Ge/Ay133 – Problem Set #2 **Due October 13^{th}, 2011**

1 Angular Momenta

(a) Verify eq. (1.1) (page 3) in Armitage, and use it to estimate the total angular momentum of the spinning sun, and how much angular momentum the sun would have if it were spinning on the verge of breakup (i.e. the outer layers are essentially in orbit). Treat the sun as a uniform sphere.
(b) Verify that the total angular momentum in the planets dwarfs that of the sun (also page 3 of Armitage) by summing the orbital angular momenta of all of the planets (ignore spins; they're insignificant). The data needed for parts (a) and (b) may be found in Table 1.1 and pages 2,3 of Armitage.
(c) Calculate the angular momentum of a spherical uniformly-rotating gas cloud that has a density of 100 hydrogen molecules per cm^3, a radius of 10 parsecs (1 parsec is 3×10^{16} m) and a rotation period of 100 million years (same formula as for the Sun).
(d) What do you make of any of this (i.e. can we use any of this information to help us understand the process of planet formation and what do you think it tells us)? Note that many of the problems in this class will have a similar "what do you make of any of this" type question. The reason is that the calculations above are designed to make you think about the bigger picture of what is happening. So here is where you should do that thinking in case you haven't yet.

2 The Minimum Mass Solar Nebula

All of the first ideas about planetary system formation were derived from properties of our own solar system. The hypothetical starting point for our solar system is called the Minimum Mass Solar Nebula (MMSN). Below, you'll derive its total mass, as well as its mass surface density.

The chemical compositions of solar system planets is a strong function of their distance from the sun. This is largely because if a chemical is vaporized, it is not available for constructing the solid body of the planet. Thus solar system materials are often categorized as either 'gas', 'ice' or 'rock', based on their volatility. It is reasonable to assume that planets retain all of their original 'rock', but have lost some fraction of their original 'ice' and 'gas'. The table below shows the real or estimated rock masses of the solar system planets, in Earth masses.

Body	Rock Mass	Ice mass	Gas mass	Total original mass	Total current mass
All Terrestrial	2				
Jupiter	10				
Saturn	10				
Uranus	3				
Neptune	3				

(a) The table below shows the cosmic abundances, by number, of the most common elements. We'll assume these represent the original abundances in the solar system, and use these numbers to calculate the original mass of the solar system. Assume all of the O, Mg, Si and Fe go into silicate rocks (use Mg_2SiO_4, Fe_2SiO_4, and SiO_2 for simplicity) and that the remaining O, as well as all available C and N, go into H_2O, CH_4 and NH_3 (ices). Calculate cosmic ice/rock, He/rock and H/rock *mass* fractions and use these to fill in the next 3 columns of the chart, in Earth masses. (These are approximations, so don't sweat the details.)

Atom	Abundance by #
H	1
C	4×10^{-4}
N	1×10^{-4}
O	7×10^{-4}
Mg	4×10^{-5}
Si	4×10^{-5}
Fe	3×10^{-5}
He	8×10^{-2}

(b) Look up the current masses of the planets and fill in the last column. Are the differences between the last 2 columns what you expected? Explain.

(c) Now sum up the original and current masses of all of the planets and express the two totals *in solar masses*. How much mass has been lost? How do you think it was removed?

(d) Now let's estimate the MMSN surface density, Σ. Take the total original mass of each planet and spread it out into an annulus that extends halfway to each of its neighbors. Plot the result, using g/cm^2 and AU, and log-log space.

(e) Do a linear fit to the MMSN in log-log space and find the exponent, p, assuming $\Sigma = \Sigma_0 (R_{\rm AU})^p$

(f) One way to think about a physical basis for the Titius-Bode relationship is to assume that you have approximately the same total mass in each 'planet forming zone.' More quantitatively, this means that the mass within an octave of radius (a fancy way of saying a factor of two) is constant. Write down the integral expression for this statement of the Titius-Bode relationship.

(g) If the Titius-Bode relationship is true, what exponent would you derive for the mass surface density?

(h) Assuming the total mass of the MMSN is the same as that derived above (in question 3), calculate the Titius-Bode mass surface density and add it to your plot. Is it similar to the MMSN you derived?

(i) In the Chiang & Goldreich paper we'll be covering shortly, the assumed mass surface density is $\Sigma = \Sigma_0 R_{\rm AU}^{-3/2}$, where $\Sigma_0 = 10^3$ g cm^{-2} is the mass surface density at 1 AU. Add this to your plot.

(j) How different is the Chiang & Goldreich surface density from the one you derived? Can you think of any reasons why the surface density of the MMSN might have varied with time?

3 Exoplanetary Piddling

Go to `exoplanets.org`, and look around. From the various distributions you see, dig into one system, take a look at it, and tell us what's interesting about it. Be brief, and use plots as is sensible.