At the end of chapter 14, we started thinking about a model for viscous convection involving conductive heat transfer at a boundary layer that periodically "peels." Let's zoom out and develop some scalings for a whole system losing heat through a boundary layer.

Using vorticity and boundary layer conductive heat flow arguments, Dave derived two important scalings:

1. The convective velocity: 

   \[ \text{Recall } Ra = \frac{g \alpha \Delta T L^3}{\nu \kappa}, \text{ and } Ra > 10^3 \rightarrow \text{convective} \]
\[ u = 0.2 \left( \frac{R_a}{K} \right) R_a^{2/5} \quad [\text{Eq. 15.7 in text}] \]

where \( R_a \) is the Raleigh number of the whole system (i.e., using total \( \Delta T \), not just \( \Delta T \) in one boundary layer).

1. A relationship between the Nusselt number, which is the ratio of actual heat flux to the heat flux expected from conduction alone, to the Raleigh number:

\[ Nu = \frac{q}{q_{\text{cond}}} \approx 0.2 R_a^{1/3} \]

We have been implicitly assuming constant viscosity convection, but this assumption breaks down for real systems (as viscosity of most materials is very temperature-dependent). Because of this, we expect a stagnant lid model, in which there are two boundary layers: an outer "frozen" layer, with very high viscosity, and a peeling under layer, where convection occurs. However, the Earth doesn't behave like this! It has plate tectonics, which isn't understood well.

Putting this mystery aside, we can use the physics we've developed to make a model of Earth's thermal evolution. The ingredients are:

- the first law of thermodynamics (\( Q = mc \Delta T \))
- a description of heat flow from boundary-layer viscous convection
- a description of radioactive heat flow over time
- a prescription for viscosity \( \nu(T) \) (Dave used an Arrhenius relationship \( \nu \propto \exp\left(\frac{\text{const}}{kT}\right)\))

If we plug in all of these things, some insights emerge:

1. The Earth "forgets" its initial condition, i.e., no matter what temperature the Earth started at, it will converge to the same track in a few Myr.
The Earth cools over time. There's a disagreement between models & data about how much heat comes from radioactivity.

Since the Earth was hotter in the past, there was more volcanism (i.e. hydrodynamic cooling) and melting in the Precambrian period.

Convective velocities were higher in the past, meaning that there was more mantle mixing in the past.

We can use models to constrain the core cooling rate, which is important for magnetic field generation.