Ge 133 - Problem Set # 3, due Oct. 27^{th}

A) The goal of this problem is to understand Spectral Energy Distributions (SEDs), the spectra emitted by a star plus a disk. Using some simple assumptions, you'll generate your own model SED. For this problem, assume the star has the properties adopted by Chiang & Goldreich (an effective temperature of 4000 K, a mass of 0.5 solar masses, and a radius of 2.5 solar radii). For the disk, assume an opaque disk extending from the stellar surface out to 100 AU. We will also assume that the dust in the disk radiates as a perfect blackbody. For this problem, use the following form of the blackbody equation:

$$B_\lambda(T) = \frac{2\pi hc^2}{(\lambda)^5} \frac{1}{e^{hc/(\lambda kT)} - 1} \text{ [erg/s/cm}^2\text{/cm]}$$

where everything is in cgs units.

(1) Equation (4) of Chiang & Goldreich tells you what the temperature versus radius is for a flat, geometrically thin disk. Calculate this flat disk temperature at a distance of 1 AU and compare it to what you know about the Earth. Why do these numbers differ?

(2) Real disks are flared, and for the interior, optically thick part of the disk the temperaure can be approximated as T(interior)$\sim$150/R$^{3/7}$ K, where R=distance from the central star (in AU). Assuming that we are viewing the star and disk from directly above (i.e. along the pole) at a distance of 10 parsecs (a parsec is 3×10^{18} cm), use your favorite plotting program to plot the total emission seen from the star alone, the disk alone, and the star plus disk at wavelengths from 0.1 to 100 microns. Use a log-log scale, use cgs units for the y-axis, and use microns for the x-axis.

(3) The Spitzer Space telescope observing bands are centered at 3.6, 4.5, 5.8, 8.0, 24.0, 70.0, and 160.0 microns. For a circularly symmetric disk, write down the equation for the disk flux versus radius (distance from the central star). Call this function F(R). The "characteristic" radius for the disk emission, or first moment of the emission, would be $< R >= (\int R * F(R) dR)/\int F(R) dR$ (so things are normalized). Using your favorite mathematics package, calculate $< R >$ at 4.5, 24, and 70 microns. For disks at 140 pc, how big would a telescope have to be to provide a diffraction limit of $< R >$ at 4.5, 24, and 70 microns? In reality of course we'd have to add the stellar flux as a point source. What do these calculations tell you?

(4) Assume that there is no disk from the stellar surface until an orbital distance of 4 AU, but that the disk resumes after that. (This might happen if a giant planet has formed and cleared out part of the disk). Plot the emission as a function of wavelength for this disk geometry as well as for that calculated in part (b). What do you notice?

B) *Gravitational Focusing.* One of the most important concepts in pairwise accretion is the "gravitational focusing" caused by the deflection of a small body by a massive body. This deflection works to increase the cross-section for collision of the two bodies. Your goal is to reproduce a calculation first done by Safronov. Try this, at first, with no books, no notes, no discussion with anyone. Really. Consider a test particle approaching a body of mass M and radius R. The impact parameter is b and the velocity at infinite distance is v_∞ (i.e. in the absence of gravity, the two bodies would approach within a distance of b of each other and with relative velocity v_∞, see the figure at the end of the problem set). In the absence of gravity, the cross-section for collision would be πR^2. We would like to know by how much this cross-section increases due to the effects of gravity.

1. What is the escape velocity from the large body? (you'll need this for later).
2. Draw a diagram of a *grazing* collision between the two bodies. (A grazing collision means that if the point mass had just slightly more energy it would not collide with the massive body) Pay particular attention to where the two bodies touch.
3. For the case you just drew, use the principle of conservation of angular momentum to write an equation relating b, v_∞ , R, and v_{imp}, the velocity of the object at impact. (Hint: Write down the total angular

momentum in the 'before' picture, shown above, and the total angular momentum at the moment of impact, and set these equal)

4. Write an energy conservation equation relating v_∞ , v_{imp}, R, and M.

5. What is the maximum value for b for which an impact will occur? Write your answer in the form of $b^2 = R^2(1+f)$ where f is called the focusing or Safronov parameter, which should be expressed in terms of the escape velocity.

6. (you can start collaborating again at this point) How large does an object need to be before gravitational focusing significantly increases the impact (and thus accretion) cross-section. The answer to this question is not obvious without some additional information that you don't neccessarily possess, so do your best and make reasonable estimates where neccessary. Justify your answer carefully.

7. An important concept that we will discuss next week is that of *dynamical friction*, in which large bodies are slowed by the presence of a swarm of smaller bodies. This process leads to an equipartition of energy between small and large bodies such that $mv^2 = MV^2$ where the lowercase values refer to small bodies and the uppercase to large bodies. Explain how this dynamical friction could greatly speed up the growth of large bodies.