/\sqrt{r} + O(\sqrt{r}) \,, \tag{13}$$

where K is the stress intensity factor and r is the distance to the crack tip. Taking W to be distance where the stress is comparable to the yield strength τ_y gives

$$W \approx K^2 / (\tau_y - \tau_0)^2 \,, \tag{14a}$$

where τ_0 is a representative value of the initial stress. For our order of magnitude scaling estimates we do not consider in detail the effect of the orientation of the background stress field. The stress intensity factor scales as $K \approx k(V_r)\Delta\tau\sqrt{R/2}$, where $k(V_r)$ is a function of rupture speed V_r that decays from 1 at zero speed to 0 at the limiting speed (Rayleigh speed C_R for modes I and II and shear speed for mode III). We thus have

$$W \approx k^2 \Delta\tau^2 R / [2(\tau_y - \tau_0)^2] \,. \tag{14b}$$

Writing W in terms of the stress excess parameter $S \approx (\tau_y - \tau_0)/(\tau_0 - \tau_d)$, where τ_d is the dynamic friction strength, gives

$$W \approx k^2 R/(2S^2)\,. \tag{14c}$$

This estimate is consistent with numerical simulations of dynamic crack-like ruptures on frictional faults that produce off-fault plasticity (e.g., Andrews 2005; Templeton and Rice, 2008) or material damage (e.g., Yamashita, 2000; Ampuero et al., 2008) in regions where yielding criteria are reached. In particular, the mode II self-similar crack simulation by Andrews (2005) has S = 1.5, $V_r \approx 0.91C_R$ and W/R=0.08. Taking k = 0.4 from Freund (1990, Figure 6.6), equation (14c) underestimates W/R by a factor 2.25. We adopt this correction factor as representative of the effect of the orientation of the background stresses, which was ignored in the derivation of equation (14c).

Using the above results, the damage-related radiation for circular crack-like ruptures can be estimated as

$$d \approx |\Delta c / c| \cdot \tau_0 \cdot V_2 = 2.25 |\Delta c / c| \tau_0 k^2 \pi R^3 /(3S^2)\,. \tag{15}$$

From (12) and (15), the ratio of the standard moment and damage-related source terms for circular crack like rupture is

$$m/d \approx \frac{c}{|\Delta c|} \frac{\Delta\tau}{\tau_0} \frac{S^2}{k^2}\,. \tag{16}$$

Ruptures on faults with uniform stress and uniform frictional properties are predicted to transition to supershear speeds if S is below a critical value (Andrews, 1976). For bilateral crack-like ruptures with initial rupture speed $V_r \approx 0.9C_R$, the critical value is $S \approx$ 0.75 in 2D and $S \approx$ 0.5 in 3D (Dunham, 2007). If $\Delta\tau/\tau_0 \approx 0.1$, $|\Delta c / c| \approx 0.5$, and the initial shear stress is sufficiently high to produce a marginal supershear rupture, S = 0.5, equation (16) predicts the surprising result that $m/d \approx 0.3$. For more typical subshear ruptures S and the ratio m/d will be larger. Assuming for example that S = 1.5, 3 and 5, and using again the same values for the other parameters, give $m/d \approx$ 2.8, 11 and 31, respectively. If the dynamic modulus reduction is 25% or 100%, instead of the assumed 50%, the above estimates should be multiplied or divided by 2, respectively.

For pulse-like ruptures the scaling of equation (14c) still holds provided that R is replaced by the pulse width H (the size of the zone actively slipping at a given time). Equation (16) must therefore be multiplied for pulse-like ruptures by R/H. In addition, a 2/3 factor is required for steady-state pulses, for which H and hence W are approximately constant (see Ben-Zion and Shi, 2005, for examples). Seismological inferences of rise times (e.g., Heaton, 1990) suggest that H is typically much smaller than R. Pulse-like ruptures on a fault with velocity-weakening friction that is inversely proportional at high slip rate to the sliding velocity propagate at subshear speed if S > 1.5 (see Figure D1 of Ampuero and Ben-Zion, 2008). For subshear pulse-like ruptures, m/d < 10 only if the pulse width H is larger than a third of the shortest fault dimension R. This may be a typical situation for geometrically controlled pulses, which are generated by barriers or by the finite depth of the seismogenic zone, as opposed to self-healing pulses generated by frictional weakening.

4. Decomposition of the radiation from the damage process

To understand better the seismic radiation from sources that sustain dynamic changes of elastic moduli, we decompose the integrand in the right term of equation (9) to isotropic and deviatoric terms, and the deviatoric term to double-couple and CLVD components (Knopoff and Randall, 1970). In the material below we neglect the temporal fluctuations of the elastic moduli and strain field in the source region and provide a simple analysis in terms of the final elastic strain and moduli.

We denote the principal components of the final elastic strain in the source region as ε_1, ε_2 and ε_3, ordered such that $|\varepsilon_1 - \bar{\varepsilon}| > |\varepsilon_2 - \bar{\varepsilon}| > |\varepsilon_3 - \bar{\varepsilon}|$ with $\bar{\varepsilon} = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)/3$. The principal components of the damage-related tensor d_{ij} are written for the isotropic case of equation (9) as $d_k = 3\Delta\lambda\bar{\varepsilon} + 2\Delta\mu\varepsilon_k$ with k = 1, 2, 3. Following Knopoff and Randall (1970), the diagonal tensor d_{ij} with the principal components $[d_1, d_2, d_3]$ is decomposed as

$$[d_1, d_2, d_3] = d^V[1,1,1] + d^{DC}[1,-1,0] + d^{CLVD}[1,-1/2,-1/2] \,, \tag{17a}$$

where d^V is the amplitude of the isotropic component given by

$$d^V = (3\Delta\lambda + 2\Delta\mu)\bar{\varepsilon} \,, \tag{17b}$$

d^{DC} is the amplitude of the double-couple component given by

$$d^{DC} = 2\Delta\mu(\varepsilon_3 - \varepsilon_2) \,, \tag{17c}$$

and d^{CLVD} is the amplitude of a compensated linear vector dipole given by

$$d^{CLVD} = -4\Delta\mu(\varepsilon_3 - \bar{\varepsilon}) \,. \tag{17d}$$

The above decomposition of the deviatoric part is not unique (e.g., Julian et al., 1998), but can be used to illustrate the expected properties of the radiation.

As an example of the relative contributions of the above components, we consider strike-slip faulting in an isotropic elastic medium. In this case the intermediate principal stress is vertical, having the lowest absolute deviatoric stress, and $\varepsilon_3 = \bar{\varepsilon}$. The various components of the set (17) are then given by

$$d^V = (\frac{3}{2}\Delta\lambda + \Delta\mu)(\varepsilon_1 + \varepsilon_2) \,, \tag{18a}$$

$$d^{DC} = \Delta\mu(\varepsilon_1 - \varepsilon_2) \,, \tag{18b}$$

$$d^{CLVD} = 0 \,. \tag{18c}$$

This predicts for the damage-related radiation a larger isotropic component than the *DC* component. We note that observed source mechanisms of acoustic emission in laboratory fracturing experiments include considerable isotropic (and also *CLVD*) components (e.g., Stanchits et al., 2006; Thompson et al., 2009). The source mechanism of each emission event is associated with a single microcrack, but the entire set of mechanisms in the approach to brittle instability can be interpreted collectively as reflecting rock damage in the source region of the macroscopic failure zone.

As a numerical example, we assume that the initial Young's modulus and Poisson's ratio are E = 100 GPa and ν = 0.25, respectively, and that the damage process lead to 50% drop in E and 30% increase in ν during the brittle instability. In terms of the Lamé parameters, this corresponds to $\Delta\lambda$ = −5 GPa and $\Delta\mu$ = −21 GPa. Assuming that the principal stress components are σ_1 = 100 MPa, σ_2 = 50 MPa, $\sigma_3 = (\sigma_1 + \sigma_2)/2$, gives ε_1 = 6.875 10^{-4} and ε_2 = 6.25 10^{-5}. The resulting components of the set (18) are d^V = −21.4 MPa, d^{DC} = 13.2 MPa, and d^{CLVD} = 0 MPa. As suggested by the analysis of section 3, these values are of the same order as the assumed background shear stress. Other faulting types with states of strain $\varepsilon_3 \neq \bar{\varepsilon}$ may also have a damage-related radiation with nonzero *CLVD* component. The entire radiation, which includes also the traditional moment source (equations 8 and 9), may still be dominated by the *DC* term in cases (section 3) where $m/d >> 1$.

5. Discussion

We derive and analyze results associated with a seismic representation theorem for the radiation from regions sustaining, during brittle deformation episodes, rapid inelastic strain and changes of elastic moduli. The former produces the traditional moment source, while the latter leads to an additional radiation term that is proportional to the changes of the moduli multiplied by the elastic strains in the source region. This can be understood intuitively by recognizing that rapid changes of the elastic moduli in a given reference strain field (e.g., the one existing right after the brittle deformation event) will change rapidly the elastic strain energy in the source volume by an amount given by the integral of $\varepsilon_{ij}(c^f_{ijkl} - c^i_{ijkl})d\varepsilon_{kl}$, and should therefore modify the radiated energy.

Our analysis ignores the physical processes associated with the evolution of the elastic moduli in the source region before, during and after brittle failure episodes. Reducing the elastic moduli in the source region will absorb strain energy from the bulk (or chemical energy in the case of explosion), and following the brittle instability (or explosion) the material in the source volume will partially heal (or partially reassemble) through processes (e.g., plastic yielding at asperity contacts and mineral deposition) that will again reduce the chemical or strain energy in the bulk. The details of such processes are important for understanding the involved physics and overall energy budget, but they do not affect our basic result that the representation of radiation from sources sustaining brittle instabilities should include a damage-related term as given by (8).

The ratio of the radiation from the moment and damage-related terms scales (section 3) as $m/d \approx (c/|\Delta c|)(\Delta\tau/\tau)(V_1/V_2)$. It is thus seen that, unless the ratio of volumes associated with the moment and damage-related terms is very large, the radiation from the evolving elastic moduli can be a significant portion of the entire field. This can be especially the case during brittle instabilities of low-porosity rocks that may be accompanied by a transient moduli reduction of 50% or more (e.g., Lockner et al., 1977; 1992), and explosions where the dynamic moduli reduction can be essentially complete. On the other hand, increasing moduli in the source regions, as may occur during compaction of high-porosity rocks (e.g., Wong et al., 2002; Baud et al., 2004) will decrease the seismic radiation to the bulk.

The decomposition analysis of section 4 indicates that the damage-related radiation has both double-couple and non-double-couple terms, and that the latter can include a significant isotropic component. Standard derivations of earthquake source parameters based solely on the moment representation map the double-couple component of the damage-related radiation onto slip, and may therefore produce either overestimates (for earthquakes in low-porosity rocks) or underestimates (for earthquakes in high-porosity rocks) of the actual fault slip values. Isotropic components of radiation in source inversions of earthquakes in regular non-extensive (volcanic or geothermal) environments may perhaps be used as diagnostic of material damage.

A significant fraction of the events in the USGS and Harvard moment tensor global catalogs have large CLVD components (Frohlich, 1994). While there are various physical and analysis artifact sources that can produce CLVD components (e.g., Frohlich, 1994; Julian et al., 1998), the routine moment tensor inversions assume pure deviatoric moments, and hence map any physical isotropic radiation onto the CLVD component.

Isotropic radiation is generally not observed in moment tensor inversions of shallow earthquakes, which are essentially low frequency results. However, general moment tensor inversions applied to dense and high quality earthquake data may include isotropic components (e.g., Dufumier and Rivera, 1997; Ford et al., 2009). Moreover, the observed P/S amplitude ratios at high frequencies can be larger than predicted by shear faulting (e.g. Castro et al, 1991) and may reflect isotropic radiation. There are also indications of reduced directivity effects at high frequency waves (e.g., Spudich and Chiou, 2008), which again may reflect isotropic radiation. Nevertheless, distinguishing between damage-related and other possible kinematic sources of isotropic moment components is likely to be difficult. For instance, the earthquakes with large isotropic component studied by Ford et al. (2009, Figure 5) are also compatible with a combination of shear and tensile cracking (Julian et al., 2008, Figure 12), and rupture on non-planar faults should generate some motion in the normal direction to the nominal fault plane (e.g., Castro et al., 1992).

The estimates of the various contributions to radiation that are given in sections 3 and 4 are very approximate and they neglect the dynamic changes of the strain field in the source region as well as the detailed evolution of the elastic moduli. More precise estimates require using a damage rheology framework that can account for the brittle damage process (e.g., Lyakhovsky and Ben-Zion, 2008, and references therein) and performing detailed numerical calculations. This will be the subject of future work.

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