0. About this Book

1 Introduction

Most textbooks are too long; this one is not. Most textbooks are not very readable; the main text of this book is designed to be read. It is not a recipe book and it is not a source book. The index should guide you back to a place you’ve already visited; it is not the right method for finding what you want to know. This text does not attempt to document all the important work. References are kept to a minimum. It does not have lots of tables and figures. It concentrates on the ideas and not the details of how they are applied. Order of magnitude arguments are used extensively to frame the issues. Without an understanding of these, detailed derivations or results have little worth. The details can be found elsewhere. Basic concepts are often not spelled out adequately (if at all) in existing texts, yet these matter most. Why would you believe (or even pay attention to) somebody’s model or data interpretation if you don’t have a firm appreciation of the ideas and approximations that motivate it?

The text is divided into three categories: (1) The main text, which is sequentially readable. (2) The supplementary text, which provides some background to relevant basic physics and is confined to the appendices at the ends of some chapters, beginning with Chapter 2. (3) The problems, half of which are given sketch solutions. The main text is written in a Socratic style. It poses questions and guides the reader to likely answers. Things we don’t know are given at least as much attention as the things we do know, as befits their central importance to scientific research—knowledgeable uncertainty is more important than certainty. Parameters are explained from basic principles—good science might be done by inserting parameters (e.g., specific heat, thermal conductivity or whatever) into some calculation, but excellent science is more likely to result if one knows the fundamental principles that govern them and whether the chosen values should be believed.
The appendices are usually less than sufficient for a complete understanding of the underlying principle (e.g., the origin of exchange in quantum mechanics), but are constructed as a bridge to basic texts in other fields. In some cases (e.g., fluid dynamics) this material is quite systematic because the alternative texts are deficient for the purpose at hand. The questions are usually not routine applications of the material presented. In some cases, they lack precise answers or an endpoint—defined as the point where you think you’ve done what is expected. This is just like real science.

2 Comments on Units, Dimensions, Dimensionless Numbers, Scaling and Numerical Constants

Units are almost a religious issue (like PC vs. Mac), but pantheism is best. Despite the tendency towards universal use of SI units in journals, there are advantages to other unit systems and therefore merit in learning the (mostly easy) translation back and forth. Astrophysicists often use cgs. Special units—those in which the quantity is of order unity—are particularly useful within their domain of application. Examples include the bar (a unit of pressure roughly equal to atmospheric pressure at sea level on earth) and the electron volt (a unit of energy).

This text uses mostly cgs but also makes frequent use of special units. The author learned physics in SI (based on kilograms and meters) and discovered the pleasures of cgs (based on grams and cm) only much later. (This invalidates the perception some have that cgs is somehow an “older” set of units, supplanted more recently by SI.) Cgs was originally constructed so that densities are of order unity for zero-pressure condensed matter, particularly water, and this is nice in planetary calculations. Normal densities are many thousands in SI units. The occasional claim (stemming from freshman physics classes?) that cgs gives very large values for things is silly: Why would $1.6 \times 10^{-12}$ erg be somehow less attractive or intuitive than $1.6 \times 10^{-19}$ Joules? (These are both equal to 1eV, a very natural energy scale in atomic physics.) It is remarkable that in cgs, the natural unit of action (velocity times distance) is $\hbar/m_e$ and of order unity (where $\hbar$ is Planck’s constant divided by two pi and $m_e$ is the electron mass) and the luminosity to mass ratio of solar mass stars is also of order unity. They are both very
far from unity in SI. If God exists, perhaps (s)he used cgs.

Consistency is good within a given context, but not a general virtue. For example, there is some merit to using SI in some areas of electromagnetism because our everyday experience (amps and volts) corresponds to SI, whereas the cgs equivalents are unfamiliar. Remarkably, the most useful cgs unit for magnetic field strength, the Gauss, also gives values of order unity in many planetary applications. However, the part of this text that deals with electromagnetic issues is mostly SI. Of course, one should always have an awareness of the units in use when doing a calculation or reading the text.

Dimensions should always be checked in an equation. However, it is not necessary to remember the dimensions of fundamental constants, because these are defined by their governing equations. For example, you should never bother to learn the dimensions of G, the gravitational constant; instead, you should remember that the gravitational force is just \( GM^2/R^2 \) (where \( M \) is a mass and \( R \) is a distance), or that gravitational acceleration is \( GM/R^2 \). Using these defining relations, we can easily get the units for \( G \), by remembering the familiar dimensions are \( ML/T^2 \) for force and \( L/T^2 \) for acceleration. (Dimensions are intuitively symbolized using \( M \) for mass, \( L \) for length, and \( T \) for time). Similarly, the Stefan-Boltzmann’s constant is another constant with strange dimensions, but if we only recall that it must be paired with the appropriate number of powers of temperature, we can relate to the familiar dimensions of energy flux. This method of sanity checking an equation is called dimensional analysis, and we will use it frequently throughout this text (as should you).

Dimensionless numbers are very valuable and should be sought whenever possible. Their value is independent of the units used (unlike the dimensional examples cited above for the merits of cgs relative to SI). For example, one can learn a lot by asking “what is the ratio of thermal energy to electronic energy?” or “what is the ratio of internal pressure to bulk modulus?” or “what is the ratio of incident solar flux to internal heat flux?” In some areas of science, especially fluid dynamics, dimensionless numbers are so central to our understanding that they are given names (e.g. Reynolds number, Rayleigh number and so on.) But the fact that some dimensionless ratio has no name does not necessarily diminish its usefulness.

Scaling is an immensely valuable technique. It can save you a lot of time, reduce the number of parameters you have to remember and give you additional insight into how things work. For example, the fact that mass goes as radius cubed together with the information that a solar radius is
about 10 Jupiter radii and about 100 Earth radii tells you that the mass ratio of Sun:Jupiter:Earth would be $10^6 : 10^3 : 1$ if they had similar densities. (Actually, Sun’s density is similar to Jupiter density and 4 times less than Earth density, hence the actual mass ratio of 300,000:300:1.) Here is a more useful example: Earth receives 5000 times more energy from the Sun than the geothermal heat flow from within. Since solar flux falls off as orbital distance squared, you would have to place Earth at further than $5000^{1/2}$ AU, which is about 70AU, in order that the surface temperature of Earth would be dominated by radioactive heat from within rather than sunlight. Additionally, the effective temperature of Earth would be lower by $5000^{1/4}$, a factor of 8, so the effective T would be $240/8 = 30$K. Notice that you can get approximate results here without knowing fundamental constants or doing any substantial calculation. Approximate answers are an essential part of doing science. Always do that first to establish the regime of interest in a problem. I give statements like this emphasis because they are so often ignored.

Physical Constants: With a modest investment of effort you can memorize just about all that you need know; the others are derivable from them. Of course if you want to do an accurate calculation, you can look up more significant figures. Here are the most important ones (given approximately):

$G$ (gravitational constant) = $6.67 \times 10^{-8}$ cgs
$c$ (speed of light) = $3.0 \times 10^{10}$ cm/s.
$m_p$(mass of proton) = $1.67 \times 10^{-24}$ g
$m_e$(mass of electron) = $9.11 \times 10^{-28}$ g
$e$ (electronic charge) = $4.80 \times 10^{-10}$ esu
$h$ (Planck’s constant divided by two pi) = $1.05 \times 10^{-27}$ erg.sec
$\sigma$(Stefan-Boltzmann constant) = $5.67 \times 10^{-5} erg/cm^2.s.K^4$
$k_B$(Boltzmann’s constant) = $1.38 \times 10^{-16}$ erg/K

It is also useful to know the mass of Earth ($6 \times 10^{27}$ g), the radius of Earth ($6.4 \times 10^8$ cm), the distance from earth to Sun (1AU = $1.5 \times 10^{13}$ cm), etc. One good sourcebook is The Planetary Scientist’s Companion, Lodders and Fegley (Oxford Un. Press).