

# ESE/Ge 148b – Atmosphere and Ocean Circulations

## Review for midterm

### 1 Properties of fluids

#### 1.1 Equation of state

The equation of state gives the relationship between state variables: density ( $\rho$ ), pressure ( $p$ ), temperature ( $T$ ), and salinity ( $S$ ).

For the ATMOSPHERE, we use the ideal gas law

$$p = \rho RT \quad (1)$$

where  $R = 287 \text{ J/kg/K}$  is the gas constant for the dry atmosphere (equal to the universal gas constant divided by the molecular weight of dry air).

For the OCEAN, we use tables, plots, or numerical algorithms to obtain seawater density, which is a complex function of temperature, salinity, and pressure.

In the range of  $T$  and  $S$  values that are relevant for ocean water, density increases with salinity and decreases with increasing temperature. Typical values of  $\rho$  are between  $1025$  and  $1035 \text{ kg m}^{-3}$ . The density anomaly ( $\sigma$ ) is defined as

$$\sigma = \rho - \rho_{\text{ref}} \quad (2)$$

where  $\rho_{\text{ref}} = 1000 \text{ kg m}^{-3}$  is the reference density.

Small changes in density can be approximated using the linearization of the equation of state around  $\sigma_0(T_0, S_0)$ :

$$\sigma = \sigma_0 + \rho_{\text{ref}}(-\alpha_T[T - T_0] + \beta_S[S - S_0]) \quad (3)$$

where  $\alpha_T$  is the thermal expansion coefficient and  $\beta_S$  is the saline dependence coefficient (both depend on pressure).

#### 1.2 Potential temperature

Temperature is not conserved when a fluid's pressure changes under adiabatic conditions. Air or seawater gets warmer as it moves adiabatically downward (toward increasing pressure). The potential temperature,  $\theta$ , is defined as the temperature that the fluid would have if brought adiabatically to surface pressure.

In the ATMOSPHERE (not saturated with water vapor), adiabatic displacements change the temperature according to

$$\frac{dT}{dz} = -\frac{g}{c_p} = -\Gamma_d \quad (4)$$

where  $\Gamma_d \approx -10 \text{ K km}^{-1}$  is the dry adiabatic lapse rate.

Potential temperature is

$$\theta = T \left( \frac{p_0}{p} \right)^{R/c_p} \quad (5)$$

where  $c_p$  is the specific heat at constant pressure and  $p_0$  is the reference pressure (usually 1000 mbar); the ratio  $R/c_p = 2/7$ .

For air that is saturated with water vapor, cooling causes condensation, which releases latent heat. This partly offsets the adiabatic cooling, and the temperature change for adiabatic displacements becomes

$$\left(\frac{dT}{dz}\right)_{\text{sat}} = -\Gamma_s \quad (6)$$

where  $\Gamma_s \approx -7 \text{ K km}^{-1}$  is the saturated adiabatic lapse rate. We can define an equivalent potential temperature,  $\theta_E$ , to account for the latent heat of moist air.

Unlike temperature, potential temperature (or more generally  $\theta_E$ ) is conserved for adiabatic motion.

In the OCEAN, the adiabatic lapse rate (rate of temperature change with depth) is a function of temperature, salinity, and pressure; the potential temperature is calculated from tables or numerical algorithms.

### 1.3 Moisture

The specific humidity of air,  $q$ , is the ratio of the mass of water vapor to the mass of air per unit volume. The relative humidity is  $U = q/q_* \times 100\%$ , where  $q_*$  is the saturation specific humidity (a function of temperature). Condensation occurs when the air is saturated with water vapor ( $U = 100\%$ ).

## 2 Equations of fluid motion

The velocity of a fluid is  $\vec{u} = (u, v, w)$  where  $u$  is the zonal velocity (positive toward the East),  $v$  is the meridional velocity (positive toward the North) and  $w$  is the vertical velocity (positive upward).

### 2.1 Lagrangian and Eulerian

Eulerian derivative: the rate of change at a fixed point in space

$$\left.\frac{\partial}{\partial t}\right|_{\text{fixed point}} \quad (7)$$

Lagrangian derivative: the rate of change following a moving fluid parcel

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \vec{u} \cdot \nabla \quad (8)$$

where  $\frac{\partial}{\partial t}$  is the Eulerian derivative and  $\vec{u} \cdot \nabla = u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}$  is the advection.

### 2.2 Momentum equations

#### 2.2.1 A useful set of momentum equations

In the local cartesian coordinate system ( $x$  toward the East,  $y$  toward the North, and  $z$  upward), the horizontal momentum equations are

$$\begin{aligned}\frac{Du}{Dt} - fv &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \mathcal{F}_x \\ \frac{Dv}{Dt} + fu &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \mathcal{F}_y\end{aligned}\tag{9}$$

We define the Coriolis parameter as  $f = 2\Omega \sin \phi$ , where  $\Omega$  is the Earth's rotation rate and  $\phi$  is latitude. For motion on a relatively small scale we can assume a constant value of  $f$ ; this is called the “ $f$ -plane” approximation.

The hydrostatic approximation to the vertical momentum equation is adequate for most atmospheric and oceanic flows:

$$\frac{\partial p}{\partial z} = -\rho g\tag{10}$$

### 2.2.2 A more general formulation

Conservation of momentum, for a fluid in the rotating frame of reference, is written in vector form as

$$\frac{D\vec{u}}{Dt} + 2\vec{\Omega} \times \vec{u} = -\nabla\Phi - \frac{\nabla p}{\rho} + \vec{\mathcal{F}}\tag{11}$$

where  $\vec{\Omega}$  is the angular velocity (positive if anti-clockwise rotation) and  $\Phi$  is the geopotential (includes the effects of gravity and centrifugal acceleration). On the sphere,

$$\nabla\Phi = \vec{g} + \Omega^2 \vec{r}\tag{12}$$

where  $\vec{g}$  is a vector pointing toward the center of the Earth and  $\vec{r}$  is the vector distance from the rotation axis (pointing outward).

### 2.2.3 Forces

The forces acting on a fluid, in the rotating frame, are

(i) gravity

(ii) the centrifugal force

It is responsible for the bulging figure of the Earth (radius is larger at the equator than at the poles). We bundle up the centrifugal acceleration of a fluid at rest with the gravitational acceleration; the resulting vector  $\nabla\Phi$  is in the direction of the local vertical (perpendicular to the surface, on which  $\Phi$  is constant) and its magnitude is approximately equal to  $g$ .

(iii) the Coriolis force

It is always perpendicular to the motion, therefore causes changes in direction but no change in speed (it does no work). To the right (left) of the flow in the northern (southern) hemisphere. The vertical component of the Coriolis force is negligible compared to gravity.

(iv) the pressure gradient force

Fluids are pushed from high to low pressure (down the pressure gradient).

(v) friction

It is negligibly small except close to the boundaries.

### 2.3 Continuity (conservation of mass)

In a fixed volume, the total change in mass must be equal to the flux of mass through the volume. This can be expressed as a conservation equation:

$$\frac{D\rho}{Dt} + \rho(\nabla \cdot \vec{u}) = 0 \quad (13)$$

For an incompressible fluid (water is mostly incompressible), this reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (14)$$

which means that stretching a fluid parcel in one direction must be compensated by compressing it in another direction, in such a way that the volume is unchanged. A flow that satisfies this is said to be non-divergent.

### 2.4 Thermodynamic equation

Conservation of thermodynamic energy is expressed, for the atmosphere, as

$$\frac{D\theta}{Dt} = \left(\frac{p}{p_0}\right)^{-R/c_p} \frac{\dot{Q}}{c_p} \quad (15)$$

where  $\dot{Q}$  is the rate of diabatic heating,  $\theta$  is given by equation 5 and other variables are defined in Section 1.

## 3 Balanced flows

### 3.1 Hydrostatic balance

Comparing the magnitude of the different terms in the vertical momentum equation (vertical component of equation 11), we find that the acceleration, the vertical component of the Coriolis force, and the friction terms are small compared to gravity and the pressure gradient. The hydrostatic balance

$$\frac{\partial p}{\partial z} = -\rho g \quad (16)$$

is a very good approximation for most atmospheric and oceanic flows (it breaks down when strong vertical flows for which acceleration terms cannot be neglected).

### 3.2 Gradient wind

When friction can be neglected, the horizontal momentum equations are reduced to a balance between the acceleration, Coriolis, and pressure gradient terms. Special cases are inertial motions and geostrophy.

### 3.3 Inertial motion

A flow that is not experiencing external forces (pressure gradient, friction) will keep moving because of its inertia (Newton's first law of motion). In the rotating frame, the equations are

$$\begin{aligned} \frac{Du}{Dt} - fv &= 0 \\ \frac{Dv}{Dt} + fu &= 0 \end{aligned} \quad (17)$$

with a non-zero velocity as initial condition. The resulting trajectory is a circle, and the period of oscillation is  $2\pi/f$  (the inertial period).

### 3.4 Geostrophy

Geostrophy is a balance between the pressure gradient force and the Coriolis force. Since the Coriolis force is perpendicular to the velocity, the flow must be perpendicular to the pressure gradient so that the two forces can balance exactly.

#### 3.4.1 Rossby number

Acceleration ( $D\vec{u}/Dt$ ) can be neglected when the motion is slow compared to the rotation period of the Earth; in that case, the Coriolis force is dominant over the acceleration term. The ratio of these is given by the Rossby number (a non-dimensional parameter)

$$Ro = \frac{U}{fL} \quad (18)$$

where  $U$  is the magnitude of the horizontal flow, and  $L$  the length scale of the motion ( $f$  is the Coriolis parameter). Rotation is dominant when  $Ro < 1$ .

Typical flows in the atmosphere at midlatitudes have a small Rossby number, around  $10^{-1}$ . In the ocean, the Rossby number is even smaller, around  $10^{-3}$ .

Note that  $f \rightarrow 0$  as we get toward the equator, hence geostrophy does not hold near the equator.

#### 3.4.2 Geostrophic balance in the ocean

When the Rossby number is small, we can neglect the acceleration terms in the horizontal momentum equations, which yields the geostrophic balance.

For the OCEAN, we can approximate  $\rho \approx \rho_{\text{ref}}$  and write

$$\begin{aligned} fu_g &= -\frac{1}{\rho_{\text{ref}}} \frac{\partial p}{\partial y} \\ fv_g &= \frac{1}{\rho_{\text{ref}}} \frac{\partial p}{\partial x} \end{aligned} \quad (19)$$

where  $u_g$  and  $v_g$  are the geostrophic velocities.

On the  $f$ -plane ( $f$  constant), we find (by differentiating and adding the equations above) that

$$\frac{\partial u_g}{\partial x} + \frac{\partial v_g}{\partial y} = 0 \quad (20)$$

i.e. the flow is horizontally non-divergent (stretching in the  $x$  direction must be compensated by compressing in the  $y$  direction and vice versa). By continuity, and noting that the vertical flow vanishes at the surface, we infer that the geostrophic flow does not have a vertical component, i.e. it is purely horizontal.

We can define a geostrophic streamfunction  $\Psi_g = \frac{p}{\rho f}$  such that  $\vec{u}_g = \hat{z} \times \nabla \Psi_g$ . The geostrophic flow is parallel to the streamlines (from the equation above,  $\vec{u}_g$  is perpendicular to  $\nabla \Psi_g$ ), hence parallel to contours of constant pressure.

### 3.4.3 Geostrophic balance in the atmosphere

We introduce pressure coordinates in order to simplify our description of geostrophic flows in a compressible fluid (for which  $\rho$  cannot be assumed constant). This is done by taking  $p$  as the independent variable and  $z$  as a dependent variable.

For the ATMOSPHERE, we can then write the geostrophic equations as

$$\begin{aligned} f u_g &= -g \left. \frac{\partial z}{\partial y} \right|_p \\ f v_g &= g \left. \frac{\partial z}{\partial x} \right|_p \end{aligned} \tag{21}$$

where  $z$  is the geopotential height (the height of a pressure level) and the derivatives are taken on surfaces of constant pressure.

The geostrophic flow is horizontally non-divergent in the pressure coordinate system. We can then define the geostrophic streamfunction as  $\Psi_g = \frac{gz}{f}$  such that  $\bar{u}_g = \hat{z} \times \nabla_p \Psi_g$ . (The operator  $\nabla_p$  takes the horizontal gradients on surfaces of constant pressure.) The geostrophic flow is now parallel to contours of constant geopotential height.

### 3.4.4 Flow around highs and lows

When contours of  $z$  or  $p$  are curved, the geostrophic flow turns. If contours form closed loops then the flow is around a center of either high or low pressure (corresponding to high or low geopotential height  $z$ ).

Cyclonic flows are in the sense of rotation of the Earth, i.e. anti-clockwise in the northern hemisphere and clockwise in the southern hemisphere (as we see it by looking from above the North pole and South pole, respectively). Anti-cyclonic flows are in the opposite direction.

The geostrophic flow is cyclonic around low-pressure centers. The pressure gradient force is toward the center, and for geostrophic balance to be satisfied the Coriolis force must be equal and opposite; since the Coriolis force is to the right (left) of the velocity in the northern (southern) hemisphere, the flow must be anti-clockwise (clockwise). The geostrophic flow is anti-cyclonic around high-pressure centers (by a similar argument).

The magnitude of the flow is proportional to the pressure gradient, i.e. it is stronger where contours (of  $p$  or  $z$ ) are closer together.

## 3.5 Thermal wind

If the geostrophic balance and the hydrostatic balance are satisfied, then the thermal wind relationship describes the vertical shear associated with horizontal density gradients.

For the ATMOSPHERE, we have (in pressure coordinates)

$$\begin{aligned} \frac{\partial u_g}{\partial \ln p} &= \frac{R}{f} \left. \frac{\partial T}{\partial y} \right|_p \\ \frac{\partial v_g}{\partial \ln p} &= -\frac{R}{f} \left. \frac{\partial T}{\partial x} \right|_p \end{aligned} \tag{22}$$

where  $R$  is the gas constant for air (see Section 1).

Horizontal temperature gradients imply that the thickness of a pressure layer (the difference in height between two pressure surfaces) also vary horizontally; thus there is a geostrophic flow. If we consider the atmosphere as a stack of pressure layers, the cumulative effect of temperature on the layers' thickness imply that the magnitude of the flow increases with altitude. (We assume that, because of the effect of friction, the geostrophic flow vanishes at the surface).

For the OCEAN, we have (in height coordinates)

$$\begin{aligned}\frac{\partial u_g}{\partial z} &= \frac{1}{\rho_{\text{ref}} f} \frac{\partial \sigma}{\partial y} \\ \frac{\partial v_g}{\partial z} &= -\frac{1}{\rho_{\text{ref}} f} \frac{\partial \sigma}{\partial x}\end{aligned}\tag{23}$$

where the density anomaly  $\sigma$  is obtained from the equation of state.

### 3.5.1 Taylor-Proudman theorem

In the special case where density is a function of pressure only, such that constant density surfaces are parallel to constant pressure surfaces (barotropic fluid), horizontal density gradients vanish and thus there is no vertical shear. The Taylor-Proudman theorem states that

$$\frac{\partial \vec{u}}{\partial z} = 0\tag{24}$$

for a barotropic fluid (on the  $f$ -plane), i.e. the flow is independent of height.

The fluid behaves as columns (called Taylor columns) that cannot be tilted (since  $\partial u/\partial x = \partial v/\partial y = 0$ ) or stretched (since  $\partial w/\partial z = 0$ ).

## 3.6 Subgeostrophic flow

Although geostrophy is a good approximation in the atmosphere and ocean, real flows are not always geostrophic. In particular, near the surface friction cannot be ignored. We can write the subgeostrophic flow as

$$\vec{u} = \vec{u}_g + \vec{u}_a\tag{25}$$

where  $\vec{u}_a$  is the ageostrophic component.

Friction acting as a “drag” is always opposite to the flow. If acceleration terms can be neglected, there is a balance between the pressure gradient force, the Coriolis force, and friction. Since the Coriolis force is always perpendicular to the flow and friction is always parallel (but opposite) to the flow, the motion cannot be along lines of constant pressure anymore. Instead, the subgeostrophic flow is partly down the pressure gradient.

As a result of friction, flows spiral in to low-pressure centers, causing convergence, and out of high-pressure centers, causing divergence. By continuity, this is associated with vertical motions. In the atmosphere, low pressures are associated with upward flow, which can result in condensation and possible rain. High pressures are associated with downward flow, and mostly clear skies.

## 4 Atmosphere and ocean structure

### 4.1 Vertical structure

#### 4.1.1 Atmospheric layers

The atmosphere can be separated into 4 layers. Separations between layers occur (roughly) where the sign the  $dT/dz$  changes. From the surface to the top, we have (i) the troposphere, (ii) the stratosphere, (iii) the mesosphere, and (iv) the thermosphere.

In the troposphere, temperature decreases with height. However this cooling is due to the change in pressure, and the potential temperature actually increases with height (otherwise the atmosphere would not be stably stratified). Thus if a parcel of (cold) air from the tropopause (the top of the troposphere) was brought adiabatically to the surface, it would be warmer than the surface air.

The amount of water vapor that air can hold is a function of temperature. Because cold air can hold less than warm air, the concentration of water vapor decreases rapidly with height.

#### 4.1.2 Ocean layers

In the ocean we have, from the surface to the bottom: (i) the mixed layer, where the water is well mixed (ii) the thermocline, where the water is strongly stratified (the temperature gradient is strong), (iii) the deep ocean, where the flow is very weak, and (iv) the abyss, in places where the ocean floor is particularly deep.

Since the ocean is heated from above, water temperature tends to decrease with depth (although in the deep ocean pressure can cause a weak reversal of the temperature gradient). Salinity profiles depend on the atmospheric forcing (evaporation – precipitation) and on the deep currents. Potential density (density calculated from potential temperature) increases with depth if the water column is stable.

#### 4.1.3 Pressure

The vertical profile of pressure is obtained by integrating the hydrostatic balance (from the surface, where  $p = p_s$ , to an arbitrary height  $z$ ).

In the ATMOSPHERE, the hydrostatic balance can be expressed (by making use of the equation of state) as

$$\frac{1}{p} \frac{\partial p}{\partial z} = -\frac{g}{RT} \quad (26)$$

where  $H = \frac{RT}{g}$  is the scale height.

In an isothermal atmosphere ( $T$  constant), pressure decreases exponentially with height:

$$p(z) = p_s \exp\left(-\frac{z}{H}\right) \quad (27)$$

In the OCEAN, pressure varies as

$$p(z) = p_s + g \langle \rho \rangle (\eta - z) \quad (28)$$

where  $\eta$  is the sea surface height (relative to mean sea level),  $\langle \rho \rangle$  is the mean density between  $z$  and  $\eta$ , and the surface pressure can be approximated as a constant.



Neglecting variations in sea surface height ( $\eta \approx 0$ ) and assuming constant density ( $\rho \approx \rho_{\text{ref}}$ ), we find that pressure increases linearly with depth.

#### 4.1.4 Convection

When fluid moves adiabatically upward, its temperature decreases because of the decline in pressure. Convection occurs in a fluid when the stratification is unstable. We can evaluate the stability of a column of air or seawater by considering adiabatic displacement of parcels of fluid within that column. Either the parcels gain buoyancy (become more dense than the environment) when displaced upwards, and return to their initial height (after oscillations): in this case the stratification is stable. Or the parcels lose buoyancy and keep moving upward; in that case the stratification is unstable.

In the ATMOSPHERE, we must compare the lapse rate (temperature change with height) of the environment and the temperature change of a parcel moving adiabatically. Parcels of air that is not saturated with water vapor follow the dry adiabatic lapse rate (equation 4). The condition for instability (dry convection) is

$$\left(\frac{\partial T}{\partial z}\right)_{\text{environment}} < -\Gamma_d \quad (29)$$

If the air is saturated with water vapor, rising motions cause condensation and the release of latent heat; as a result the cooling occurs slower. Parcels follows the saturated adiabatic lapse rate (equation 6) and the condition for instability (moist convection) is

$$\left(\frac{\partial T}{\partial z}\right)_{\text{environment}} < -\Gamma_s \quad (30)$$

Atmospheric convection can be induced by warming the surface air. A warmed parcel of air at the surface becomes more buoyant than the environment; it rises following the dry adiabatic lapse rate until (a) it cools enough that it becomes saturated with water vapor, and from that level on follows the saturated adiabatic lapse rate, or (b) it becomes neutrally buoyant, i.e. its temperature becomes the same as the environment (the parcel's pressure is always the same as the environment).

In the OCEAN, the condition is

$$\frac{\partial \sigma_\theta}{\partial z} < 0 \quad (31)$$

where  $\sigma_\theta$  is the potential density anomaly (calculated with  $\theta$  instead of  $T$ ).

If the fluid column is unstable, convection occurs. Convection mixes the fluid and homogenizes  $\theta$  (or for moist air, the equivalent potential temperature,  $\theta_E$ ).

## 4.2 Meridional structure

### 4.2.1 Temperature

The atmosphere and the ocean are warmest near the equator and coldest near the poles. This temperature gradient arises because the low latitudes receive more solar radiation than high latitudes (a consequence of the spherical shape of the planet) and the snow and ice-covered poles reflect a large fraction of the incoming radiation (they have a large albedo).

However, the observed equator-to-pole temperature gradient is less than what is expected from a local energy balance where the incoming radiation is balanced (separately at each latitude) by the outgoing radiation (which is proportional to the temperature). This is due to meridional transport

of heat by the atmosphere and the ocean, cooling the tropics and heating the high latitudes.

Because of the temperature gradient, surfaces of constant pressure are higher at low latitudes (where columns of warm air “expand”) than at high latitudes (where columns of cold air “shrink”). This implies that the thickness of pressure “layers” decreases from equator to pole.

### 4.2.2 Winds

[Note: easterly wind = from the East; westerly wind = from the West.]

From the thermal wind relationship, we expect a vertical shear in zonal winds on the planetary scale. The westerly winds in the subtropical jets indeed get stronger with height; these jets are located at latitudes where meridional temperature gradients are very strong.

The zonally-averaged circulation shows three circulation “cells” in each hemisphere. Air rises near the equator and flows poleward, then sinks at about  $30^\circ$  latitude and returns toward the equator (the Hadley circulation). The air moving equatorward near the surface is deflected by the Coriolis force; this causes easterly winds (the Trade winds). The air moving poleward aloft gains speed if its angular momentum is conserved (hence the above-mentioned westerly winds).

Circulations in the Ferrel cell (roughly  $30^\circ$  to  $60^\circ$  latitude) and polar cell (poleward of  $60^\circ$ ) are weaker.

### 4.2.3 Precipitation

Deep atmospheric convection occurs near the equator, removing the vertical gradients in  $\theta$  and especially  $\theta_E$ . It is associated with abundant precipitation.

Salinity is controlled at the sea surface by precipitation (addition of freshwater) and evaporation (which leaves the salt behind). Near the equator, high precipitation (due to rising moist air) cause relatively fresh surface waters. In the subtropics, there is low precipitation (due to subsiding air) and warm temperature causing evaporation and salty surface waters. In subpolar regions, precipitation is high and cold temperature cause little evaporation, thus surface waters are relatively fresh.

## 5 Ocean circulation

In the ocean, pressure varies horizontally due to the combined effects of tilting of the sea surface and horizontal density gradients.

The geostrophic flow is, from equations 19 and 28,

$$\vec{u}_g = \frac{g}{f} \hat{z} \times \nabla \eta + \frac{g}{\rho_{\text{ref}} f} (\eta - z) \hat{z} \times \nabla \langle \rho \rangle \quad (32)$$

Near the surface, the height of the water column ( $\eta - z$ ) is small and thus the surface flow is determined by the slope of the sea surface:

$$\begin{aligned} u_{g \text{ surface}} &= -\frac{g}{f} \frac{\partial \eta}{\partial y} \\ v_{g \text{ surface}} &= \frac{g}{f} \frac{\partial \eta}{\partial x} \end{aligned} \quad (33)$$

Note that if there are no horizontal density gradients (barotropic ocean), then the deep flow is equal to the surface flow (i.e. determined by sea surface height). This is a consequence of the Taylor-Proudman theorem.

If however the surfaces of constant density are not parallel to surfaces of constant pressure (baroclinic ocean), then a vertical shear exists, as determined by the thermal wind relationship. We can make use of this relationship to calculate geostrophic currents. Integrating equation 23 from an arbitrary depth  $z$  to a reference level  $z_1$  at which the flow is known, we have

$$\begin{aligned} u_g(z) &= u_g(z_1) + \frac{1}{\rho_{\text{ref}} f} \frac{\partial}{\partial y} \int_z^{z_1} \sigma dz' \\ v_g(z) &= v_g(z_1) - \frac{1}{\rho_{\text{ref}} f} \frac{\partial}{\partial x} \int_z^{z_1} \sigma dz' \end{aligned} \tag{34}$$

A convenient choice for the reference level is  $z_1 = \eta$  if the sea surface height, and hence geostrophic surface flow, is known. Alternatively, one can assume a “level of no motion”, i.e. assume that  $\vec{u}_g = 0$  for an arbitrarily large value of  $z_1$  (for example,  $z_1 = 2000$  m).

## Important concepts

- 1** The importance of rotation: Rossby number, Coriolis effect
- 2** Geostrophy (calculate the geostrophic flow from gradients of  $p$ ,  $z$ ; sketch the direction of the geostrophic flow; understand how friction affects the geostrophic flow)
- 3** Thermal wind (calculate the wind shear from gradients of  $T$ ,  $\sigma$ )
- 4** Hydrostatic balance (you should know this equation by heart; calculate vertical profiles of  $p$  in the atmosphere and ocean, understand the concept of height of surface pressures)
- 5** Stability of vertical profiles (understand the concepts of potential temperature, adiabatic lapse rate, convective instability)
- 6** Equation of state (you should know the equation for the atmosphere); moisture, condensation
- 7** Meridional structure of  $T$ , humidity, winds, surface salinity
- 8** Homework problems (except what we did not emphasize in class)