In class, we discussed the one-dimensional diffusion equation

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2} + F$$

where $T$ is temperature, $z$ is depth measured downward from the Earth’s surface, $t$ is time, $\kappa$ is thermal diffusivity, and $F$ is the internal heat production. If we have thermal steady-state, then $\frac{\partial T}{\partial t} = 0$. If any value is omitted from the specific question below, please refer to the list of parameters on the next page.

1. Using the 1-D diffusion equation develop an expression for a geotherm assuming a steady-state, a constant heat production $F$, a surface temperature of $T_s$, and a surface heat flux of $q_s$. Plot your geotherm down to 500 km while assuming the following values, $T_s=0^\circ C$, $q_s=70$ mW m$^{-2}$ and a heat production of $6.2 \times 10^{-12}$ W kg$^{-1}$. This heat production is a model value for the earth’s mantle. In what way could this result be inconsistent with what you know about the earth or rocks from your other classes? Suggest one or more reasons what may be wrong with our assumptions.

2. Using the 1-D diffusion equation, develop an expression for a geotherm in the continental crust assuming a steady-state and that the heat production decays as a function of depth according to

$$F = F_s e^{-z/h}.$$

Assume that the heat flux at great depth (i.e. $z \to \infty$) is $q_m$ and that the surface temperature is $T_s$.

3. Show that your result from problem 2 predicts a linear relation between surface heat flux and the surface heat production. What is the slope of this relationship?

4. Using the data in Table 1 for the Sierra Nevada Mountains, estimate numerical values for $q_m$ and $h$. Plot a geotherm down to 40 km using these inferred values at one surface location from Table 1.
Surface heat flow (mW m\(^{-2}\))                  Surface heat production (\(\mu\)W m\(^{-3}\))

\begin{tabular}{|c|c|}
\hline
18 & 0.3 \\
25 & 0.8 \\
25 & 0.9 \\
29 & 1.3 \\
31 & 1.5 \\
34 & 2.0 \\
42 & 2.6 \\
54 & 3.7 \\
\hline
\end{tabular}

Table 1. Surface heat flow and heat production data for the Sierra Nevada Mountains

Some additional numerical values:

- Diffusivity, \(\kappa=10^{-6} \text{ m}^2 \text{ s}^{-1}\)
- Conductivity, \(k=3 \text{ W m}^{-1} \text{ K}^{-1}\)
- Density, \(\rho=3,000 \text{ kg m}^{-3}\)
- Specific heat, \(C_p=10^3 \text{ J kg}^{-1} \text{ K}^{-1}\)