Almost all young stars are found in multiple systems. This suggests that protostellar cores almost always fragment into multiple objects. The observed properties of multiple systems such as their separation distribution and mass ratios provide strong constraints on star formation theories. We review the observed properties of young and old multiple systems and find that the multiplicity of stars changes. Such an evolution is probably due to (a) the dynamical decay of small-N systems and/or (b) the destruction of multiple systems within dense clusters. We review simulations of the fragmentation of rotating and turbulent molecular cores. Such models almost always produce multiple systems, however the properties of those systems do not match observations at all well. Magnetic fields appear to suppress fragmentation, perhaps suggesting that they are not dynamically important in the formation of multiple systems. We finish by discussing possible reasons why theory fails to match observation, and the future prospects for this field.

1. INTRODUCTION.

Correctly predicting the properties of young multiple systems is one of the most challenging tests of any theory of star formation. In this chapter we discuss the current understanding of how dense prestellar cores fragment into multiple stars including brown dwarfs (by ‘stars’ we generally mean both stars and brown dwarfs).

Firstly, we will discuss the important observed properties of both young and old binary systems and the differences between them. Then we will describe the possible origins of the differences between the young and old systems and hopefully convince the reader that almost all stars must form in multiple systems. This initial binary population must form from the fragmentation of star-forming dense molecular cores and so we discuss the observed properties of these cores that may influence their ability to form multiple systems. We will then review the current models of core fragmentation, with an emphasis on turbulence as the mechanism that promotes fragmentation. Finally we will examine why theory currently fails to correctly predict binary properties.

2. THE PROPERTIES OF MULTIPLE SYSTEMS.

There has been an extensive study of binary properties over the past two decades with the modern study often marked as beginning with the detailed survey by Duquennoy and Mayor (1991, hereafter DM91). Multiple systems in the field are by far the best studied due to the availability of local samples whose completeness is easier to estimate, and the properties of the field provide the benchmark against which younger samples are measured.

2.1 Multiple Systems in the field.

2.1.1 Multiplicity fraction. The fraction of field stars in multiple systems is found to be high and increases with increasing primary mass. Many different measures are used to quantify multiplicity which can become very confusing. Most important is the ‘multiplicity frequency’ $f_{\text{mult}} = (B+T+Q+...)/(S+B+T+Q+...)$, where $S$, $B$, $T$ and $Q$ are the numbers of single, binary, triple and quadruple systems respectively, thus $f_{\text{mult}}$ represents the probability that any system is a multiple system (see Reipurth and Zinnecker, 1993).

The raw value of the field G-dwarf multiplicity frequency found by DM91 is 0.49, when corrected for incompleteness this rises to 0.58. However, recent studies using Hipparcos data have shown that it may even be higher than this (Quist and Lindegren, 2000; Söderhjelm, 2000).

It should be noted that the brown dwarf (BD) binary fraction has generally been considered to be much lower than that of stars at $\sim 0.1 - 0.2$ (Bouy et al., 2003; Close et al.,...
2.2 Pre-Main Sequence Multiple Systems.

The properties of pre-main sequence (PMS) multiple systems are much harder to determine than those in the field. We refer the reader to the chapter by Duchêne et al. for a detailed review of the observations of PMS multiple systems and the inherent problems.

The separation distribution of PMS stars also appears different to that in the field with an over-abundance of binaries with separations of a few hundred AU (Mathieu, 1994; Patience et al., 2002; Fig. 1). More specifically, the binary frequency in the separation range \( \sim 100 \) – 1000 AU is a factor of \( \sim 2 \) higher than in the field (Mathieu, 1994; Patience et al., 2002; Duchêne et al., 2004). Extrapolating this increase across the whole separation range implies that \( f_{\text{mult}} \) for PMS stars could be as high as 100%. (It appears that in Taurus the binary frequency is \( \sim 100\% \) for stars > 0.3 \( M_{\odot} \), Leinert et al., 1993; Köhler and Leinert, 1998).

The mass-ratio distribution of PMS stars is similar to the field population. A detailed comparison is not yet possible because low-mass companions to pre-main sequence primaries are very difficult to observe as the available results depend mostly on direct imaging or speckle interferometry, while for main-sequence systems radial-velocity surveys have been done over decades (DM91). Thus, using near-infrared speckle interferometry observations to obtain resolved JHK-photometry for the components of 58 young binary systems, Wyatt et al. (2001) found that the mass-ratio distribution is flat for mass ratios \( q \geq 0.2 \) which is
consistent with random pairing from the IMF, i.e. fragmentation processes rather than common-accretion (Fig. 2).

The eccentricity distribution of PMS stars also is similar to the field, with a thermalised distribution except at low separations where tidal circularisation has occurred rapidly (e.g., Kroupa, 1995b; White and Ghez, 2001). Finally, the ages of components in young multiple systems appear to be very similar (White and Ghez, 2001).

It is currently unclear what the proportion of higher-order multiples is in young systems (see the chapter by Duchêne et al.). We will revisit this question in the next section.

2.3 The evolution of binary properties.

The observations described in Sections 2.1 and 2.2 clearly show that at least the binary fraction and separation distributions evolve significantly between young stellar populations and the field.

Indeed, binary properties are seen to change even within populations in star forming regions. The binary fraction is found to vary between embedded and (older) non-embedded sources in both Taurus and ρ Oph (Duchêne et al., 2004; Haïsch et al., 2004). Also, the mass ratio distributions of massive stars appear to depend on the age of the cluster with those in young clusters being consistent with random sampling from the IMF and those in dynamically evolved populations favouring equal-mass companions (Section 2.4).

The evolution of binary properties has been ascribed to two mechanisms. Firstly, the rapid dynamical decay of young small-N clusters within cluster cores (e.g., Reipurth and Clarke, 2001; Sterzik and Durisen, 1998, 2003; Durisen et al., 2001; Hubber and Whitworth, 2005; Goodwin and Kroupa, 2005; Umbreit et al., 2005), and secondly, the dynamical destruction of multiples by interactions in a clustered environment can modify an initial PMS-like distribution into a field-like distribution (Kroupa, 1995a, b; Kroupa et al., 2003; Figs. 1, 3).

2.3.1 Small-N decay. Multiple systems containing \( N \geq 3 \) stars are unstable to dynamical decay unless they form in a strongly hierarchical configuration (stability criteria for \( N \geq 2 \) systems are provided by Eggleton and Kiseleva, 1995). Generally, a triple system is unstable to decay with a half-life of

\[
t_{\text{decay}} = 14 \left( \frac{R}{\text{AU}} \right)^{3/2} \left( \frac{M_{\text{stars}}}{M_\odot} \right)^{-1/2} \text{ yrs}
\]

(1)

where \( R \) is the size of the system, and \( M_{\text{stars}} \) is the mass of the components (Anosova, 1986). The decay time for \( R = 250 \text{ AU} \) and \( M_{\text{stars}} = 1M_\odot \) is \( \sim 55 \) kyr which is of order the duration of the embedded phases of young stars, thus ejections should mainly occur during the main Class 0 accretion phase of PMS objects (e.g., Reipurth, 2000). Indeed, one such early dynamical decay appears to have been observed by Gómez et al. (2006), and this is probably

the process at work to reduce the binary fraction between the embedded and non-embedded stars seen by Duchêne et al. (2004) and Haïsch et al. (2004). These early dynamical processes cause embedded protostars to be ejected from their natal envelopes, possibly causing abrupt transitions of objects from class 0/I to class II/III (Reipurth, 2000; Goodwin and Kroupa, 2005).

Significant numbers of small-N decays will dilute any initial high multiplicity fraction very rapidly to a small binary fraction for the whole population (e.g., \( N = 5 \) systems would lead to a population with a binary fraction \( f = 1/5 \) within \( < 10^5 \) yr). The observed high binary fraction in about 1 Myr old populations thus suggests that the formation of \( N > 2 \) systems is the exception rather than the rule (Goodwin and Kroupa, 2005). Ejections would occur mostly during the very early Class 0 stage such that ejected embryos later appear as free-floating single very low-mass stars and BDs (Reipurth and Clarke, 2001). However, the small ratio of the number of BDs per star, \( \approx 0.25 \) (Munech et al., 2002; Kroupa et al., 2003; Kroupa and Bouvier, 2003b; Luhman, 2004), again suggests this not to be a very common process even if all BDs form from ejections.

Ejections have two main consequences: a significant reduction in the semi-major axis of the remaining stars (Anosova, 1986; Reipurth, 2000; Umbreit et al., 2005), and the preferential ejection of the lowest mass component (Anosova, 1986; Sterzik and Durisen, 2003). The early ejection of the lowest-mass component forms the basis of the embryo ejection scenario of BD formation (Reipurth and Clarke, 2001; Bate et al., 2002).

The \( N \)-body statistics of the decay of small-N systems has been studied by a number of authors (Anosova, 1986; Sterzik and Durisen, 1998, 2003; Durisen et al., 2001; Goodwin et al., 2005; Hubber and Whitworth, 2005). However, only Umbreit et al. (2005) have attempted to include the effects of accretion on the \( N \)-body dynamics which appear to have a significant effect - especially on the degree of hardening of the binary after ejection. Goodwin et al. (2004a, b) and Delgado Donate et al. (2004a, b) have simulated ensembles of cores including the full hydrodynamics of star formation, however proper statistical conclusions about the effects of ejections are difficult to draw due to the different numbers of stars forming in each ensemble (which there is no way of controlling a priori), and the smaller number of ensembles that may be run in a fully hydrodynamic context. However, some conclusions appear from these and other studies (Whitworth et al., 1995; Bate and Bonnell, 1997; Bate et al., 2003; Delgado Donate et al., 2003).

Firstly, that early ejections are very effective at hardening the remaining stars (c.f. Umbreit et al., 2005). Secondly, this early hardening tends to push the mass ratios of close binaries towards unity. This occurs as the low-mass component has a higher specific angular momentum than the primary and so is more able to accrete mass from the high angular momentum circumstellar material (see Whitworth et al., 1995; Bate and Bonnell, 1997). However, Ochi et al. (2005) find in detailed 2D simulations of accretion onto bi-
naries that the gas accretes mainly onto the primary due to shocks removing angular momentum.

One significant caveat to the previous discussion is that the gradual formation (over ~ 0.1 Myr) of stars allows far more stable triples and higher-order multiples to form than expected observationally, which are stable for at least 10 Myr (Delgado Donate et al., 2004a, b). Indeed, simulations that form a large number of stars often form very hierarchical higher-order multiples (often quadruples and quintuples formed when even larger systems decay) which are not observed (Delgado Donate et al., 2003, 2004a, b; Goodwin et al., 2004a, b). Such systems would probably be destroyed during the cluster destruction phase (see below), but not dilute the binary fraction on very short timescales.

2.3.2 Dynamical destruction in clusters. In the highly-clustered environments in which most stars are thought to form (e.g., Lada and Lada, 2003) dynamical interactions will be common and may disrupt many initially binary systems.

Binaries can be sub-divided into three dynamical groups: (i) the wide, or soft, binaries, (ii) the dynamically active binaries, and (iii) the tight or hard binaries.

Wide binaries have orbital velocities much smaller than the velocity dispersion, σ, in a cluster and are easily disrupted. This is best seen by a gedanken experiment, where we construct a reduced-mass particle (a test particle orbiting in a fixed potential with total mass, eccentricity and orbital period equal to that of the binary in question) in a heat bath of field particles (the cluster stars). If the orbital velocity of the test particle is smaller than the typical velocities of the field particles (v_{orb} \ll σ), then the test particle will gain kinetic energy by encounters, i.e. by the principle of energy equipartition, until its orbital velocity surpasses the binding energy of the binary. The binary consequently gets disrupted. Energy conservation requires the heat bath to cool; cluster cooling has been seen in N-body computations by Kroupa et al., (1999), but the effect is not significant for cluster dynamics. The general effect of this process is that binaries with weak binding energies are disrupted (i.e., binaries with long periods and/or small mass-ratios).

Hard binaries, on the other hand, can be represented by a reduced-mass binary in which the test particle has v_{orb} \gg σ, so that energy equipartition leads to a reduction of v_{orb} and to an increase of σ (cluster heating). This increases the binding energy of the binary which heats up further (v_{orb} increases as the test particle falls towards the potential minimum). This run-away process only stops because either the binary merges when it is so tight that the constituent stars touch (forming a blue straggler), or because the hardened binary receives a re-coil expelling it from the cluster, or the hardening binary evolves to a cross-section so small that the binary becomes essentially unresolved in further interactions.

Heggie (1975) and Hills (1975) studied the details of these processes and formulated the Heggie–Hills law of binary evolution in clusters: “soft binaries soften and hard binaries harden”. An important implication of this law is that hard binaries can absorb the entire binding energy of a cluster and drive the evolution of the core of a massive cluster.

Not accessible to analytical work are active binaries with intermediate binding energies. Only full-scale N-body computations can treat the dynamics of the interactions accurately (e.g., Heggie et al., 1996). Such binaries couple efficiently to the cluster, and efficiently exchange energy with it. The binary–binary and binary–single-star interactions form complex resonances and short-lived higher-order configurations that decay by expelling typically the least massive member. Active binaries are thus quite efficient in exchanging partners, but more work needs to be done in order to quantify the exchange rates for typical Galactic star-forming clusters.

The dynamical interactions between binaries or single stars will continue to alter the binary properties of the population as long as it remains relatively dense (i.e. until the cluster dissolves or expands significantly after residual gas removal). In a series of papers (Kroupa, 1995a, b; Kroupa et al., 2001) it has been shown that a population initially composed entirely of binary systems with a PMS binary separation distribution can evolve into the field-like distribution through dynamical encounters, as is exemplified by the evolution of the mass-ratio distribution (Fig. 2).

Importantly, however, dynamical interactions in a cluster cannot form a significant number of binaries from an initially single star population, or widen an initially narrow separation distribution (Kroupa and Burkert, 2001). Also, the clusters within which most Galactic-field stars form do not sufficiently harden an initially wide separation distribution to be consistent with the number of tight binaries. Such clusters that would lead to significant hardening would disrupt too many of the wide binaries diluting the Galactic-field binary-star population.

This indicates that the observed broad period distribution (Fig. 1) is already imprinted at the time of binary formation. The existing N-body simulations of young clusters tend to assume a relatively well-mixed and relaxed initial distribution of stars. However, real young clusters tend to be lumpy and unrelaxed which alters the binary-binary interaction rate (e.g., Goodwin and Whitworth, 2004), but is not expected to change the general outcome (a smaller-N system with a smaller radius is dynamically equivalent to a larger-N system with a larger radius, Kroupa, 1995a).

2.3.3 The effect of dynamics on the binary population. Small-N decay will act to modify the birth binary population (i.e. that produced by star formation) by reducing the overall binary fraction and by hardening the remaining binaries on a timescale of < 10^5 yrs. Small-N decay cannot occur very often as it would produce too many single PMS stars and too many hard binaries (and quite possibly too many BDs). However, it is unclear if small-N decay is rare because cores usually form only binaries and only sometimes triples, or because higher order multiples are formed in stable, hierarchical systems.

The initial binary population (i.e. the population af-
Fig. 2.— The mass-ratio distribution of all late-type primary stars. The solid dots are observational data by Reid and Gizis (1997), while the expected initial distribution is shown as the dashed histogram. In a typical star cluster it evolves to the solid histogram which reproduces the observed data quite well (from Kroupa et al., 2003).

After small-\(N\) decay and internal energy re-distribution processes, “eigenevolution”-see below, has modified it) is further changed by dynamical interactions within a cluster on a timescale of a Myr for typical Galactic star-forming clusters (Kroupa, 1995a, b; Adams and Myers, 2001). Encounters will destroy soft and active binaries leaving mostly the hard binary population unchanged. Crucially it cannot produce more binaries in any significant numbers.

Thus both of the processes that act to modify the birth binary population into the field binary population reduce the binary fraction. We are led to the conclusion that the birth binary fraction must be higher than that of the field. This conclusions agrees well with observations.

Goodwin and Kroupa (2005) argue that the birth binary fraction must be almost unity for all stars, and that the low binary fraction amongst M-dwarfs is due to the preferential destruction of low-mass binaries. Lada (2006), however, argues that most stars form as single stars, as most M dwarfs are single and most stars are M dwarfs. The crucial issue here is how many initial M dwarf binaries decay? We note that in a model that assumes that stars are born with a 60 % binary fraction with companion masses selected randomly from the IMF and without dynamical dissolution of the binaries leads to a population with the observed binary fraction-spectral type relationship (Kroupa et al., 1993, their Fig. 11). In models that assume a 100 % initial binary fraction, processing through an ‘average’ cluster (Lada and Lada, 2003) also converts the binary population into the observed field population. Observations of the binarity of a very young (embedded?) M star population in an average cluster (as opposed to Taurus which is atypical) are required to fully resolve this issue. However, for higher-mass stars, it seems almost certain that the initial binary fraction must be significantly higher than the field binary fraction due to their current relatively high binary fraction. It is worth noting that the observed population of high-order multiples may be formed from the final handful of stars that remain at the end of cluster dissolution which tend to be in high-order hierarchical systems (see Goodwin and Kroupa, 2005).

Thus, whilst more work needs to be done, especially on the effects of the cluster density and on the initial distribution in star-forming regions with very different physical properties, a picture has emerged in which the observed high multiplicity PMS population can be modified by secular dynamical evolution to produce the field population. Most (especially K-dwarfs and later) stars therefore form in multiple - probably binary and triple - systems with a very wide separation distribution and a relatively flat mass-ratio distribution.

2.4 The initial binary population.

Given these results, a useful working hypothesis appears to be that the initial binary-star properties are invariant to star-formation conditions. The observed differences between binary populations result from different secular dynamical histories of the respective populations: ie. due to different cluster masses and densities.
Kroupa (1995b) suggests that the field binary properties can be understood if the birth binary population has the following semi-empirical distribution functions:

1. Companion masses are chosen randomly from the IMF;
2. The distribution of periods is independent of primary mass, and can be described with the following functional form,

\[ f_{lP} = \frac{lP - 1}{45 + (lP - 1)^2}, \]  

where \( dN_{lP} = N_{\text{tot}} f_{lP} dlP \) is the number of binaries with periods in the range \( lP, lP + dlP \) (\( lP = \log_{10} P, P \) in days) and \( N_{\text{tot}} \) is the number of single-star and binary-star systems in the sample under consideration;
3. The eccentricity distribution is thermal (all binding energies are equally occupied),

\[ dN = f_o f(e) de = f_o 2 e de, \]  

being the number of binary systems with eccentricity in the range \( e, e + de \).

These birth distributions need to be modified for short-period binaries (\( lP \lesssim 3 \)) through the evolution of the binding energy and angular momentum owing to dissipative processes within the young binary system termed collectively as pre-main sequence eigenevolution. This then gives the initial distributions which are evident in dynamically unevolved populations (e.g., Taurus, Kroupa and Bouvier, 2003a) and can be used as the initial binary-star population in \( N \)-body modelling of stellar populations.

The distribution over binding energies and specific-angular momenta can be evaluated readily given the above distribution functions. Fig. 4 shows that the distribution of specific angular momenta of molecular cloud cores forms a natural extension to the distribution of specific angular momenta of the initial binary stellar population, possibly suggesting an evolutionary connection.

These semi-empirical distribution functions have been formulated for late-type stars (primary mass \( m_p \lesssim 1 M_\odot \)) as it is for these that we have the best observational constraints. It is not clear yet if they are also applicable to massive binaries. Baines et al. (2006) report a very high (\( f \approx 0.7 \pm 0.1 \)) binary fraction among Herbig Ae/Be stars with the binary fraction increasing with increasing primary mass. Furthermore, they find that the circumbinary discs and the companions appear to be co-planar thereby supporting a fragmentation origin rather than collisions or capture as the origin of massive binaries. Most O stars are believed to exist as short-period binaries with \( q \approx 1 \) (García and Mermilliod, 2001), at least in rich clusters, while small \( q \) appear to be favoured in less substantial clusters such as the Orion Nebula Cluster (ONC), being consistent there with random pairing (Preibisch et al., 1999). Kouwenhoven et al. (2005) report the A and late-type B binaries in the Scorpius OB2 association to have a mass-ratio distribution not consistent with random pairing. The lower limit on the binary fraction is 0.52. Perhaps the massive binaries in the ONC represent the primordial population, whereas in rich clusters and in OB associations the population has already dynamically evolved through hardening and companion exchanges to that observed there. This possibility needs to be investigated using high-precision \( N \)-body computations of young star clusters.

Given such reasonably-well quantified estimates of the distribution functions of orbital elements of the primordial binary population, the problem remains as to how these distributions functions can be understood theoretically as a result of the star-formation process. Fisher (2004) notes that distribution functions similar to the ones derived above, and in particular a wide mass-ratio distribution, a very wide period range, and a thermal eccentricity distribution, are obtained quite naturally from a turbulent molecular cloud (see also Burkert and Bodenheimer, 2000).

3. THE PROPERTIES OF PRESTELLAR CORES.

The gas that is just-about-ready to form stars arranges itself into denser structures often called prestellar ‘cores’ (e.g., Myers and Benson, 1983). Often, a ‘typical’ prestellar core is described as having a radius \( \sim 0.1 \) pc, density \( \gtrsim 10^4 \) cm\(^{-3} \), and velocity dispersion \( \sim 0.5 \) km s\(^{-1} \). In fact though, the idea that such cores are ‘typical’ primarily arises from the relative ease with which nearby, isolated dense cores, that will each form fewer than a handful of stars, can be observed and modelled. It is in fact likely that accounting for the diversity in core properties is crucial to improving the match between theory and observations of the conversion of gas to (binary) stars.

3.1 What is a ‘core’?
In general, observations and theory have concentrated on isolated and coherent prestellar cores such as those found in low-mass star forming regions such as Taurus (due to the relative ease of observing such cores). It is not yet clear if legitimate analogs to these cores exist within the dense concentrations of gas that form the clusters (e.g., Goodman et al., 1998). In particular, the \(~ 0.1 \) pc size of isolated cores would result in them having multiple dynamical encounters in the dense environment that forms ‘typical’ clusters such as Orion (e.g., Lada and Lada, 2003).

That we have not observed any \(0.1\) pc core analogues in dense clusters is not surprising. Even in very nearby clusters like NGC1333 in Perseus (at \( \sim 300 \) pc), \(0.1\) pc is 1 arcmin, typical of single-dish resolution for tracers like \(\text{NH}_3\) and \(\text{N}_2\text{H}^+\), which map out gas with density \(\gtrsim 10^4\) (e.g., Benson and Myers, 1989; Evans, 1999). Thus, to find meaningful dense structures on scales significantly less than 1 arcmin, interferometry is required. Interferometers have definitively revealed sub-structure in the gas within clusters, but this substructure does not offer a one-to-one gas clump-to-star match the way observations of isolated cores do. Instead, regions forming many stars are associated with more dense gas than those forming fewer. This lack of one-to-one correspondence suggests that long-lived blobs associated with the formation of individual cores within clusters do not exist.

Given this, is it reasonable to extend observations and simulations of isolated cores to more typical clustered star forming ‘cores’? The answer is possibly. Whilst large isolated cores cannot exist in clustered star forming regions, the regions in which stars are thought to form are far smaller than the size of a whole core. Observationally, the size of binary systems is <a few hundred AU in agreement with theoretical expectations of the scale of fragmentation (see Section 4.1). Systems of this scale would be expected to interact on timescales of > Myr in a typical cluster which is significantly longer than the star formation timescale is thought to be (see the chapters by Di Francesco et al. and Ward-Thompson et al.). So, while the details of the first stages of collapse in isolated cores are probably not applicable to most star formation, the details of the final stages of fragmentation and star formation occurring on few hundred AU scales quite possibly occur in relative isolation. However, continued accretion onto cores may significantly effect the evolution of the inner proto-stellar system depending on the details and timescale of core and star formation in clusters.

With this in mind, we will continue to review the properties of isolated pre-stellar cores.

### 3.2 Rotation.

Clearly, for cores to fragment some angular momentum must be present otherwise the cores will collapse onto a single, central point. The simplest source of this angular momentum is due to bulk rotation of the core. It is a relatively straightforward procedure to estimate the component of solid-body rotation present in a dense core by fitting for the gradient in observed line-of-sight velocity over the face of a core (e.g., Goodman et al., 1993; Barranco & Goodman, 1998; Caselli et al., 2002). The results of this fitting (see Fig. 4) have been used as input values of ‘initial angular momentum’ in many calculations. While the estimates of the component of solid-body rotation made in this way are sound, and are thus fine to use as inputs, it is important to appreciate that cores do not really rotate as solid-bodies (Burkert and Bodenheimer, 2000). When the velocity measurements are put into the context of measurements of velocities on larger scales, both observations (Schnee et al., 2005) and simulations (Burkert and Bodenheimer, 2000), show that the “rotation” is often just an artifact created by larger-scale turbulent motions.

Fig. 4 gives a summary of the measured specific angular momentum for core rotation for the 29 dense cores from Goodman et al. (1993) which show significant rotation. The majority of cores included in the Goodman et al. (1993) study are isolated, low-mass cores: one should keep in mind that the rotational properties of smaller fragments that may form inside those cores as true(r) precursors to protostars remain largely unmeasured.

#### 3.3 Non-thermal line widths.

It has been known for many years (Larson, 1981; Myers, 1983; Solomon et al., 1987; see also Elmegreen and Scalo, 2004a and references therein) that the line widths inside even the most quiescent of dense cores are more than thermal. The coldest isolated dense cores have gas temperatures of order 10 K, and dust temperatures as low as 6 K (see the chapters by Di Francesco et al. and Ward-Thompson et al.). A gas temperature of 10 K implies an H\(_2\) 1\r velocity dispersion of only 0.2 km s\(^{-1}\). Observed dispersions have a distribution from \(\sim 0.2\) to 1 km s\(^{-1}\) with a peak at \(\sim 0.4\) km s\(^{-1}\), never quite reaching down to the thermal value (e.g., see the catalogue of Jijina et al., 1999).

The origin of the non-thermal line width in dense cores is the subject of an extensive literature, but it is fair to say that a consensus exists that ‘turbulence’ is responsible (see e.g., Myers, 1983; Barranco & Goodman, 1998; Goodman et al., 1998; Elmegreen and Scalo, 2004a). Significant levels of turbulence in cores are important for fragmentation as they may provide the angular momentum to form multiple systems (see Section 4.2.2) and, as mentioned above, could also be responsible for the observed rotation.

#### 3.4 Magnetic fields.

The role of magnetic fields in supporting cores against collapse is a subject of much debate. Magnetic fields are not thought to be dynamically dominant in cores as was once thought (e.g., Shu et al., 1987; or indeed in the ISM as a whole Elmegreen and Scalo, 2004a). However, they may be very important as the role of magnetic fields in the fragmentation of cores is poorly understood (see Section 4.2.6).
Isolated cores are found to be statistically triaxial with a tendency towards being prolate (Jones et al., 2001; Goodwin et al., 2002). In contrast to this, magnetic support would tend to produce oblate cores. In addition these magnetically-supported oblate cores would tend to rotate around their short axis which is not observed (Goodman et al., 1993). This is supported by observations of the magnetic field which show that cores are not magnetically critical (Crutcher et al., 1999; see Bourke and Goodman, 2004 and references therein).

4. THE FRAGMENTATION OF PRESTELLAR CORES.

4.1 The Physics of Collapsing Cores.

During the early stages of the collapse of a prestellar core, the rate of compressional heating is low and the gas is able to cool radiatively, either by molecular line emission, or, when $\rho > 10^{-19}$ g cm$^{-3}$, by thermally coupling to the dust. The gas is therefore approximately isothermal (at $\sim 10$ K) with an equation of state $P \propto \rho$.

Eventually the rate of compressional heating becomes so high (due to the acceleration of the collapse), and the rate of radiative cooling so low (due to the increasing column density and dust optical depth), that the gas switches to being approximately adiabatic, with $P \propto \rho^\gamma$ (where $\gamma = 5/3$ initially for a monatomic gas, and then $\gamma = 7/5$ above $\sim 300$ K when $H_2$ becomes rotationally excited).

This behaviour has been studied in detail (Larson, 1969; Tohline, 1982; Masunaga et al., 1998; Masunaga and Inutsuka, 2000) for cores in the range $1 - 10M_\odot$ with an initial temperature of 10 K. These authors find that the switch between isothermality and adiabaticity occurs at a critical density of $\rho_{\text{crit}} \sim 10^{-13}$ g cm$^{-3}$. (See Fig. 1 in Bate 1998 or Fig. 2 in Tohline, 2002). Thus, as contraction proceeds and the density, $\rho$, increases the Jeans mass, $M_J \propto \rho^{-1/2} T^{3/2}$, decreases as long as the gas can retain the same temperature, $T$, while it is optically thin. Once the opacity increases such that the gas core heats up, $M_J$ increases. The most important result of this thermal behaviour is therefore that there is a minimum Jeans mass that is reached at $\sim \rho_{\text{crit}}$ of order $M_{\text{min}} \sim 10^{-2}M_\odot$. This is often referred to as the opacity limit for fragmentation.

There is an even lower minimum mass that occurs during a later isothermal phase at $\rho \sim 10^{-3}$ g cm$^{-3}$ when molecular hydrogen dissociates at a few thousand K. It is possible that a further fragmentation episode can occur at these densities which may account for some close binaries.

Fragmentation in cores is expected to occur at around $\rho_{\text{crit}}$ as at lower or higher densities the Jeans mass increases (although how rapidly it rises above $\rho_{\text{crit}}$ does depend sensitively on the $\gamma$ used in the adiabatic equation of state). Thus we expect multiple systems to be formed with a typical length scale $R_{\text{form}}$ of

$$R_{\text{form}} \lesssim \left( \frac{3M_{\text{core}}}{4\pi\rho_{\text{crit}}} \right)^{1/3} \sim 125 \left( \frac{M_{\text{core}}}{M_\odot} \right)^{1/3} \text{AU} \quad (4)$$

where $M_{\text{core}}$ is the mass of the core. Interestingly, this scale matches the observed peak in the T Tauri separation distribution (see also Sterzik et al., 2003).

There is a minimum separation in this picture of $\sim 30$ AU which is the separation of two fragments of $M_{\text{min}}$ at $\rho_{\text{crit}}$. It may - or may not - be significant that this is the average binary separation (DM91; see also Sterzik et al., 2003). However, it would appear difficult to form binaries closer than $\sim 20 - 30$ AU without some hardening mechanism or a secondary fragmentation phase.

It should be noted that the length scales of star formation of less than a few hundred AU are several orders of magnitude smaller than the thousands of AU scales on which core properties have been observed.

4.2 Fragmentation mechanisms.

In this section we examine the main mechanisms that have been proposed to explain multiple formation. Given the complex and highly non-linear nature of the physics in most models, numerical simulations are the main route by which the mechanisms for fragmentation have been investigated. Bulk rotation and turbulence are the two main mechanisms that have been considered to provide the angular momentum required for fragmentation to occur and we review the theoretical work and simulations conducted on both of these mechanisms. In addition, we discuss the possible role of magnetic fields, disc fragmentation and ‘secondary fragmentation’.

4.2.1 Rotational fragmentation. The simplest situation in which fragmentation may well occur is in a spherical cloud with solid-body rotation and an isothermal equation of state. Tohline (1981), using semi-analytic arguments concluded that all such clouds should fragment. A number of simulations have shown that such clouds do fragment if $\alpha_{\text{therm}}/\beta_{\text{rot}} \lesssim 0.12 - 0.15$, where $\alpha_{\text{therm}} = E_{\text{therm}}/|\Omega|$ is the initial thermal virial ratio (where $E_{\text{therm}}$ is the thermal kinetic and $\Omega$ is the gravitational potential energy), and $\beta_{\text{rot}} = E_{\text{rot}}/|\Omega|$ the initial rotational virial ratio (where $E_{\text{rot}}$ is the rotational kinetic energy) (Miyama et al., 1984; Hachisu and Eriguchi, 1984, 1985; Miyama, 1992; see also Tsuribe and Inutsuka, 1999a, b; Tohline, 2002).

Boss and Bodenheimer (1979) added an $m = 2$ azimuthal density perturbation to a standard rotating cloud (effectively creating an elongated cloud more similar to those observed than purely spherical clouds; see Section 3.4). They found that with a perturbation of amplitude $A = 0.5$ the cloud fragments into a binary system. This simulation was repeated by Burkert and Bodenheimer (1993) who also found that when $A = 0.1$ a filament connecting the two components of the binary fragments into several smaller fragments. However the connecting filament should not fragment as predicted by Inutsuka and
Miyama (1992) and demonstrated by Truelove et al. (1997). Indeed the ’Boss and Bodenheimer test’ has become a standard test for the accuracy of codes (e.g., Truelove et al., 1997 for adaptive mesh refinement - AMR - and Kitsionias and Whitworth, 2002 for smoothed particle hydrodynamics - SPH). However, it is a rather unsatisfactory test as, whilst the Truelove et al. simulations are generally considered to have converged, no analytic solution to the problem exists. An alternative test based on the original analysis of Jeans is presented by Hubber et al. (2006).

The simulation of rotating clouds can be made more physical by including an adiabatic (e.g., Tohline, 1981; Miyama, 1992) or barotropic (e.g., Bonnell, 1994; Bate and Burkert, 1997; Boss et al., 2000; Cha and Whitworth, 2003) equation of state (eos). Bate and Burkert (1997) showed that the Boss and Bodenheimer test does produce a line of fragments with a barotropic eos, but not if it remains isothermal. In addition, Boss et al. (2000) simulated a cloud with an $n = 2$, $\Delta = 0.1$ perturbation using a barotropic eos and also with radiation transport; the second case producing a binary whilst the first did not: despite the similarity of the pressure-temperature relations. Both of these results suggest that fragmentation is highly sensitive to thermal inertia and radiation transport effects.

Other authors have modified the initial conditions to include effects such as different density profiles (e.g., Myhill and Kaula, 1992; Burkert et al., 1997; Boss, 1996; Boss and Myhill, 1995; Burkert and Bodenheimer, 1996; Boss et al., 2000; Boss, 1993), differential rotation (which tends to promote fragmentation: Myhill and Kaula, 1992; Boss and Myhill, 1995; Cha and Whitworth, 2003) and non-spherical shapes (e.g., Bastien, 1983; Bonnell and Bastien, 1991; Bonnell et al., 1991; Nelson and Papaloizou, 1993; Boss, 1993; Sigalotti and Klapp, 1997). The effect increasing external pressure on the collapse of rotating cores have been investigated by Hennebelle et al. (2003, 2004, 2006).

4.2.2 Turbulent fragmentation. Recently a picture of star formation as a rapid and highly dynamic process has appeared (e.g., Elmegreen, 2000; Vázquez-Semadeni et al., 2000; Larson, 2003; Elmegreen and Scalo, 2004b) as opposed to a quasi-static process (e.g., Shu et al., 1987). In particular, the idea of cores evolving slowly via ambipolar diffusion (e.g., Basu and Mouschovias, 1994, 1995a, b; Ciolek and Mouschovias, 1993, 1994, 1995; Ciolek and Basu, 2000) has been replaced by one in which cores form in converging flows in a highly turbulent molecular cloud. This is rather good news for fragmentation, as the main effects of a quasi-static evolution are to delay fragmentation and reduce the angular momentum and turbulence in a core and organise material so that its collapse is well focused onto a central point (see also Section 3.2). Simulations of core formation in a turbulent medium suggest that cores form with significant amounts of turbulence. Turbulent, rapidly formed cores also reproduce many of the observed properties of cores (e.g., Burkert and Bodenheimer, 2000; Ballesteros-Paredes et al., 2003; Jappsen and Klessen, 2004) and have a mass spectrum not dissimilar to the observed core mass spectrum (e.g., Padoan and Nordlund, 2002, 2004; Klessen et al., 2005).

Simulations of the effects of turbulence in cores focus on two different regimes: high-velocity (Bate et al., 2002, 2003; Bate and Bonnell, 2005; Delgado Donate et al., 2004a, b), and low-velocity (Goodwin et al., 2004a, b). Also see Fisher (2004) for a semi-analytic approach to multiple formation with turbulence. The level of turbulence is usually quantified as a turbulent virial ratio $\alpha_{\text{turb}} = E_{\text{turb}}/|\Omega|$, where $E_{\text{turb}}$ is the kinetic energy in turbulent motions and $|\Omega|$ is the gravitational potential energy (note - not any rotational property). Highly turbulent simulations focus on $\alpha_{\text{turb}} = 1$ in $50M_\odot$ (Bate et al., 2002, 2003; Bate and Bonnell, 2005) and $5M_\odot$ (Delgado Donate et al., 2004a, b) cores. Simulations of slightly turbulent cores range between $\alpha_{\text{turb}} = 0 - 0.25$ in $5.4M_\odot$ cores (Goodwin et al., 2004a, b). In all of these simulations turbulent motions are modelled using a Gaussian divergence-free random velocity field $P(k) \propto k^{-n}$ where $n$ is usually taken to be 4 to match observations of cores for which $n = 3$–4 provides a good fit to the Larson relations (Burkert and Bodenheimer, 2000). It should be noted that the random chaotic effects introduced by variations in the initial turbulent velocity field can be very important. Therefore a statistical approach is desirable utilising large ensembles of simulations (e.g., Larson, 2002).

These simulations of turbulence are different to those of turbulence in molecular clouds which concentrate on the formation of dense cores and massive stars (e.g., Klessen and Burkert, 2000). This is largely due to computational limitations which do not allow the resolution of the opacity limit for fragmentation in the large-scale context of giant molecular clouds. A mass resolution of $\sim 10^{-2} M_\odot$ is required to resolve the opacity limit for fragmentation in SPH. In AMR the problem is even worse as the Jeans length continues to fall (albeit more slowly) after the minimum Jeans mass is reached and codes must resolve few AU scales to capture the lowest-mass fragments.

Even very low levels of turbulence ($\alpha_{\text{turb}} \sim 0.025$) are enough to allow cores to fragment: that is, for most cores in an ensemble to form more than one star (Goodwin et al., 2004b). As the level of turbulence is increased, the average number of stars that form in a core increases (Goodwin et al., 2004b). It has been suggested that approximately one star forms per initial Jeans mass: $\sim 1M_\odot$ for these initial temperatures and densities (cf. Bate et al., 2003; Delgado Donate et al., 2004a). This seems to hold in highly-turbulent cores, however the number of stars forming falls with decreasing turbulence (Goodwin et al., 2004b) and so this at best probably only represents a (statistical) asymptotic behaviour.

In highly-turbulent cores, the supersonic turbulent velocity field creates a number of condensations in shocked, converging regions which become Jeans unstable and collapse (see Bate et al., 2003; Delgado Donate et al., 2004a, b). However, it is unclear if this mode of fragmentation is realistic in small (certainly $< 1$ few $M_\odot$) cores as the
observed levels of non-thermal motions rule-out significant highly supersonic turbulence in these cores.

In Fig. 5 we show the formation of a fragment in a mildly-turbulent $5.4 M_\odot$ core with $\alpha_{\text{turb}} = 0.05$ based on observations of the isolated core L1544 (from Goodwin et al., 2004a). Fragmentation occurs in a ‘disc-like’ mode in circumstellar accretion regions (we avoid the use of ‘disc’ to describe these regions as they are not rotationally supported structures) (CARs) which form around the first star. CARs are highly unstable structures as there is non-uniform (in space, time and angular momentum) inflow onto them. Complex spiral instabilities form in the CAR due to this inhomogeneous infall of material. We note that these instabilities are seen in both SPH and AMR simulations of the same situation (Gawryszczak et al., 2006; see also Section 4.4). Fragmentation occurs if the density in spiral waves becomes high enough that the Jeans length falls to the typical width of a spiral wave and the collapse time falls to a low enough fraction of the local rotation period that it may escape shredding by differential rotation. In these simulations, fragmentation occurs for some (but not all) regions that exceed $\sim 10^{-12}$ g cm$^{-3}$ in density (equating to a Jeans length of $\sim 20$ AU) beyond $\sim 50 - 100$ AU from the central star. We note that the highly unstable nature of CARs makes usual applications of instability criteria such as the Toomre Q-parameter impossible.

Such a mode of fragmentation is highly sensitive to the equation of state that has been adopted. It has been found that the number of fragments that form increases if $\gamma$ is changed from 5/3 to 7/5 (Goodwin et al., in prep.). This is due to the sensitivity of the Jeans length with density and so to the ease with which fragmentation can occur in CARs.

The process of fragmentation in CARs is highly chaotic, relying as it does on a certain degree of ‘luck’ in being able to reach a high-enough density and avoiding shredding whilst collapsing. Thus it is no surprise that anywhere between 1 and 12 stars form in each core depending entirely on the details of the initial turbulent velocity field (Goodwin et al., 2004a, b).

In summary, it is found that turbulent cores generally fragment into several stars: approximately one per initial Jeans mass ($\sim 1 M_\odot$) in the core. The number of stars that form increases with increasing turbulence and is also highly sensitive to the details of the turbulent velocity field. However, only relatively high-mass cores ($> 5 M_\odot$) have been investigated in turbulent simulations so far. The effect of turbulence in lower-mass cores must be investigated, as lower-mass cores appear to dominate the core mass function (Motte et al., 1998; Testi and Sargent, 1998; Motte et al., 2001).

4.2.3 Disc fragmentation. Disc fragmentation is a mechanism by which low-mass stars and BDs may be formed. In the dense environments of clusters close encounters between stars can disturb the circumstellar discs promoting instabilities which can lead to the fragmentation of otherwise stable discs. (Note that this is rather different to the turbulent disc-like scenario described above as these proto-planetary discs are much less massive than CARs and are also stable, rotationally supported discs as opposed to CARs).

In a series of papers, Boffin et al. (1998) and Watkins (1998a,b) found that most star-disc interactions will lead to gravitational instabilities which form new low-mass companions. These simulations generally considered massive discs where $M_{\text{star}} = M_{\text{disc}} = 0.5 M_\odot$. Bate et al. (2003) find that star-disc encounters play an important role in forming binaries and also truncating discs. Star-disc encounters are also thought to play an important role in forming angular momentum in proto-planetary discs even if they do not cause further fragmentation (Larson, 2002; Pfalzner, 2004; Pfalzner et al., 2005).

Star-disc encounters probably play a role in star formation, and may lead to the formation of BD (or even planetary) mass companions (Thies et al., 2005). However, they are probably not a significant contributor to the primordial stellar binary population. This is due to the requirement that the encounters occur early in the star formation process - during the Class 0 phase when the disc mass is still very large compared to the stellar mass - a phase which lasts for only $\sim 10^5$ yrs, leaving only a small time for encounters to occur. However, the role of disc fragmentation in planet formation may well be important.

4.2.4 The role of magnetic fields. The treatments of collapse and fragmentation discussed above do not include magnetic fields. The new picture of rapid, turbulence-driven star formation combined with the lack of observational evidence for magnetically critical cores suggests that magnetic fields are not dynamically dominant. In addition, fragmentation is expected to occur at densities $\gtrsim 10^{-13}$ g cm$^{-3}$, densities at which the magnetic field is expected to be decoupled from the gas due to the extremely low fractional ionisation (see Tohline, 2002). However, possibly one of the main reasons for neglecting magnetic fields is the difficulty in including them in SPH simulations. (although this is improving, see esp. Hosking and Whitworth, 2004a, b; Price and Monaghan, 2004a, b). Magnetic fields are clearly present in (many) cores, even if they are not dynamically dominant, and their effects may be very important.

Grid-based simulations which include magnetic fields in rotating clouds show that fragmentation can occur in these clouds, although magnetic fields appear to have a tendency to suppress fragmentation (e.g., Hosking and Whitworth, 2004b; Machida et al., 2005b), although Boss (2002;2004) claims the opposite. Sigalotti and Klapp (2000) find binary and higher-order multiple formation in slowly rotating $\sim 1 M_\odot$ clouds which includes a model for ambipolar diffusion.

Possibly the most extensive investigation of the effects of magnetic fields on fragmentation has been made by Machida et al. (2005a,b). They find that fragmentation occurs in $\sim 50\%$ of their rotating, magnetised clouds when either the rotation is relatively high or magnetic field strength relatively low. In particular fragmentation always occurs in magnetised clouds if $\beta_{\text{rot}} > 0.05$, but it almost never
timescale runs from 66.6 to 67.8 kyr after the start of the simulation in steps of 400 yrs. The grey-scale bar gives the column density in g cm$^{-2}$. The spiral features can become self-gravitating if their density exceeds $\sim 10^{-12}$ g cm$^{-3}$ as their Jeans length falls to $\sim 20$ AU which can allow collapse without being shredded. This does not always occur, in the first two panels a dense knot can be seen being shredded and accreted onto the central object. Very similar evolution is also seen in AMR simulations (Gawryszczak et al., 2006).

occurs below this limit (see Fig. 10 from Machida et al., 2005b). Indeed, Burkert and Balsara (2001) conclude that once magnetic fields are strong enough to affect the dynamical evolution they will also efficiently suppress fragmentation which means that magnetic fields cannot be important as we know that fragmentation must occur.

4.3 ‘Secondary’ fragmentation.

As briefly mentioned in Section 4.1, there is a second isothermal phase in the evolution of gas towards stellar densities. This occurs at a temperature of $\sim 2000$ K and a density of $\sim 10^{-3}$ g cm$^{-3}$ when molecular hydrogen dissociates into atomic hydrogen. This phase occurs in the hydrostatic protostar when its radius is $\sim 1$ AU and - if fragmentation can occur at this stage - it may explain very close binaries.

Both Boss (1989) and Bonnell and Bate (1994) simulated the collapse of a rotating hydrostatic first object to high densities. They found that fragmentation can occur in axisymmetric instabilities or a ring formed by a centrifugal bounce. However, Bate (1998) found that spiral instabilities remove angular momentum and suppress further fragmentation. Recent 2D simulations by Saigo and Tomisaka (2006) suggest that the angular momentum of the first core is a crucial factor in determining if fragmentation will occur during the second collapse.

Thus it is unclear if a secondary fragmentation phase occurs. However, we suggest that such a phase could well be responsible for the apparently high incidence of very close BD-BD binary systems (Pinfield et al., 2003; Maxted and Jeffries, 2005) as the evolution of BD-mass hydrostatic objects occurs on a longer timescale than in stellar-mass objects. This possibility is being investigated by SG without any firm conclusions as yet.

4.4 Simulations vs. Observations

A summary of the simulations to date suggests that collapsing cores are easily able to fragment. However, no detailed model is currently able to correctly predict all of the observed binary properties.

A successful model of star formation must produce multiple systems which generally have only 2 or 3 stars with a wide range of separations from $<< 1$ AU to a peak at $\sim 100 - 200$ AU. At all separations, most stars must usually have quite different masses, but avoiding BDs within at least 5 AU of the primary (the BD desert).

Possibly the most significant problem at the moment, is that simulations seem to form too many single stars (see Bouvier et al., 2001; Duchêne et al., 2004; Goodwin and Kroupa, 2005). As described in Section 2.3.1 systems with $N \geq 3$ are generally unstable and decay by ejecting their lowest mass member and hardening the remaining multiple. The ejection of members of small-$N$ multiple systems dilutes the multiplicity of stars, as ejected stars tend to be single. Thus, many ejections will result in a far lower multiplicity fraction than is observed in young star forming regions. Goodwin and Kroupa (2005) suggest that the observed multiplicity frequencies can be explained if roughly half of cores form 2 stars, and half form 3 stars. However, these numbers are far lower than are usually found in core fragmentation simulations.

The inclusion of magnetic fields produces the opposite problem that too few binaries are produced. Machida et al. (2005b) find the fragmentation does not occur in rotating, magnetised clouds when $\beta_{\text{rot}} \lesssim 0.04$ - a higher level of rotation than is observed in many cores. This problem becomes especially acute when we consider that much of the observed rotation in cores could well be due to turbulent motions rather than a bulk rotation.

A related problem is posed by the existence of a significant number ($\sim 20\%$ of field G-dwarfs; DM91) of close, unequal mass binary systems. It appears difficult to form stars much closer together than $\sim 30$ AU. In order to obtain hard binary systems, a further hardening mechanism is required. In many simulations of turbulent star formation, this hardening mechanism is provided by the ejection of low-mass components (e.g., Bate et al., 2003; Delgado Donate et al., 2004a, b; Goodwin et al., 2004a, b; Um-
Fig. 6.— Comparison of an AMR (left) and a SPH (right) simulation of a collapsing, turbulent 5.4$M_\odot$ core showing the first ‘disc’ fragmentation episode from Gawryszczak et al. (2006). The bound fragments can be seen at (0,125) AU in the AMR simulation (left) and (-200,0) AU in the SPH simulation (right). The scale and orientation of both views are identical.

breit et al., 2005; Hubber and Whitworth, 2005; see Ochi et al., 2005 for a caveat). However, ejections appear to occur rapidly, during the main accretion phase, producing many close, equal-mass binary systems (see above) in contradiction to observations of a relatively flat mass ratio distribution (Mazeh et al., 1992). White and Ghez (2001) do find many roughly equal-mass PMS binaries < 100 au in Taurus, however there is no trend to more equal mass ratios at very low separations. Fisher et al. (2005) do find a bias towards equal-mass binaries among local field spectroscopic binaries (with separations  $\lesssim$ 1 AU).

In general, simulations that produce a small number of stars consistent with limits on ejection do not produce hard binaries. These binaries are difficult to form in significant numbers through later dynamical interactions in a clustered environment, which tend to disrupt wide binaries but not harden them. But simulations that produce many stars tend to form too many close, equal-mass binaries and very high-order multiple systems (many quadruples and quintuples) and also dilute the multiplicity fraction too much through ejections.

4.5 Numerical issues

As already discussed in this volume by Klein et al., there is some debate about the ability of simulations to correctly resolve fragmentation. Simulations of star formation are usually conducted using SPH as opposed to AMR schemes due to the Lagrangian nature of SPH (see Gawryszczak et al., 2006 for details).

No numerical scheme is perfect, and both SPH and AMR have their advantages and disadvantages. It is worth noting that a recent study by Gawryszczak et al. (2006) has shown that AMR and SPH converge when simulating the collapse of a slightly turbulent core. Fig. 6 shows a snapshot of both the SPH and AMR simulations at roughly 76 kyr from the start of the simulation showing that in both numerical schemes a highly unstable CAR forms that fragments in both simulations. Gawryszczak et al. found that AMR is significantly more computationally intensive than SPH for an identical simulation. This result is not surprising as when simulating gravitational collapse the Lagrangian nature of SPH should prove highly efficient. The agreement of two very different methods when applied to the same physical situation increases our confidence that the results are not dominated but numerical effects.

Both SPH and AMR suffer from problems with artificial angular momentum transport. In AMR, rotation in a poorly resolved Cartesian grid is likely to transport angular momentum outwards. In SPH, the use of artificial viscosity to reduce particle inter-penetration in shocks produces an inward transport of angular momentum with rotation. These problems are probably responsible for the different CAR (disc) sizes between SPH and AMR seen by Gawryszczak et al. (2006; see also Fig. 6).

It should be noted that Hubber et al. (2006) have shown that SPH supresses artificial fragmentation rather than promoting it which suggests that the current generation of SPH simulations could underestimate the number of fragments that form.

We would conclude that the computational situation is not perfect and many problems with both SPH and AMR remain. However, we think that the conflict between simulation and observation is more probably due to missing and/or incorrect physics, rather than any fundamental numerical difficulties.

The computational situation is made more problematic by the need to perform ensembles of simulations to get the statistical properties of multiple systems. Even in cases where the result for any single set of parameters might be expected to converge there is a large parameter space to cover, and small differences in initial conditions may make
a significant difference to the result. More realistically, situations where fragmentation occurs due to non-linear instabilities, or with turbulence (when the details of the velocity field vary) require large ensembles even for the same region of parameter space. No matter how detailed or correct any single simulation is, it can only be a snapshot of the outcome of a particular initial configuration. This vastly increases the computational effort required to model fragmentation and star formation.

4.6 Missing physics

One of the greatest problems facing the simulation of core fragmentation is to correctly model the thermal physics of cores. It is not possible in the foreseeable future that we will be able to conduct hydrodynamical simulations that include a proper treatment of radiation transport as the computational expense is just too great. However, as shown by Boss et al. (2000) thermal effects can be very important. The use of the barotropic equation of state is at best a first approximation, and it is clear that varying the adiabatic exponent can have significant effects on fragmentation (Goodwin et al., in prep.). In particular, the barotropic equation of state is based on simulations that have used (necessarily) simplistic, spherically symmetric assumptions. It is not clear to what extent these can be applied to highly inhomogeneous cores including discs and local density peaks. Improvements are being made, including using a flux-limited diffusion approximation (Whitehouse and Bate, 2004) and an approximation based on the local potential as a guide to optical depth (Stamatellos et al., in prep.). However a problem remains with any approximation in that, while it should obviously match the fully detailed radiative transfer simulations of simple situations, it is not clear if it is correct in more complex situations.

Most simulations (especially SPH simulations) do not include the effects of magnetic fields. Yet those simulations which do include magnetic fields seem to suggest that fragmentation is suppressed. This could well be a very important conclusion given that non-magnetic models seem to over-produce stars in cores. However, the very efficient suppression of fragmentation by magnetic fields may rule-out the importance of magnetic fields in the fragmentation process as we know that cores must fragment.

Given that fragmentation appears to occur in disc-like structures, the proper treatment of these is vital. Both AMR and SPH have problems with the artificial transport of angular momentum (see Gawryszczak et al., 2006) which will affect their ability to correctly model discs. Discs are also a situation in which magnetic fields may play an important role.

Finally, very few simulations attempt to model the effects of feedback from stars as jets or through their radiation field (for some first attempts to deal with these problems see Stamatellos et al., 2005; Dale et al., 2005). In particular, Stamatellos et al. (2005) find that the inclusion of jets may inhibit fragmentation by decreasing the inflow rate onto the disc and forcing that inflow to occur away from the poles.

Many of the physical situations which may result in fragmentation are rather complex and often chaotic (turbulence being the most obvious example). Such situations will not produce any single, unique answer. Indeed, given the variety of multiple systems such a situation would not be expected. However, this does require that a statistical approach be taken when performing simulations. This vastly increases the computational effort required, as any ‘single’ region of an already huge parameter space will require an ensemble of simulations to investigate it.

5. CONCLUSIONS

Almost all young stars are found in multiple systems with a very wide separation distribution and a fairly flat mass ratio distribution. Thus prestellar cores must fragment into multiple stars and/or BDs with these properties.

The dynamical decay of small-$N$ systems would rapidly produce a large single-star pre-main sequence population if large numbers of unstable systems form with $N > 2$ or 3. This decay would also result in large numbers of very close binary systems. Neither of these are observed, leading to the conclusion that cores must usually form only 2 to 4 stars in hierarchical systems (for $N > 2$).

Simulations show that most cores which contain some angular momentum - either in bulk rotation, or in turbulence - are able to fragment into multiple objects. However, these simulations have been unsuccessful in matching their results to the observed young multiple population. In particular, the distributions of separations and mass ratios from simulations tend not to fit well.

The future is somewhat rosier, however. The inclusion of more detailed physics and more realistic initial conditions may well yield better fits to observations.

Acknowledgements. SPG is supported by a UK Astrophysical Fluids Facility (UKAFF) fellowship.

REFERENCES