Ay/Ge 133 – Problem Set #8
Due December 1^{st}, 2011

(1) The Jeans formula governing atmospheric escape due to thermal evaporation is:

$$\phi \;=\; n_i < v > \quad .$$

The flux of escaping particles where n_i is the number density of the species of interest and $< v >$ is given by

$$< v > \;=\; \frac{u}{2\pi^{1/2}}(1 \;+\; \lambda)e^{-\lambda} \quad , \qquad \lambda \;=\; \frac{GMm_i}{k\mathrm{TR}} \quad ,$$

where $u \;=\; (2k\mathrm{T}/m_i)^{1/2}$, the most probable velocity of a Maxwell-Boltzmann distribution.

(a) What two forces compete to determine if a particle escapes, and how do they manifest themselves in the formula above?

(b) Given a solar composition for the primordial atmosphere of the earth, and assuming the atmosphere is well mixed, will H_2 and He escape? If so, how long will it take to deplete the atmosphere of 99% of each species? You can determine the initial abundance by assuming that the amount of N_2 has not changed. The thermal equilibrium temperature of the stratosphere is close to 200 K, but rises steeply at higher altitudes. First assume T = 200 K, and then calculate the loss for 1000 K (this is close to the temperature of what is called the exobase for the current atmosphere).

(c) What is the flux from Jupiter of H_2 and He? (here only for 1000 K) How does this compare to the mass of each in the atmosphere, assuming solar composition, and what are their residence times (that is, the reservoir mass divided by the loss) if escape is constant and the dominant loss process?

(2) Now let's think about the opposite extreme for the early Earth - an atmosphere determined by the chemical buffering of the mantle. That is, consider a closed system containing a gas phase and the solid QFM (quartz-fayalite-magnetite) buffer in one case and the solid QFI (quartz-fayalite-iron) buffer in a second case, at a temperature of 1100°C. The former comes close to the current state of affairs on the Earth, while the latter is the most reducing buffer conceivable (free iron in the mantle). The total pressure on the gas is 1.0 atm, and it consists of a mixture of C-H-O species. What are the partial pressures of each of those species if the system is at equilibrium and the bulk atomic ratio C:H is 0.05?

To make the question more tractable, assume for the purposes of illustration that there are only five gases in the mixture: O_2, H_2, H_2O, CO, and CO_2. Further facts you will need to know include: a) The activity of O_2 defined by the QFM and QFI buffers are commonly specified by the empirical equations

$$log\ f_{O_2}(QFM) \;=\; 9.00 \;-\; (25738/T)$$
$$log\ f_{O_2}(QFI) \;=\; 7.51 \;-\; (29382/T) \quad ,$$

(so there is part of the answer for you!), b) The equilibrium constant for CO_2 breaking up into CO and O_2 is 3.79 x 10^{-13} at the relevant conditions, and c) Similarly, the equilibrium constant for the dissociation of H_2O into H_2 and O_2 is approximately 8.82 x 10^{-14}.

The above facts give you five equations for five unknowns (remember, you know the pressure and the C:H ratio). Assume all gases are ideal, i.e. that the partial pressures equal the activities. Given the composition of the atmosphere you calculate and the answers to problem #(1), what do you conclude about the oxidation state of the atmosphere of the early Earth and the likelyhood of Uery-Miller type synthesis of organic compounds?