Problem Set #1

Ge/Ay 133

Due Thursday, 6 October 2011

1. Consider a planet of mass M_p that orbits a star of mass M_* at orbital distance a, or, more precisely, the star and the planet go around their common center of mass. For a star some R parsecs distant, use Kepler's laws to derive the velocity of the planet and the maximum radial velocity of the star. How large is the velocity for the Sun and Jupiter (the sun is 2×10^{33}g, Jupiter is 1000 times less massive, and Jupiter orbits at 5.2 AU). Now imagine looking at the sun from 10 parsecs away (the distance of typical *nearby* stars, where 1 pc is 3×10^{16} m). By what angle does the Sun move due to Jupiter? What are the velocity and angular movement of the Sun due to the earth (1 AU, 6×10^{27}g)?

2. If the Jupiter-mass planet around the sun-like star 10pc away orbits precisely edge on to us, it will, once an orbit, pass directly in front of the star, block out a small amount of light from the star, and cause a perceptible dimming of the star.

a. Transit depth. How does the transit depth scale with orbital distance from the star? The radius of the Sun is easily remembered as 2 light seconds. The radius of Jupiter is 10 times smaller. What is the magnitude of the dimming of the Sun? How does the magnitude of the dimming change if the Jupiter-sized planet is at 1 AU instead of 5 AU? What if the star is moved to 100pc instead of 10pc?

b. Transit timing. Show, for an equatorial transit of the star, that

$$\tau = \left(\frac{R_p}{a}\right)\frac{P}{\pi}$$

where a, P are the planet semi-major axis and period, and R_p is the radius of the planet; and that

$$T = \left(\frac{R_*}{a}\right)\frac{P}{\pi}$$

where T is the full-width-half-maximum of the transit and R_* is the radius of the star. Here, the following additional times are defined: The full transit from first to last contact (t_T). The transit time over which the planet fully occults the star (t_F). The ingress/egress time ($\tau = (t_T - t_F)/2$).

For Jupiter at 5 AU, what are the values of t_T, t_F, and τ? The last tells you about the cadence you'd like to take data with. Can the relative values of t_T and t_F tell you at what "latitude" the planet crosses the star?

Finally, usng scaling relations (that is, do not worry about all the constants, just worry about the functional forms of various relationships), show that

$$g_P \sim \frac{K_*}{P}\left(\frac{a}{R_p}\right)^2 \sim \frac{K_* P}{\tau^2}$$

$$\rho_* \sim \left(\frac{a}{R_*}\right)^3 P^{-2} \sim \frac{P}{T^3}$$

where K_* is the speed of the star as it moves in orbit around the center-of-mass of the star-planet system (from problem #1). To do this, you'll need Kepler's Third Law in the limit that $m_p \ll M_*$

Interestingly, this shows that from timing alone the *model independent* parameters that can be derived are the surface gravity of the planet and the (bulk) stellar density!